Greek Reflections on the Nature of Music

Flora R. Levin

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GREEK REFLECTIONS ON THE NATURE OF MUSIC

In this book, Flora R. Levin explores how and why music was so important to the ancient Greeks. She examines the distinctions that they drew between the theory of music as an art ruled by number and the theory wherein number is held to be ruled by the art of music. These perspectives generated more expansive theories, particularly the idea that the cosmos is a mirror-image of music's structural elements and, conversely, that music by virtue of its cosmic elements – time, motion, and the continuum – is itself a mirror-image of the cosmos. These opposing perspectives gave rise to two opposing schools of thought, the Pythagorean and the Aristoxenian. Levin argues that the clash between these two schools could never be reconciled because the inherent conflict arises from two different worlds of mathematics. Her book shows how the Greeks' appreciation of the profundity of music's interconnections with philosophy, mathematics, and logic led to groundbreaking intellectual achievements that no civilization has ever matched.

Flora R. Levin is an independent scholar of the classical world. She is the author of two monographs on Nicomachus of Gerasa and has contributed to *TAPA*, *Hermes*, and *The New Grove Dictionary of Music*.

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To Sam

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Preface

This book owes its inception to the teachings of Dr. Seymour Bernstein: distinguished pianist, composer, author, lecturer, and master teacher. Dr. Bernstein is known and appreciated for the masterclasses in the art of the piano, which he conducts throughout the United States, Canada, Europe, and Asia. I count myself fortunate to have been granted admission to a number of these classes in New York City.

I was impressed early on in these classes by the way in which Dr. Bernstein approached the practical knowledge that must be acquired and implemented in the performance of music on the piano. Even more impressive to me was Dr. Bernstein's ability to demonstrate the transformation that must be worked on musical sound by the art of musicians. For his treatment of this transformation was, as I understood it, philosophy in action. It was to resurrect the dream of Socrates that urged the practice and composition of music as an imperative of philosophy. And since Socrates regarded philosophy as "the greatest music," he felt that by spending his life working on all aspects of music, he was also practicing philosophy in the highest degrees (Plato *Phaedo* 61A3–8).

The philosophical component of Dr. Bernstein's teachings made me think of music even as the ancient Greeks did: as something that tends to unity, like the course of human reason, while reaching for diversity, like the manifold forces of nature. The unity of reason organizes and sets limits to things musical, while the forces of human nature create things musical and set them free. These are the two principles that Dr. Bernstein emphasized in his teachings. According to him, they interpenetrate all musical thought, all musical creation, and all musical performances. Given these principles, I was prompted to think of music

as something manifold but unified, as something whose foundations are in the human soul, not in matter; they are, rather, as something next to which all particulars and partialities are dwarfed by the moving forces of melody. This is to think of music in the way of nature. This is to think of music in the way of Aristoxenus of Tarentum, a student of Aristotle, and the greatest musician of antiquity.

Aristotle's famous dictum has it that musical sound is a living sound that originates in the human voice, and that all instruments, being inanimate objects, are built to imitate the sound of the singing voice (*De anima* 420b5–6). This finds strong confirmation in the teaching of Dr. Bernstein. But, as he demonstrated, the piano, owing to its physical construction, presents a paradox of philosophical dimensions: How can the discrete pitches produced by the piano be made to imitate the living continuity of the singing voice? The sustaining pedal goes far in overcoming this discontinuity of pitch. But something more basic is needed if true artistry is to be achieved. To this end, Dr. Bernstein guided us to concepts of musical function and musical space, of melodic tension and resolution, of melodic motion and stasis – concepts that revolve around the primary axis of Aristoxenian thought. Dr. Bernstein managed to lift such concepts as these out of the textbooks and off the musical scores by demonstrating them in living sound on the piano. He did this, much as Aristoxenus must have done some twenty-five hundred years ago, by using music as a symbol of itself. And, in the process, he revealed, as complete musicians always succeed in doing, the composite nature of music in all its flowing forms and multiforms.

In Dr. Bernstein's classes, the truth of Aristoxenus' teachings was first revealed to me, namely, that the ultimate factor in making music is the intellectual process; it is this intellectual process that presides over the activity of the hands on the keyboard and is their determining principle. When, therefore, I would hear Dr. Bernstein speak of the logic of a resolution, or the function of a particular note, or the tension between two notes in a melodic phrase, I knew that he was releasing Aristoxenus' own concepts from out of the past and disposing them anew. My gratitude to Dr. Bernstein is best expressed by the content of this book.

Many years have passed since I first began to think about the woman scholar, Ptolemaïs of Cyrene, who appears in various contexts

throughout this work. I wondered first of all who she was and when she might have lived. Most important, she impressed me, even though her words as quoted by Porphyry are all too few, as being exceptionally astute where Aristoxenian theory is concerned. And since Aristoxenus had few enough partisans in antiquity to champion his views on music with any depth of understanding, whatever she had to say in his behalf invited my closest study. I was encouraged in this inquiry by the late, great, and good scholar, Professor Gilbert Highet, Anthon Professor of Latin (Columbia University), who observed in what was to be his last letter to me, "Her name alone intrigues for its history." Coming as it did from one whose instinctive recognition of a workable hypothesis I had long since learned to trust, this observation sparked my imagination and led me to speculate on the kind of woman Ptolemaïs might have been. I hope that the results of my inquiry are compatible with all that Professor Highet had intuited from her name. I was also encouraged in this pursuit by the late Professor of Latin and Ancient History, William C. McDermott (University of Pennsylvania). I regret that my expression of gratitude to him for guiding me through the intricacies of Hellenistic history must come too late for him to receive it.

I wish to express my deep appreciation of the late Professor Emeritus of English, Comparative Literature, and Classical Studies, Albert Cook (Brown University). His many contributions to the world of scholarship in such diverse fields as Biblical Studies, History, Poetics, and Philosophy have inspired and sustained me over the course of many years. Professor Cook's writings on Plato are especially compelling to me, not least for being full of dialectical arguments; but above all, for their acute appraisal of the poetic and musical aspects of Plato's style. For Professor Cook, Plato was the Beethoven of Philosophy. He demonstrated this most vividly in his analysis of Plato's use of the Greek particles – "the riot of particles," as he so aptly called them (in *The Stance of Plato)* – which make for the powerfully polyphonic texture of the Platonic dialogues. Professor Cook's scholarly originality and versatility, coupled with his extraordinary breadth of knowledge, have earned my everlasting respect, admiration, and, most of all, my gratitude for his help.

I am particularly indebted to my musically eloquent friend, Norma Hurlburt, who placed at my disposal her comprehensive knowledge of

the piano literature, especially that of Beethoven and Schubert. I owe her thanks for spending many an hour with me speaking of music – the art – and music – the epitome of logic. To this, she added many more hours playing for me things that are more definite to musicians than the meaning of words. Her ideas, both practical and theoretical, helped to set this work in motion.

My sincere thanks are extended to Dr. Baylis Thomas, whose stimulating observations, drawn from his well-appointed knowledge of song, convinced me that music, by its nature, has an inbuilt resistance to theory. It is this that protects music from being demystified.

My obligations to others for their generous help are many: to Dr. Alison Thomas for her contributions to this project through her computer skills, which she so generously placed at my disposal. Her expertise in this critical area is matched only by her pianistic gifts; to the Near-Eastern Archaeologist and Historian, Dr. Oscar White Muscarella, who supplied me with articles and special studies on the history of, and excavations at, Cyrene; to Professor Emeritus of English and Comparative Literature, William Sylvester (State University of New York at Buffalo), with whom I enjoyed many lively discussions on the acerbic views of the philosopher-poet, Philodemus of Gadara, for whom the art of music was on a par with the art of cooking; to Professor Emeritus of Indian History, Stanley Wolpert (University of California, Los Angeles), who, with his wife, Dorothy, read various sections of this work and offered valuable insights; to Professor of Classics, Jacob Stern (Graduate Center, CUNY), for his help in checking the Greek text. From the methods and experience of these erudite friends and scholars, I have learned much.

I must also thank for their many kindnesses Sheran Maitland and Diane Allen. My deep gratitude goes also to Beatrice Rehl, Publication Director of Humanities at Cambridge University Press, and to Laura Lawrie, Production Editor for Cambridge University Press.

One final debt, the greatest of all, is acknowledged in the dedication.

Introduction

The peoples of ancient Greece surrounded themselves with music; they immersed themselves in music; they were in fact imbued with music. Scarcely any social or human function, whether public or private, urban or rural, took place without its musical accompaniment. Marriages, banquets, harvestings, funerals – all had their distinctive cadences. Boatmen rowed to the song of the aulos (the double-reed oboe-like wind instrument), gymnasts exercised to music's pulse, the spirits of soldiers were sustained by its rhythmic lilt as they marched off to battle. Instrumental music accompanied libations, sacrifices, supplications, religious processions, and ceremonial rites of all sort. Musical contests drew throngs of knowing listeners. Singer-composers, who set great numbers of poetic texts to song, which they then performed from memory to the accompaniment of wind and stringed instruments, were esteemed as repositories of knowledge. Solo instrumentalists could stand as high in the public's estimation as any athlete returning victorious from the Pan-hellenic games. In Attic tragedy, the recurring motifs of the choral song not only unified the action on stage, but served also the same virtuoso function as the divisions in a modern aria da capo. In Attic comedy, the joy of life was celebrated in the ecstatic outpourings of licentious song, the chorus encircled by dancers whirling in the drunken revelry of the lascivious *kordax* (a deliberately vulgar and at times indecent dance). In sum, music was for the Greeks more, indeed, much more than a pleasant preoccupation or source of amusement. It was a significant part of life itself. That this was so is because the ancient Greek language was itself a form of melodious expression.

The melodious patterns of the ancient tongue were the products of the pitch-accents that were integral to the meanings of the words. These accents and melodious patterns were learned by the Greeks from infancy on, undoubtedly leading to their heightened perception and retention of pitch-differences in song and speech. As we learn from the fourth-century B.C. musician and theorist Aristoxenus of Tarentum, there was a kind of songful melody in everyday speech (λογῶδές τι μέλος).[1] To distort this pitch-accent was tantamount to committing an egregious grammatical error. A common example of this kinship between pitch-accent and meaning is one that students meet early on in their study of the ancient tongue, involving the difference in meaning between the two otherwise identical words, βίος, βιός (respectively, "life" and "bow"). As W. B. Stanford has pointed out in his ground-breaking study, *The Sound of Greek*,[2] "There were thousands of such words in ancient Greek if we count the verbal inflexions which had different accentuations as well as the nouns, pronouns, verbs, and adverbs."

All classical Greek authors were thus composing for the ear as well as for the mind; the meanings of their words depended in the fullest sense on the semantic nature of their accompanying pitch-accents. This was true no matter what the content or subject matter of their writings, be it poetry, history, or even science and mathematics.[3] Most important, the Greek ear was trained to recognize the most subtle intonations in song and speech. Their ability in this respect was apparently as remarkable as that of people today who are possessed of absolute pitch.[4] Sound was in fact everything in antiquity and, not surprisingly, reciting aloud – more often than not from memory – was the norm rather than the exception. When it came to sound, therefore, the resources of the Greeks were incalculable and superb. This bespeaks an acutely sensitive and highly developed auditory sense on the part of performers as well as auditors. Evidence that this was in fact so is unambiguous and voluminous.

[1] *Harm. El.* 1. 18 (Da Rios 23. 14).

[2] W. B. Stanford, *The Sound of Greek*, pp. 30–31.

[3] See Stanford (note 2), pp. 8–9.

[4] Absolute pitch is the miraculous ability to identify any pitch out of a melodic context, to name it, and even to reproduce it without mechanical aid of any sort. The most famous example of this truly mysterious faculty is, of course, W. A. Mozart.

This evidence, in addition to being massive and diverse, suggests the intriguing possibility that the Greeks may indeed have had absolute pitch. For research in this area has shown quite convincingly that the acquisition of a tonal language may be one of the unusual conditions leading to the retention of and heightened sensitivity to pitch distinctions.[5] To be sure, nothing can be proven on this point, as the ancient tonal systems were different from our own standards of pitch. But, given the possibility, this would account for the Greeks' ability to discriminate between the most subtle colorations of pitch imaginable: differences such as quarter-tones, thirds of tones, even the lowering of a note by three-quarters of a tone (*eklysis*), or the raising of a note by five quarter-tones (*ekbolē*). As their writings on music show, every pitch range of the keys of transposition (*tonoi*), every mode (*tropos*), every genus (*genos*) possessed its own meaningful character (*ethos*). Some sequences of notes were even defined by their "colors" or nuances (*chroai*). Individual notes as the *lichanos* (finger-note) were recognized for their distinctive quality, their "*lichanos*-ness" (*lichanoid*), while other notes were felt to have masculine or feminine characteristics.[6] In short, this type of acute sensitivity to sound bespeaks a whole other realm of perception.

So deep a penetration of music into almost every aspect of life presupposes a musically gifted public and a long tradition of musical education. The evidence appears in fact to depict a society concerned with music more than anything else. The truth is, of course, that music was only one of the myriad products of the Greek genius. What they achieved in all else – poetry, drama, history, architecture, sculpture – scaling heights that later civilizations have never surpassed – is familiar to everyone. What is more, almost everything, music included, seems to have begun with them.[7] Mathematics and science were their inventions,

[5] See Oliver Sacks, *Musicophilia: Tales of Music and the Brain*, pp. 113–14.

[6] This is discussed by Aristides Quintilianus *De mus*, III, 21 (Winnington-Ingram 122.22 123. 4), in which Aristides assigns male or female notes to the planets according to their associative qualities.

[7] Thus, Bertrand Russell, *A History of Western Philosophy*, p. 3: "What they [the Greeks] achieved in art and literature is familiar to everybody, but what they did in the purely intellectual realm is even more exceptional. They invented

and philosophy, that most eloquent witness to the mind of man, was their creation. When it came to music, the Greeks showed the same organic point of view, the same instinct for formulating laws governing reality that appears in every phase of their culture and art. As we learn from the evidence presented to us, the Greeks were the first to intuit music's essence, and the first to discover the universal laws governing its structure. They were the first to perceive the elements of music not as isolated entities detached from one another but as integral parts of an organic whole from which each part derived its meaning and position.

This book is an inquiry into the diverse ways in which the ancient Greeks contemplated and dealt with the nature of music. My purpose is to exhibit music as an integral part of their philosophical, mathematical, and cosmological pursuits. As their writings show, music was not an isolated art whose sole purpose was to amuse and accompany secular and religious activities. On the contrary, music was considered by them to be as necessary as language and as rational as thought itself. As such, it was regarded as powerfully paideutic, and productive of knowledge for its own sake. Moreover, it was seen to be a genuine molder of human character. What they achieved in music and musicology, although comparable to their accomplishments in literature, art and science, philosophy, history, mathematics, and cosmology, has gained them far less attention.

Acoustical theory is universally accepted to have begun with Pythagoras of Samos (6th century B.C.). Deductive reasoning from general principles as applied to music was, as I argue, an innovation of Aristoxenus of Tarentum (4th century B.C.), the leading figure in this study. This method, together with Aristoxenus' original and creative use of mathematics, founded a centuries-long tradition, the main tenets of which persist to this day.

Pythagorean harmonics was geometrical, not dynamic, whereas Aristoxenus' theory was not geometrical, but dynamic, by being rooted in the continuity of infinite number. It was this dynamic that made Aristoxenus' theory a true Science (*Epistēmē*) of Melody. By contrasting

mathematics and science and philosophy; they first wrote history as opposed to mere annals. . . . What occurred was so astonishing that, until very recent times, men were content to gape and talk about the Greek Genius."

Aristoxenus' unified theory with that of other specialists in the field, it is possible to account for its peculiar meaning in regard to the nature of music itself. To this end, translations from Ptolemy's *Harmonica*, from Porphyry's *Commentary on Ptolemy's Harmonics*, and from the fragments of *The Pythagorean Doctrine of the Elements of Music* by the little-known Ptolemaïs of Cyrene have been cast into the form of a dialogue. This results in an interesting discussion among three experts on the virtues and limitations of the various theories under examination. Of the three, it is Ptolemaïs who seems to me to have grasped the uniqueness of Aristoxenus' Aristotelian type of theoretical logistic. To her credit, Ptolemaïs demonstrated that the geometrical method of the Pythagoreans appealed solely to the eyes but that Aristoxenus' system was designed solely for the ears.

As I argue, Aristoxenus' method is in essence a profoundly dialectic one from which he obtained a fixed constant of measurement.[8] This enabled him to deal with problems of attunement that could not be solved by traditional methods of arithmetic and elementary geometry. Through this technique, Aristoxenus arrived at the concept of continuity by observing the surrounding dense (*pykna*) melodic media. The deep-lying power of Aristoxenus' method is that it enriches the study of interrelations among discrete integers. In so doing, he summoned to the aid of theorists new relations among continuous magnitudes. In short, Aristoxenus, I believe, was practicing analytic number theory centuries before its foundations were laid by such luminaries as Peter Gustav Lejeune Dirichler, Bernhard Riemann, Georg Cantor, Leopold Kronecker, and Karl Weierstrass.

[8] Cf. F. R. Levin, "*Apeiria* in Aristoxenian Theory," *Hermes* 135 (2007), 406–28.

Abbreviations

AJAH	*American Journal of Ancient History*
AJPh	*American Journal of Philology*
Barker, I	Barker, A., *Greek Musical Writing: I. The Musician and His Art*. Cambridge University Press 1984.
Barker, II	Barker, A., *Greek Musical Writings: II Harmonic and Acoustic Theory.* Cambridge University Press 1989.
Barker, *Ptolemy*	Barker, A., *Scientific Method in Ptolemy: "Harmonics."* Cambridge University Press 2000.
Bélis, *Aristoxène*	Bélis, A., *Aristoxène et Aristote: Le Traite d'harmonique*. Paris 1986.
BSA	*Annual of the British School at Athens*
CPh	*Classical Philology*
CQ	*Classical Quarterly*
JHS	*Journal of Hellenic Studies*
Laloy, *Aristoxène*	Laloy, L., *Aristoxène de Tarente. Disciple d'Aristote et la Musique de l'Antiquité*. Paris 1904.
Macran	Macran, H. S., *The Harmonics of Aristoxenus.* Oxford: Clarendon 1902.
Mathiesen, *Apollo's Lyre*	Mathiesen, Th. J., *Apollo's Lyre. Greek Music and Music Theory in Antiquity and the Middle Ages*. University of Nebraska Press 1999.

Michaelides	Michaelides, S., *The Music of Ancient Greece: An Encyclopedia*. London 1978.
PCPS	*Proceedings of the Cambridge Philological Society*
REG	*Revue des Études grecques*
Solomon, Ptolemy	Solomon, J., *Ptolemy: Harmonics Translation and Commentary*. Leiden 2000.
White, *The Continuous and the Discrete*	White, M. J., *The Continuous and the Discrete. Ancient Physical Theories from a Contemporary Perspective*. Oxford: Clarendon 1992.

Texts

Caspar	Caspar, M., *Ioannis Keppleri Harmonices Mundi Libri V.* Munich 1940.
Cousin	Cousin, V., *Opera Petri Abaelardi*. 2 vols. Paris 1849–59.
Da Rios	Da Rios, R., *Aristoxeni Elementa Harmonica*. Rome 1954.
Deubner	Deubner, L., *Iamblichi De Vita Pythagorica Liber*. Leipzig 1937.
Düring	Düring, I., *Die Harmonielehre des Klaudios Ptolemaios*. Göteborg 1930 (Högskolas Årsskrift, 36/1); rep. 1982.
Düring	Düring, I., *Porphyrios Kommentar zur Harmonielehre des Ptolemaios*. Göteborg 1932 (Göteborg 1932, Göteborgs Högskolas Årsskrift); rep. 1978.
Düring	Düring, I., *Ptolemaios und Porphyrios Über die Musik*. Göteborg 1934 (Göteborgs Högskolas Årsskrift, 40/1).
Friedlein	Friedlein, G., *Anicii Manlii Torquatii Severini Boetii De Institutione Musica Libri Quinque*. Leipzig 1867; rep. 1966
Heath	Heath, Sir Th. L. *The Thirteen Books of Euclid's Elements*. 3 vols. Cambridge 1925; rep. 1956.
Heiberg	Heiberg, J. L., *Claudii Ptolemii Syntaxis Mathematica*. 2 vols. Leipzig 1898–1903.

Hiller — Hiller, E., *Theonis Smyrnaei Philosophi Platonici Expositio Rerum Mathematicarum ad Legendum Platonem Utiliam.* Leipzig 1878.

Hoche — Hoche, R., *Nicomachi Geraseni Pythagorei Introdoctionis Arithmeticae Libri II*. Leipzig 1866.

Jan — Jan, K. von, *Musici Scriptores Graeci*. Leipzig 1895; rep. 1962.

Jonker — Jonker, G. H., *The Harmonics of Manuel Bryennius*. Groningen 1970.

Kemke — Kemke, I., *Philodemus De Musica Librorum Quae Exstant*. Leipzig 1884.

Meibom — Meibom, M., *Antiquae Musicae Auctores Septem*. 2 vols. Amsterdam 1652; rep. 1977.

Najock — Najock, D., *Drei anonyme griechische Traktate Über die Musik. Eine kommentierte Neuausgabe des Bellermannschen Anonymus*. Göttinger Musikwissenschaftliche Arbeiten, vol. 2. Göttingen 1972.

Pearson — Pearson, L., *Aristoxenus Elementa. The Fragment of Book II and the Additional Evidence of Aristoxenean Rhythmic Theory*. Oxford: Clarendon 1990.

Pistelli — Pistelli, H., *Iamblichus In Nicomachi Arithmeticam Introductionem Liber*. Stuttgart 1894; rep. addendis et corrigendis U. Klein 1975.

Ross — Ross, Sir D., *Aristotle's Physics*. Oxford: Clarendon 1955.

Vincent — Vincent, A. J. H., "Notice sur divers manuscrits grecs relatifs à la musique, comprenant une traduction francaise et des commentaires," *Notices ex extraits des manuscrits de la bibliotheque du Roi et autres bibliotheques*, vol. 16/2. Paris 1847.

Vors. — Diels, H., and Kranz W., *Die Fragmenta der Vorsokratiker*. Dublin/Zürich 1966.

Wachsmuth — Wachsmuth, T. C., and Hense, O., *Stobaei Eclogae,* vols. I–III. Leipzig 1884.

Wehrli	Wehrli, F., "Aristoxenus," *Die Schule des Aristoles*. Basle 1945.
Willis	Willis, J., *Martianus Capella*. Leipzig 1983.
Winnington-Ingram	Winnington-Ingram, R. P., *Aristidis Quintiliani De Musica Libri Tres*. Leipzig 1963.
Ziegler-Pohlenz	Ziegler, K., and Pohlenz, M., *Plutarchi Moralis*, vol. VI, Fasc. 3. Leipzig 1953.

1 All Deep Things Are Song*

This world is not conclusion;
A sequel stands beyond,
Invisible, as music,
But positive, as sound. Emily Dickinson

IT IS NOW ABOVE SEVENTEEN HUNDRED YEARS SINCE BACCHIUS the Elder (as he was then called),[1] a Greek writer on music, opened his *Introduction to the Art of Music* with the seemingly artless question: "What is music?"[2] The answer given by Bacchius to that question

[1] Bacchius *Geron*, like other ancient musicologists – Cleonides, Gaudentius, Alypius, Aristides Quintilianus, *et alii* – is today an unknown and obscure figure, leaving no posterity. Yet, he mattered once, so much so, it seems, as to have had his work on music studied by no less a figure than the Emperor Constantine the Great (285–337 A.D.). A curious epigram attached by one Dionysius (himself unknown), to several manuscripts containing Bacchius' treatise on music makes for this interesting possibility. It states: "Bacchius the Elder compiled the keys, modes, melodies and consonances of the art of music. Writing in agreement with him, Dionysius explains that the almighty emperor Constantine was a learned devotee of the arts. For, being a discoverer and dispenser of all the learned disciplines, it is befitting that he was in no wise a stranger to this one." Everything that can be known of Bacchius is reviewed and assessed by Thomas J. Mathiesen, *Apollo's Lyre*, pp. 583–93.

[2] Bacchius' *Introduction to the Art of Music* (*Eisagōgē Technēs Mousikēs*) was first published in 1623 by Frederic Morellus and, in the same year, by Marin Mersenne.

* Thomas Carlyle

illustrates wonderfully the ancient Greeks' passion for logic: "It is a conceptual knowledge of melody and all that pertains to melody."[3] There is in Bacchius' answer a dynamic reciprocity: melody provides the conditions for music's existence, while music at the same time subsists by virtue of melody. After detailing all the elements that pertain to melody – pitch, interval, consonance, dissonance, scales, modes, modulation, keys, rhythm, and much more – Bacchius arrived at not one but two different definitions of melody. The first is deliberately circular: "It is the fall and rise generated by melodious notes."[4] Such a definition is tantamount to asserting that something is a melody because its constituents are melodious. If Bacchius' definition is in fact tautological, it seems to be so consciously, in order that it be correct on purely musical grounds. It arises from Bacchius' belief that nothing in the world outside of melody can be invoked or enlisted to define anything that

The treatise is written in a question and answer format like that of the Ps.-Aristotle *Problems* (Book 11 *On the Voice* and Book 19 *On Harmonia*) and *The Pythagorean Doctrine of the Elements of Music* by the remarkable Ptolemaïs of Cyrene, of whose work only fragments remain. See Chapters 6 and 7. Bacchius' treatise belongs to the school of Aristoxenus, "The Musician," as he was known in antiquity, who, unlike Bacchius, did leave a considerable posterity. The Greek text from which these translations are drawn is that of K. von Jan, *Musici Scriptores Graeci*, pp. 292–316 (hereafter referred to as Jan). On its English translation, see Chapter 2, note 2. Another has been promised by Jon Solomon, one of the few scholars who has examined the work, in his "*EKBOLE* and *EKLUSIS* in the Musical Treatise of Bacchius," *Symbolae Osloenses* 55 (1980), 122, n. 1. The question that opens the treatise appears at I . 1 (Jan, 292. 3).

[3] I. 1 (Jan, 292. 3–4). There is an unmistakably Platonic cast to Bacchius' definition of music. His term *eidēsis* (which I have rendered as "conceptual knowledge") links "knowing" (*eidōs*) with "seeing" (*eidon*) and, as such, has a strong relationship with Plato's *eidōs-eidē*, the terms canonized by Plato as "Form" and "Forms" and abstracted by him into "Ideas." In Plato, the *eidē* are the cause of true knowledge (*epistēmē*), inasmuch as they are the eternal realities that transcend the merely sensible phenomena (*aisthēta*). The *eidē* are therefore the condition of all true philosophical discourses. See *Phaedo* 65D–E; *Parmenides* 135B–C). Bacchius' definition thus recognizes the mental activity (*eidēsis*) of the knowing mind by which the melodic concept of Form (*eidōs*) is grasped.

[4] I. 19 (Jan, 297. 22–23). The words for "fall" and "rise" are in Aristoxenian terms *anesis* and *epitasis*, which have the literal meanings, "resolution" and "tension," respectively. These words are discussed below, n. 41.

lies within the precincts of melody. Bacchius' approach to the nature of melody adumbrates almost eerily what Ludwig Wittgenstein was to observe centuries later:[5] "Die Melodie ist eine Art Tautologie, sie ist in sich selbst abgeschlossen; sie befriedigt sich selbst. [Melody is a form of tautology, it is complete in itself; it satisfies itself.]"

Like Bacchius, Wittgenstein evidently contemplated music not in any sense as a language, but as an activity of some sort. And Bacchius, like Wittgenstein, seems to have treated music as an activity whose subject matter is not in any sense factual, but whose processes and results are somehow equivalent. Accordingly, where Bacchius found logic in melody, Wittgenstein found melody in logic:[6] "Die musicalischen Themen sind gewissen Sinne Sätze. Die Kenntnis des Wesen der Logik wird deshalb zur Kenntnis des Wesens der Musik führen. [Musical themes are in a certain sense propositions. Knowledge of the nature of logic will for this reason lead to knowledge of the nature of music.]"

In Wittgenstein's view, a melody, like a logical proposition, must make sense strictly according to its own terms in order to attain to the condition of music. The implication is that the condition of music, in order to be true to itself, must inhere in the logical constants of melody.

Like a mathematical proposition that uses numerical signs and symbols to designate an intricate truth, a melody employs certain logical constants, which are preserved by notational signs and symbols, in order to make musical sense. The proposition that $7 + 5 = 12$ is necessarily true for a mathematician, whether the symbols stand for apples, peaches, trees, or anything else; it holds true because it defines an unchanging relation, one that holds independently of the objects involved or of the mind that contemplates them. Thus, to ascertain that the mathematical proposition $7 + 5 = 12$ is correct, we do not have to study the universe; we have merely to check the meaning of the numerical symbols. They assert that $7 + 5$ has the same meaning as 12, and this amounts to a mathematical truth – a truth that, by its very nature, is a tautology. So, too, the pitches C–C^1, for example, define a melodic constant, one that stands for nothing in the world save itself. This holds true whether these pitches be struck on a piano, bowed on a violin string, or blown

[5] *Notebooks*, 1914–16, p. 40, 4.3.15.
[6] *Notebooks*, 1914–16, p. 40, 7.2.15.

on an ancient Greek *aulos*. The melodic constant defined by the pitches C–C¹ is an unchanging relation – that of an octave. C–C¹ means "octave" to a musician and, as such, is a form of tautology.[7]

In the case of mathematics, we perceive the unchanging relation in an arithmetic proposition such as that given earlier, not in virtue of the symbols as phenomenal entities in themselves but by their intervention as symbols standing *for* phenomenal entities; in the case of music, we perceive the unchanging relation not by the intervention of the symbols representing something else but as phenomenal entities in themselves.[8] In other words, the octave C–C¹, whether written in alphabetic notation (as here) or as notes on a staff, *is* the octave C–C¹ at that specific pitch range. Numbers, unlike the musical notes that make up a melody, are needed not only to compose mathematical propositions, but also, if they are to have a specific meaning, to stand for or apply to common objects in the world. It is only when numbers are abstracted from the objects of the world or understood independently of the world that they begin to assume the characteristics of musical notes. Like a musical note, the number 2, say, is not the same as or identical with anything in the world. It *is* 2 and, as such, is not identical with a duo of musicians; rather, the duo of musicians is an instance of the number, 2; and the number, 2, is an instance of itself. The note C, is in this sense an instance of itself.[9] Given such an instance, the subject matter of music, like that of pure

[7] The convention for indicating pitch adopted here is that of Carl E. Seashore, *Psychology of Music*, p. 73. This tautological type of formulation is seen by many as a logical trick that is a favorite device of mathematicians. For example, Susan Langer, *Philosophy in a New Key*, p. 237: "Musical form, they reply, is its own content; it means itself. This evasion was suggested by [Eduard] Hanslick when he said, 'The theme of a musical composition is its essential content.'"

[8] Thus Langer, *op. cit.*, p. 19: "Mathematical constructions are only symbols; they have meanings in terms of relationships, not of substance; something in reality answers to them, but they are not supposed to be items in that reality. To the true mathematician, numbers do not 'inhere in' denumerable things, nor do circular objects 'contain' degrees. Numbers and degrees and all their ilk only *mean* the real properties of real objects."

[9] This is another way of saying that music is non-representational. Because it exhibits its own pure form as its own essence, it is altogether untranslatable into any other medium save itself. It is in this sense wholly tautological, as Wittgenstein observed (see note 5). Cf. Peter Kivy, *Music Alone*, pp. 66–67.

mathematics, belongs to a realm of idealized abstractions in which the composer of music or the mathematician performs specific operations with an extrawordly creative freedom.

It was perhaps his intuition of this curious affiliation between music and mathematics that led Pythagoras of Samos (sixth century B.C.), the most influential mathematician of antiquity, to make music a matter for serious philosophical reflection. As reported by Athenaeus of Naucratis (160–230 A.D.), an authority on the musical lore of antiquity:[10] "Pythagoras, who occupied so very great a position in philosophy, stands out among the many for having taken up music not as an avocation; indeed, he explains the very being of the whole universe as bound together by music." Pythagoras' intuition along these lines was to produce one of the most momentous discoveries of all time: musical sound is ruled by number. This meant nothing less than that the whole universe, as being bound together by music, must itself be ruled by number.[11]

[10] Athenaeus of Naucratis in Egypt wrote a monumental work entitled *Deipnosophistai* (Sophists at Dinner) sometime after the death of the Emperor Commodus (180–92 A.D.). The work consisted originally of fifteen volumes. Much is lost, but what remains of Books IV and XIV in particular is valuable for preserving information on music and musical practices from much earlier sources such as Pindar (c. 522–c. 446 B.C.), Bacchylides (c. 520–c. 450 B.C.), Damon, the teacher of Socrates (5th century B.C.), Hesiod (c. 700 B.C.), Aristotle (384–322 B.C.), Aristoxenus of Tarentum (b.c. 375 B.C.; his date of death is unknown), the leading musician and musical theorist of antiquity. The passage quoted here is from Book XIV, 632b.

[11] The literature on Pythagoras and his discovery of the mathematical ratios underlying the production of musical pitches is vast enough to make up an entire library. Some of the comprehensive studies are: P.-H. Michel, *De Pythagore à Euclide*; P. Kucharski, *Étude sur la doctrine pythagoricienne de la tétrade*; E. Frank, *Plato und die sogennanten Pythagoreer*; A. E. Chaignet, *Pythagore et la philosophie pythagoricienne*; A. Delatte, *Études sur la littérature pythagoricienne*; H. Thesleff, *An Introduction to the Pythagorean Writings of the Hellenistic Period*; H. Thesleff, *The Pythagorean Texts of the Hellenistic Period*; W. Burkert, *Lore and Science in Ancient Pythagoreanism*; W. K. C. Guthrie, *A History of Greek Philosophy*, Vol. 1: *The Earlier Presocratics and the Pythagoreans*; J. A. Philip, *Pythagoras and Early Pythagoreanism*; K. S. Guthrie, *The Pythagorean Sourcebook and Library*; J. Godwin, *The Harmony of the Spheres: A Sourcebook of the Pythagorean Tradition*. Pythagoras' reputation is summed up accordingly by G. E. Owen, *The Universe of the Mind*,

On the basis of the truths arrived at by mathematical means, Pythagoras and his followers could think of music's elements as concrete realities linked by number to nature's own divine proportions. At the same time, they could think of music itself as the expression in sound of those same proportions by which nature asserts her divine symmetry. This being the case, the universal order of things could be said to have its counterpart in the underlying structures of harmonic theory. The notion that music owes its life to mathematics, and that the universe, by the same agency, owes its soul to *harmonia* – the attunement of opposites – took hold of human imagination from its first utterance and has transfixed it for the millennia.[12]

It was during that brilliantly fecund period when Aeschylus was producing his dramas and Pindar his *Odes* that Pythagoras made the discovery about music, a discovery that reverberates to this day. As far as we can ascertain, no Greek-speaking person had ever committed the story of the discovery to writing before Nicomachus of Gerasa, the "Pythagorean," so-called, a once famous mathematician living at the turn of the century, 100 A.D.[13] According to Nicomachus, Pythagoras had long been pondering the problem of how to translate the musical sounds he produced on the strings of his lyre into some sort of concrete form that his eye could see and his mind could contemplate. While deliberating about this problem, he happened to be walking by a smithy

p. 26: "Born about 573 B.C.E. on Samos, according to legend Pythagoras became the ideal of mathematics, a philosopher and a prophet, apotheosized in his own lifetime by his own society."

[12] The moment Pythagoras discovered that the lengths of a vibrating string sounding a fundamental pitch, its fifth, its fourth, and its octave, are in the ratios 2: 3: 4, he heard the *harmonia* of the universe and defined the ordering of its elements (*stoicheia*) in numerical terms. From now on, nature and all its properties were to be found in the science of number. Cf. H. E. Huntley, *The Divine Proportion*, pp. 51–56. It is thus to Pythagoras that we owe the first conception of the universe as a harmony patterned on music. For because sounds were shown to be the embodiment of numbers, it was conceivable that *mundum regunt numeri*, that the world was in fact ruled by number.

[13] Nicomachus tells the story of Pythagoras' discovery in his *Harmonikon Enchiridion* (*Manual of Harmonics*), Ch. 6 (Jan, 245.19–248). *The Manual*, together with ten *Excerpta* from a lost work of Nicomachus, is translated by Levin, *The Manual of Harmonics of Nicomachus the Pythagorean*.

where, by sheer good fortune, he heard the smith's hammers beating out on the anvil a whole medley of pitches. These registered on his ear as the same consonances that he could produce on his lyre-strings – the octave, the fifth, and the fourth – as well as the dissonance separating the fourth from the fifth – the whole-tone.

Intuiting that the size of the sounding body had something to do with the differences between the pitches he heard, Pythagoras ran into the smithy, conducted a series of experiments, which he repeated at home, and came upon the elegantly simple truth about musical sound: the pitch of a musical sound from a plucked string depends upon the length of the string. This led him to discover that the octave, the fifth and the fourth, as well as the whole-tone, are to each other as the ratios of the whole numbers. These, the harmonic ratios, as they came to be called, are all comprehended in a single construct: 6:8 :: 9:12. This means that the octave may be represented by 12:6 or 2:1; the fifth by 12:8 or 3:2; the fourth by 12:9 or 4:3. Moreover, the fifth may also be represented by 9:6 = 3:2, and the fourth by 8:6 = 4:3. The whole-tone, being the difference between the fifth and the fourth, is represented by 9:8, a ratio that cannot on division yield a pair of whole numbers. That is, dividing 9:8 gives $3:2\sqrt{2}$, an irrational or *alogos* number.

In the deceptively simple construct – 6:8 :: 9:12 – there are contained all the primary constituents of music's elemental structure and, inferentially, the harmonic symmetry of the universe. What is more, there is embodied in this set of ratios the original Pythagorean *tetraktys*, the ensemble of the four primary numbers – 1, 2, 3, 4 – that became the corner-stone of the Pythagorean philosophy of number. From the intrinsic properties of this – the *tetraktys* of the *decad* – the sum of whose terms equals the number 10, there were harvested in turn the theory of irrationals, the theory of means and proportions, the study of incommensurables, cosmology, astronomy, and the science of acoustics.[14] Because

[14] The original Pythagorean *tetraktys* or quaternary represented the number 10 in the shape of a perfect triangle composed of four points on each side. It showed at a glance how the numbers 1, 2, 3, 4 add up to ten. The number 10 was thus regarded as sacred, since it comprehended the equivalent in each of its terms to the universal components of the point (= 1), the line (= 2), the triangle (= 3), the pyramid (= 4). What is more, it contained in its terms the basic elements

"truth is truth to the end of reckoning," the harmonic properties of these numbers are as true today as they were for Pythagoras when he first discovered them. But a by-product of Pythagoras' discovery turns out to be the most stubborn problem in the science of acoustics: the incommensurability of the whole-tone. Interestingly enough, the same irrational number that appears on the division of the whole-tone was found by Pythagoras to obtain between the side of a square and its diagonal. In the one case, that of the whole-tone in the ratio 9:8, the division for obtaining a semi-tone, or one half of a whole-tone, produces the square root of 2. In the case of the geometric square, the Pythagorean theorem demonstrates that if the length of the side is 1 inch, the number of inches in the diagonal is also the square root of 2. As George Owen explains, this discovery was greeted with little joy by the Pythagoreans:[15]

> Having founded their order on the purity of number, the Pythagoreans were dismayed to discover the existence of the irrational number. Such numbers may well have played an important role in their mystery rites. Soon after the death of Pythagoras, Hippasus [an acoustical expert from Metapontum] ... communicated his [Pythagoras'] views and some of the Pythagorean doctrines to outsiders. For this he and his followers were expelled from the society.

The problem of irrationality for mathematicians was eventually resolved by a feat of intellectual genius: the reduction of arithmetic to logic.[16]

of music: 2:1 (octave), 3:2 (fifth), 4:3 (fourth), 4:1 (double octave). Cf. Owen (above, n. 11), pp. 27–28. The literature on the *tetraktys* is vast. For pertinent references, see F. R. Levin, *The Harmonics of Nicomachus and the Pythagorean Tradition*, p. 65, n. 62.

[15] Owen (note 11), p. 32. Just as no fraction will express exactly the length of the diagonal of a square, so too, no fraction will express exactly the size of the musical interval that registers on the ear as a semi-tone. The division of both the diagonal and the whole-tone yield the same irrational. As Bertrand Russell, *Introduction to Mathematical Philosophy*, p. 67 observed: "This seems like a challenge thrown out by nature to arithmetic. However the mathematician may boast (as Pythagoras did) about the power of numbers, nature seems able to baffle him by exhibiting lengths which no numbers can estimate in terms of the unit."

[16] The fundamental thesis to which Bertrand Russell devoted his *Principles of Mathematics* (1903) is that all the constants that occur in pure mathematics are

For musicians, the incommensurability of the whole-tone was resolved by an intellectual feat of no less brilliance: the well-tempered system of tuning.[17]

Because of the deep and far-reaching implications that Pythagoras' discovery had for such fundamental branches of knowledge as mathematics, cosmology, and astronomy – implications that extended far beyond its immediate utility in converting the sensory distinctions of pitch and interval into objective numerical form – it was treated by the ancients as a divine revelation. The story of Pythagoras' experiments with stretched strings and various other instruments such as panpipes, monochords, auloi, and triangular harps, and the dazzling discovery to which these experiments led, having once been told by Nicomachus, was passed along through the centuries in an unbroken tradition from one (now) obscure writer to another: from the musical theorist Gaudentius, surnamed "The Philosopher" (2nd or 3rd century A.D.), to the Neo-Platonist, Iamblichus (c. 250–c. 325 A.D.), to the biographer of ancient philosophers, Diogenes Laertius (3rd century A.D.), to the Roman grammarian, Censorinus (3rd century A.D.), thence to

logical constants; that, hence, the truths of mathematics can be derived from logical truths. As Russell argued, in order to deal with the two great sources of irrational numbers – the diagonal of the square and the circumference of the circle – the logical notion of spatial continuity had to have been introduced as an axiom *ad hoc* (pp. 438–39). In thus generalizing the notion of spatial continuity to the utmost, Russell created a set of new deductive systems, in which traditional arithmetic – which had laid bare the irrationality of the diagonal of a square and the circumference of the circle – was at once dissolved and enlarged. He observes in *Mathematics and Logic*, p. 196: "... we have, in effect, created a set of new deductive systems, in which traditional arithmetic is at once dissolved and enlarged ..."

[17] It will be argued below (pp. 202ff.) that Aristoxenus, the major source of such logically-minded musical theorists as Bacchius the Elder and others, attacked the problem of the irrationality of musical space in much the same way as that described by Russell; that is, he began by introducing the logical notion of spatial continuity as a necessary axiom *ad hoc*. This led him to dissolve and enlarge what had been for him traditional mathematics – Pythagorean harmonics. Logic and the establishment of logical constants led him finally to the monumental accomplishment for which he is credited in these pages: equal temperament.

Chalcidius, honored for his commentary on Plato's *Timaeus* (4th century A.D.), to the illustrious polymath, Macrobius, known for his commentary on Cicero's *Somnium Scipionis* (4th–5th century A.D.), to Fulgentius (c. 467–532 A.D.), famed for his writings on mythology. Finally, the tale of Pythagoras' discovery was deposited with the towering scholar, Boethius (480–524 A.D.), who honored Nicomachus by translating his account into Latin. Thereafter, it was preserved for posterity by one of the most important links between the scholarship of antiquity and that of the Middle Ages, Isidore of Seville (570–636 A.D.),[18] reappearing with various embellishments in the account of the Patriarch, André de Crète, surnamed Hagiopolites (died 8th century A.D.).[19]

Pythagoras' revelation of the affinity between music, mathematics, and philosophy has lost none of its majesty in these many retellings; if anything, it has gained power and importance through the numerous scientific investigations it set into motion long after its institution in the scientific literature.[20] Indeed, according to general opinion, no

[18] Another tradition, existing quite apart from the Pythagorean, connects the legendary discovery of the concordant intervals produced on the anvil with the smith's hammers to the Idean Dactyls, so-called, the dwarfish craftsmen of ancient Phrygian chronicles – ancient *Nibelungen*, as it were – the servants of the Asian Goddess, Rhea Kybele. This tradition is discussed by Eric Werner, *The Sacred Bridge*, pp. 376–77. Still another tradition is to be found in *Genesis* 4.21, where the discovery of music and the harp is attributed to Jubal, the descendant of Cain. The inference is that the connection between music and number had been arrived at independently of Pythagoras and quite possibly long before him. This is argued by Otto Neugebauer, *The Exact Sciences in Antiquity*, pp. 35–36.

[19] The account of the Hagiopolites differs interestingly from that of Nicomachus. For whereas Nicomachus portrays Pythagoras as experimenting with various kinds of instruments, finally settling on the monochord as the most convenient for his purposes, the Hagiopolites describes Pythagoras as building a four-stringed lute-type instrument, an instrument which he named *Mousikē*. This account is to be found in one place only, that of A. J. H. Vincent, in the sixteenth volume of a very rare and beautiful book entitled *Notice sur divers manuscrits grec relatif à la musique*, pp. 266–68.

[20] Thus, Sir Thomas Heath. *Aristarchus*, pp. 46–47: "The epoch-making discovery that musical tones depend on numerical proportions, the octave representing the proportion of 2:1, the fifth 3:2, and the fourth 4:3, may with sufficient certainty be attributed to Pythagoras himself, as may the first exposition of

discovery of comparable magnitude was made until the seventeenth century, when Marin Mersenne, the French mathematician, philosopher, and scientist, first explained in his *Harmonie Universelle* the relations obtaining between tension and the frequency of vibration of a stretched string, relations subsequently codified in "Mersenne's Laws."[21] The discovery of these relations was made independently by Galileo Galilei (1564–1642), the son of the musician, Vincenzo Galilei; and subsequent investigations conducted by Isaac Newton (1642–1727), Leonard Euler (1707–83), and Daniel Bernoulli (1700–82) revealed that the harmonic ratios were not confined to string lengths, but belonged to the musical intervals produced by all instruments. Hermann Helmholtz (1821–94), a man who not only unified the practice and teaching of such sciences as medicine, physiology, physics, and anatomy, but who also related them lastingly to the art of music, was uniquely qualified to speak of Pythagoras' discoveries; he assessed them in these terms:[22]

> This relation of whole numbers to musical consonances was from all time looked upon as a wonderful mystery of deep significance. The Pythagoreans themselves made use of it in their speculations on the harmony of the spheres. From that time it remained partly the goal and partly the starting point of the strangest and most venturesome, fantastic or philosophic speculations, till in modern times the majority of investigators adapted

the theory of *means*, and of proportion in general applied to commensurable quantities, i.e. quantities the ratio between which can be expressed as a ratio of whole numbers. The all-pervading character of number being thus shown, what wonder that the Pythagoreans came to declare that number is the essence of all things? The connection so discovered between number and music would also lead not unnaturally to the idea of the 'harmony of the heavenly bodies.'" Cf. O'Meara, *Pythagoras Revived*. pp. 14ff.

[21] As Mersenne demonstrated, the peculiar relation obtaining between tension and frequency of vibration is such that the latter – the frequency of vibration – is the square root of the former – the amount of tension. The details are to be found in M. Mersenne, *First Book of String Instruments*, Prop. VII, Sixth Rule, p. 177. "Mersenne's Laws" are discussed by Alexander Wood, *The Physics of Music*, pp. 90–92 and related by him to the construction of the modern pianoforte. Cf. Levin, "*Plēgē* and *Tasis*," 206, n.2.

[22] H. Helmholtz, *On the Sensation of Tone*, p. 15.

> the notion accepted by Euler himself, that the human mind had a peculiar pleasure in simple ratios, because it could better understand them and their bearings.

Musicians have always seen powerful connections between the working dynamics of their art and the combinatorial logic of mathematics and, to this day, will invoke purely mathematical concepts to explain various aspects of their discipline. Statements such as: "music is the organization of a certain finite number of variables"; or, "sound is heard number; number is latent sound";[23] or, "a fugal theme, like a mathematical equation, is subject to inversions, augmentation, and diminution," all reflect this sensibility. Such statements seem to gratify the need for certainty that musicians crave in discussing their art. Mathematicians, by contrast, whose working facts are nothing if not certain, see their discipline as no less aesthetic in its technical processes than is the art of music in the abstract. They testify to their sensibility on this point by assigning to the working dynamics of their discipline such musical characteristics as cadences, modulations, phrases, and tempos. Morris Kline speaks of the mathematician's aesthetic sensibility in these terms:[24]

> In the domain of algebra, calculus, and advanced analysis especially, the first-rate mathematician depends on the kind of inspiration that we usually associate with the composer of music. The composer feels that he has a theme, a phrase which, when properly developed and embroidered, will produce beautiful music. Experience and a knowledge of music aid him in developing it. Similarly, the mathematician divines that he has a conclusion which will follow from the axioms.

More recently, Edward Rothstein defined the link between music and mathematics as "something that mathematics and music share with our notions of the universe and our notion of the mind and soul – and

[23] Thus, Edward Rothstein, *Emblems of Mind*, p. 23: "In this way, sounds and numbers have become intimately related. And their connection is not arbitrary. It is not a metaphor: if we interpret the words properly, sound is simply heard number, number is latent sound."

[24] Morris Kline, *Mathematics in Western Culture*, pp. 457–58.

our notion of beauty as well." To this determinedly Pythagorean view of things, he adds the following:[25]

> Both music and mathematics create order, worlds in which processes occur, relationships are established, and elements are regulated. These worlds possess structures that might be mapped into our own: they might be similar in the strongest sense the mystics allow.

Miraculously, the dialectical steps by which the Pythagoreans converted music – the most unfathomable and inscrutable of the arts – into a branch of mathematical science were in turn diverted by them to transform astronomy – the most conspicuous and mathematical of the sciences – into the unheard archetype of music. Their point of departure was motion. Thus, they argued, if motion, as of vibrating strings, air columns, and percussion instruments, is the cause of pitch variation, and the primary exemplar of motion is present in the rotations of the celestial bodies through space, it should follow that these macro-cosmic motions of the planets must themselves produce pitches.[26] Moreover, that the pitches presumed to be generated in this way are musical may be attributable to the continuous, uniform, and regular motion that the planets execute in their orbits. And because the relative pitch of any musical sound is a function of the velocity of the moving object, the pitches emitted by the planets in their courses through the ether should necessarily vary with their individual speeds. As Nicomachus explained

[25] Rothstein (note. 23), p. 30. Along these lines Rothstein quotes to excellent effect the words of the nineteenth-century mathematician, James Joseph Sylvester, from a paper on Isaac Newton: "May not music be described as the Mathematics of sense, Mathematics as Music of the reason?" (Rothstein, p. 31).

[26] In the third chapter of his *Manual* (Jan, 241–42), Nicomachus provides what may be the most ancient version of this distinctly Pythagorean-Platonic concept. He began, as did Plato, by enlisting only those propositions whose truth can be verified by mathematics. From that point on, he saw to it that his concept of celestial motion should rest upon the same mathematical bases as those underlying acoustical motion. To this end, he followed the Pythagoreans and Plato in lifting what appears to be a purely poetic notion into the precincts of science. See Burkert (above, note 11), pp. 352–55; Barker, II, p. 251, n. 17 and n. 20. Nicomachus' description of the heavenly order is analyzed in Levin (note 13), pp. 47–57.

in his account of the Pythagoreans' cosmic view, the variation in pitch imputed to the planets is a function of their mass, speed, and orbital position.[27]

With this line of reasoning, the conditions were set for so thorough a union of harmonics and astronomy that henceforth the mathematical laws underlying the one could be held to account for the harmonic perfection seen in the other. There was required only the addition of a single proposition to complete an incorruptible circle of necessity wherein harmonics and astronomy would find their common bond in number: the harmonic sectioning of the heavens. This was done by the planets, whose orbital distances from one another conformed to the concordant distribution of pitches in a well-attuned musical scale. It could now be assumed that the harmonious properties of music, discoverable in the mutual relations of number, are implicit in the planetary order. Accordingly, by virtue of their being distant from one another in the same relative degrees as the notes of a well-attuned scale – these degrees of difference dictated by the mathematical ratios determining the concordant intervals making up the octave, the perfect consonance – the planets could be shown to trace harmonic boundaries in the heavens. And, conversely, the structural elements of music could be said to imitate the heavenly paradigm in all its particulars.[28]

The task of reconciling this conception of a harmonic universe with the astronomical phenomena produced diverse theories and rationalizations, all assembled under the traditional title "Harmony of the Spheres," the ultimate expression of which is found in Plato's *Timaeus*. For Plato, the intrinsic meaning of music lay, therefore, in the concept

[27] *Manual*, Chapter 3 (Jan, 241. 12–15). As Barker, II, p. 251, n.18 sees it, "... the sense is not that the planets differ from one another in the positions of their orbits in space (i.e., in the distances of their orbits from the earth), but that their orbits have different ranges of variation."

[28] This concept of a universe animated by the same physical laws that underlie the tuning of the lyre and invested with the composite symmetry and serene regularity of a musical attunement (*harmonia*) could free man, in a metaphoric leap, from the prosaic notion that life is altogether incongruous and man himself totally insignificant. The notion of musical instruments as expressions of the universe was gloriously rendered by Robert Fludd (1574–1637), whose cosmic lute, once seen, is never to be forgotten. See Jocelyn Godwin, *Robert Fludd*, pp. 45ff.

of *harmonia*, the attunement, or proper "fitting together," of opposites.[29] That the movements in space by the heavenly bodies were, like the well-attuned pitches of a melodious movement, harmonious in their combined perfection of uniform, circular, and constantly regular courses, is an assumption that dominated astronomy from the time of Plato down to that of Kepler.[30] The various theories that derived from this assumption, whether heliocentric, like those of Aristarchus and Copernicus, or geocentric, like that of Ptolemy, found their explicit formulation

29 In the *Timaeus* 35B–36B, Plato, without having to appeal to the empirical evidence of music itself, gave music universal significance as the embodiment of the World-Soul. He accomplished this by application of the harmonic, geometric, and arithmetic progressions of mathematical science; by these means, he fixed the boundaries of the physical universe in such a way that they conformed to a (modern) diatonic scale. He began with the two geometric progressions: 1, 2, 4, 8 and 1, 3, 9, 27, each of which has in common the number 1, the ratio between their terms being 2:1 and 3:1, respectively. By inserting the harmonic and the arithmetic means between each of the terms in the two series, Plato completed the full diatonic scale. Expressed in algebraic terms, to find the harmonic mean, b, within the terms a, b, c, the formula is applied: $b = \frac{2ac}{a+c}$; for the arithmetic mean: $b = \frac{a+c}{2}$. See Levin, *Manual*, pp. 114–20. Jamie James, *The Music of the Spheres*, p. 44, has this to say of Plato's construction: "The eight pages (in translation) of the *Timaeus* that Plato devotes to the creation of the cosmos have generated thousands of pages of commentary, yet no one has ever quite managed to clarify Plato's ambiguities successfully. What the dialogue does communicate, unambiguously, is Pythagoras' final triumph. The cosmogonic vision of the *Timaeus* is the mystical Pythagorean equivalence of music, the cosmos, and mathematics brought out of the esoteric closet and thrown open for inspection by all thinking persons." Cf. L. Spitzer, *Classical and Christian Ideas of World Harmony*, pp. 10–17.

30 The diverse features of this conception that were henceforth to influence astronomical thought for centuries converged with reality in the celestial physics of Johannes Kepler (1571–1630), who could not but stand in awe of his own discovery (*Harmonikēs Mundi*, *proemium*, Book 5, 179; Caspar, p. 289. 13–19): "At last, I say, I brought to light, and beyond what I could ever hope for, I discovered the ultimate truth: that the entire nature of harmonic science in all its magnitude and with all its parts explained by me in my third book is to be found in the heavenly motions; but not in the way I had supposed – this part of my discovery being not the least of my joy – but in another utterly different way, also both most excellent and utterly perfect."

in the models provided by geometry. For by assigning to each planet a path that conformed to its visible course in space, geometry assisted astronomy to determine whether its hypotheses were consistent with the observed phenomena. These latter, being points in space – discrete and enumerative – embroidering the heavens with their uniform, continuous, and regular courses of a certain magnitude, were thereupon accounted as the visual embodiment of number in motion and governed therefore by the uniform and immutable laws of mathematics.[31]

If the harmonious perfection perceived in the motion of the heavenly bodies could be rendered knowable through the mediation of number, then the motion that is antistrophal to it and productive of the audible phenomena should be approachable by similar means. Implicit in this assumption is the notion on which the Pythagoreans based their theory of sound. This theory held that the actual, physical, and observable motion in air of vibrating bodies – a motion that produces sound – is subject to the same laws that govern the actual, physical, and observable motion of the heavenly bodies. For, as had already been shown by the Pythagoreans, in the absence of this actual and observable motion, sound could no more be said to exist than the heavenly bodies could be conceived to pursue their perdurative courses. This physical motion that produces sound was explained by them to be caused by a percussion of some sort – the striking of something against the outside air, whether from the vibrations of a plucked string, or from the infusion of breath into the mouthpiece of a wind instrument. These percussions, being conducted through the medium of air, were found to generate all the varieties of sound that the ear can

[31] The laws of planetary motion that Kepler discovered are these three: (1) the planets trace elliptical (not circular) orbits; (2) the focus of each ellipse is the sun; (3) the squares of the period of revolution of any two planets are in the same ratio as the cubes of their mean distances from the sun. Kepler says this of his discovery (*Harmonikēs Mundi*, Book 5, 202; Caspar, p. 317. 6–10): "So far, therefore, it has been proved, by means of numbers taken on the one hand from astronomy, on the other from harmonics, that there obtain between these twelve terms or motion of the six planets circling the sun up, down, and in all directions harmonic relationships (*proportiones Harmonices*), or relationships extremely close to being harmonic within an imperceptible part of the smallest consonance."

apprehend. As Plato explained it, the faster the vibratory motion, the higher the pitch produced and, conversely, the slower the motion, the lower the pitch produced.[32]

With musical pitch regarded as inseparable from the motion that is its cause, its identification with speed and so with quantity and number could be specified. If quantity could be predicated of pitch, and if pitch could then be represented by number, the differences between all the high and the low pitches of melody could be systematically formulated on the logical bases of numbers in their mutual relations. This meant that the harmonious property underlying all utterance that is recognizably musical could now be rendered knowable through the mediation of number. On this approach, the musical universe could be contemplated as the embodiment in sound of numbers in their mutual relations, and therein subject, like the visible phenomena, to the uniform and immutable laws of mathematics. The assumption of motion as the principle of audible phenomena, and the adduction of number as the primary element of musical pitch, thus supplied the *Urstoff* of the Pythagorean doctrine of harmonics. But, according to Plato, it took a Creator to infuse into the universal soul all the attributes of a mathematically true attunement and thereby to have the universal voice be "propertied with all the tuned spheres." Plato called this Creator the Demiurgus (literally, "one who works for the people") and represented his artistry as that of a Master-Musician. The affinity between music and mathematics that prompted Plato's identification of the World-Soul and music's harmonically structured foundation has been felt ever since by astronomers, mathematicians, and philosophers of all epochs. And, like the "floor of heaven" itself, poetry is "thick inlaid with patines" of its variegated implications.

By the sheer force of his intellect, Pythagoras had revealed to the coming ages worlds within worlds of unchanging and harmonious order, where reason governs everything and nothing can do or suffer

[32] In *Timaeus* 67B–C1, Plato explained that the kind of physical motion produced in the air by percussion accounts for the differences in sound: "The rapidity of the motion is what causes highness of pitch. The slower the motion, the lower the pitch. Motion that is uniform produces a sound that is even and smooth; the opposite is what causes a sound that is harsh."

wrong. It is here that mathematics and philosophy meet, each bound to the other by the power of *harmonia*, the attunement of opposing natures. Had Pythagoras ever committed any of these thoughts to writing, he would quite possibly have expressed himself as Plato did in the *Phaedo*, where he has Socrates being advised in a recurring dream to live a Pythagorean life:[33] "Compose and practice music . . . for philosophy is music in the highest."

Not surprisingly then, the works of numerous writers on music have been and are to this day almost completely dominated by the influence of Pythagorean thought. Indeed, one might say that when it comes to music, Pythagoras seems always to be presiding over the discussion, dictating the reasoning, and even reinforcing the musical analyses. Book titles bearing the Pythagorean imprimatur have continued to appear since Johannes Kepler first spoke of *De harmonikes mundi*, this being the fifth book of his classic work, *The Harmony of the World*.[34] Today, we have such representative works as *Cosmic Music: Musical Keys to the Interpretation of Reality*, edited by Joscelyn Godwin and, by the same author, *The Harmony of the Spheres: A Sourcebook of the Pythagorean Tradition in Music.* Similarly titled is *The Music of the Spheres: Music, Science and the Natural Order of the Universe* by Jamie James. Other works that are deeply imbued with Pythagorean thought and philosophy are *The Concept of Music* by Robin Maconie; *Emblems of Mind: The Inner Life of Music and Mathematics* by Edward Rothstein; and *Music, the Brain, and Ecstasy: How Music Captures Our Imagination* by Robert Jourdain.

[33] Plato *Phaedo* 61A2–4. In commenting on this passage, Albert Cook, *The Stance of Plato*, p. 1, speaks of Plato as "the Webern and the Beethoven of philosophy, an essentialist who is at the same time powerfully polyphonic." To this perception he adds, "In trying to account for the unified utterance of a whole dialogue of Plato, and still more of the body of his writings, we have to try to be anthropologists, literary critics, and responsible philosophers all at once, at the risk of putting asunder what it was his unique achievement to put in perilous equipoise together."

[34] On the plan of the *Harmonikēs Mundi Libri V*, see the analysis of J. V. Field, *Kepler's Geometrical Cosmology*, pp. 96–99. She thus observes (p. 99): "The position of the musical book immediately after the geometrical ones was probably designed to emphasize that the 'musical' ratios, long seen as arithmetical in origin, had now been given a basis in geometry." As for the title, the form used by Kepler, *Harmonikēs*, is a transliteration of the Greek genitive, the nominative of which is *Harmonikē*. Cf. Bruce Stephenson, *The Music of the Heavens*, p. 4, n. 1.

In Plato's explanation and development of the Pythagorean doctrine, there stands as a first cause, a Demiurgus or Creator, who alone knows the secret of universal harmony. This is Number in all its musical manifestations. The notion of number in music and music in number is expressed in the celebrated dictum of J. S. Bach's contemporary, Gottfried Wilhelm Leibniz:[35]

> Musica est exercitium arithmeticae occultum
> nescientis se numerare animi.
>
> Music is a hidden exercise in arithmetic of a mind that doesn't know itself to be dealing with numbers.

The mind, according to Leibniz, when dealing with music, is unconsciously engaging in some sort of arithmetic operation. And this is evidently true of the listener's mind as well as that of the composer. The inference is that when we are presented with music, it is essentially number to which we are responding. But Schopenhauer, who ranked music as philosophy in the highest, recast Leibniz' dictum so as to alter its meaning at this deepest level:[36]

> Musica est exercitium metaphysices occultum
> nescientis se philosophari animi.
>
> Music is a hidden exercise in metaphysics of a mind that does not know itself to be dealing with philosophy.

In other words, to take Leibniz' definition of music literally would be to consider not the innermost, but only the outward or exterior significance of music. As Schopenhauer says:[37]

> Therefore, from our standpoint, where the aesthetic effect is the thing we have in mind, we must attribute to music a far more serious and profound significance that refers to the innermost being of the world and of our own

[35] This is quoted by Arthur Schopenhauer from Leibniz' *Letters*, *ep*. 154 (Kortholt) in *The World as Will and Representation*, Vol. 1, p. 256, n. 46.
[36] Schopenhauer, op. cit., p. 264.
[37] Schopenhauer, op. cit., p. 256.

> self. In this regard, the numerical ratios into which it can be resolved are related not as the thing signified, but only as the sign.

Schopenhauer was surely of a mind with Bacchius the Elder, that all but forgotten musical theorist whose words open this essay. For had Bacchius believed that it is to the numbers ruling musical pitch differences that we unconsciously respond on music's presentation, he would have defined music in this way: "It is a conceptual knowledge of number and all that pertains to number." But Bacchius scrupulously and deliberately avoided any mention of number in his definition of music. In fact, number is not mentioned by him even once in the whole course of his treatise on music. When he did have occasion to use numerical terms, it was solely for such enumerative purposes as given in these examples:[38]

> How many species of musical notes are there? Three. How many inflections of melody do we say there are? Four. How many modulations do we say that there are? Seven.

The numbers 3, 4, and 7, respectively, as used here by Bacchius, are not identical with the musical notes, melodic inflections, or the modulations that they signify. These numbers are merely what each of these collections (notes, inflections, modulations) have in common and what distinguishes them from other such collections. The number 7, for example, is what characterizes the collection of modulations because, according to Bacchius, there are seven distinct modulations in his (ancient Greek) music.[39] In each of these cases, Bacchius has used

[38] *Introduction to the Art of Music*, I. 43 (Jan, 302. 7); I. 45, (Jan, 302. 16); I. 50 (Jan 304. 6).

[39] Modulation (*metabolē*) is, and always has been, one of the most important resources of musical expression. It does for music what metaphor (*metaphora*) does for language: it effects a change of profound dimensions. In the one case, that of language, the change occurs when there is transferred to one word the sense of another. To take one of Aristotle's examples in *Poetics* 21. 1457b25: "Old age is the *evening* of life." In the other case, that of music, the change occurs when there is transferred to one pitch the function (*dynamis*) of another. For example, the note C that has the function of tonic (I) in the key of C can be made to bear the function of subdominant (IV) in the key of G. The

numbers to enumerate members of a designated class. But our knowledge of modulation, to take one of his examples, and what it does in music or what it means for music cannot be derived from the number 7; nor, for that matter, can the numbers 3 and 4 enable us to understand what is meant by species (*eidē*) or inflections (*pathē*) of melody. The fact is that knowledge in regard to all such collections and all else pertaining to the composition of music cannot be gained from numbers. The reason for this is that number is not the defining property of music.

The virtue of Bacchius' definition of music – "It is a conceptual knowledge of melody and all that pertains to melody" – lies in its recognition of melody as the defining property of music – a property by whose possession music is distinguished from all else in the natural universe. In his recognition of the status of melody, Bacchius could offer what Bertrand Russell would regard as a definition by intension, that is, a definition that specifies a defining property of the thing defined. Indeed, Russell considered this kind of definition more fundamental logically than any definitions that enumerate the components or elements of a thing. According to Russell then, definitions of the latter type, so-called definitions by extension, can always be reduced to definitions by intension. It is for this reason that Russell considered intensional definitions to be the ultimate source of knowledge. Because we cannot enumerate all fractions, say, or all of any other infinite collections, be they notes, intervals, or numbers, "our knowledge," Russell says, "in regard to all such collections can only be derived from a definition by intension."[40]

Proceeding logically from his intensional definition of music, Bacchius thereupon offered a deliberately tautological definition of

subject of modulation/metaphor bristles with complexities, but this much can be said here: metaphor can only work within the framework of a standardized rule-governed language; modulation, to work at all, requires a standardized equally-tempered attunement. See below, pp. 323–24. The seven types of modulation listed by Bacchius are: by system, by genus, by key, by character (*ēthos*), by rhythm, by rhythmic tempo, by arrangement of the rhythmic composition. Here, as elsewhere, Bacchius is following a tradition set down centuries earlier by Aristoxenus. Cf. Solomon (note. 2), 122, n. 2. On modulation, see Barker, II, p. 424.

[40] Russell, *Introduction to Mathematical Philosophy*, p. 13.

melody: "It [melody] is the fall and rise generated by melodious notes."[41] Then, after examining in detail the seemingly limitless melodic permutations and combinations yielded by the fall and rise of melodious notes, Bacchius saw fit to add a second definition of melody, one which conforms interestingly to what Russell terms "extensional."[42] Bacchius' definition in this instance is extensional in that it enumerates the constituents of a class, the class in this case being that of the melodic elements. Thus Bacchius:[43] "What is melody? It is that which is composed of notes and intervals and units of time."

As Bacchius seems to have intuited, notes, intervals, and units of time themselves form an infinite collection of terms and cannot therefore be satisfactorily defined solely by enumeration or, as Russell has it, by "extension." Moreover, such collections of notes, intervals, and units of time, having a given number of terms themselves, presumably form infinite collections. That being so, it may be presumed that in music it is possible for there to be produced an infinite number of melodies. Bacchius evidently wished to define melody in such a way then, that the production of an infinite number of melodies might not only be thought a possibility, but indeed a probability, given the infinite number of terms involved in its creation. Therefore, Bacchius approached music, as he had to, in terms of an infinite number of melodic elements. At the same time, he had to define such a collection by intension, that is, by a property common to all its members. This property is melodiousness (*emmelēs*). Bacchius is wonderfully clear on this point:[44]

[41] I. 19 (Jan, 297, 19–20). The technical terms for "rise" and "fall" come from Aristoxenus' *epitasis* (tension) and *anesis* (resolution) and are defined as such by Bacchius at I. 45 (Jan, 302. 18–21): "[*Anesis*] is the motion of melody from the higher note to the lower." *Epitasis* is the motion of melody from the lower note to the higher."

[42] Russell (above, note 40), p. 12. As Russell points out (p. 13): "The vital difference between the two [definitions] consists in the fact that there is only one class having a given set of members, whereas there are always many different characteristics by which a given class may be defined."

[43] II. 78 (Jan, 309. 13–14).

[44] II. 69 (Jan, 307. 5–10).

> What sort of notes are melodious? Those which people use when singing and when they play something on musical instruments. For without this [*emmelēs*] being present, it is impossible for any of the musical elements to be defined.

To Bacchius then, music and its defining property – melodiousness – are for all practical purposes interchangeable. Or, to put it another way, music in the absence of melodiousness is no longer music. A melodious sequence, being "that which is composed of notes and intervals and units of time" (see note 43), is more, therefore, indeed, much more, than the sum of its parts.

Melody was, accordingly, music's proper subject matter for Bacchius. Without attempting to compare melody to anything in the world – to language, for example, or to mathematics, or to architecture, or to painting – Bacchius concentrated solely on identifying those melodic attributes that make music possible. In this effort, his intention was not merely to show what makes for the nature and differences between musical notes and intervals and units of time, but, more specifically, to demonstrate that musical notes, intervals, and units of time are in reality original and immutable qualities of the melodies that exhibit them. This approach to the question of music's essential nature stems from the recognition that, ultimately, music speaks for itself and will not endure translation into any other medium of expression.[45]

The question posed by Bacchius – "What is music?" – is, of course, one that has been asked ever since Homer first set Western civilization into motion with the quintessential word of music that opens the *Iliad*: *aeide* ("Sing"). It was, in fact, in song that the Homeric bards had already been keeping civilization alive for many generations in their singing "the deeds of gods and men." Indeed, song, as the ancient Greeks had long known, is something that requires exacting thought, consummate skill, keen perception and, above all, *paideia*: cultivation, in the highest sense of the word.[46]

[45] See note 9. It is Aristoxenus' recognition of this fact that makes him so worthy of modern attention.

[46] The Greeks created the ideal of *paideia*. Indeed, as Russell might have said, *paideia* is the defining property of Hellenism. Yet the word *paideia*, like *aretē*, has no sufficient equivalent in English. Cf. Werner Jaeger, *Paideia*, Vol. I, pp. 4ff. *Paideia* has to do with the training of man to fulfill the ideal of humanity as it ought to be. As Jaeger, *Paideia*, Introduction, p. xxiv says: "It starts from

By Homer's time,[47] song, accompanied on the *phorminx*,[48] the stringed instrument of the bards, was a well-evolved art. Phemius, for example, the bard of the *Odyssey*, despite his denial of formal training, had to admit that his repertory of songs was virtually inexhaustible:[49] "I am self-taught, and a god has planted all sorts of songs in my mind."

Many of Phemius' songs were the product of his own genius or, as he put it, were given him by a god. Others of his songs came down to him from countless generations of singers, and the audiences for whom he sang were familiar with them all. If one of them made Penelope weep, it may easily be imagined that the melody sung by Phemius was as much a cause of her tears as were the words of his song. As Homer describes the episode:[50]

> Then with tears did she [Penelope] address the bard: 'Phemius, you know many other enchantments for mortals – the deeds of men and gods that singers celebrate; sing one of them as you sit by these men [the suitors of Penelope] and let them drink their wine in silence; but stop singing this sorrowful song which wears away my own heart within my breast.

the ideal, not from the individual." As such, it "is the universally valid model of humanity which all individuals are bound to imitate." The role played by music in this enterprise is described by Athenaeus *Deipnosophists* XIV. 628C: "Music plays a part in the exercise and the sharpening of the mind; for that reason, every one of the Greeks and those foreigners whom we happen to know make use of it."

47 Since the seventeenth and eighteenth centuries of our era, Homer's time and place in history have been, and probably always will be, a matter of scholarly debate. Some scholars believe in the multiple authorship of the two epics, the *Iliad* and the *Odyssey*, while others go so far as to deny the existence of Homer altogether. The view accepted here is that Homer lived and worked at or near Chios before 800 B.C., that his is one of the greatest minds Greece was able to produce even before its own history commenced, and that Homer is the single author of the *Iliad* and the *Odyssey*.

48 According to Martha Maas and Jane McIntosh Snyder, *Stringed Instruments of Ancient Greece*, p. 4, *phorminx* is the Homeric term for lyre, but its linguistic origin remains a mystery. "We are left with the conclusion that the word *phorminx* was borrowed by the Greeks from some other, possibly non-Indo-European language." Cf. Anderson. *Music and Musicians in Ancient Greece*, pp. 36–37.

49 *Odyssey* 22. 347–48.

50 *Odyssey* 1. 336–42.

These words say much: that Homer must have known as well as any great composer of *lieder* has known ever since – a single line of melody produced by a "clear-toned" voice, like that of Circe (or Dawn Upshaw, Cecilia Bartoli, or Frederica von Stade), can be as emotionally searing, if not even more so, than the words of a poem.[51] According to Bacchius, it is in the interior precincts of melody that music finds its best definition.

Unlike Bacchius' definition of music, which ultimately turns on its *own* ground of determination, modern definitions of music are grounded in the sensibility it induces in human beings. These definitions focus on how the listener is affected by the music, so that what the listener feels is transferred to the object itself: music. The subjective significance of this feeling, once transferred to the music that induced it, has produced many different definitions of music. Of these, the most commonly accepted holds that "Music is the language of feeling." Because it is a definition associated directly with the cognitive faculty – the mind, where music begins – while linked at the same time to human emotions – where music finds its target – it is accounted by many to be appropriate at all levels. Thus, for example, the composer and writer on music Robin Maconie gives it the following endorsement:[52]

> Music is a field of human expression which has successfully resisted analysis in terms of conventional theory. . . . From one vantage point music can be perceived as operating in the same auditory domain as speech, being processed by the same sensory mechanisms, capable of arousing a predictable response, and available for inspection in written form. From such a viewpoint music is therefore arguably language-like.

[51] The question as to whether the beauty of melody always takes precedence over the associated text, or whether the words of the text add expressive content to the music has been debated forever, it seems. See, for example, Wayne D. Bowman, *Philosophical Perspectives on Music*, pp. 141–52. On the use of language to describe music, see Peter Kivy, *Music Alone*, pp. 97–101. The music most admired by Plato, for example, has been best described by Sir Walter Scott in *Quentin Durward*. Speaking of the Lady with the Lute, Scott wrote: "The words had neither so much sense, wit, or fancy as to withdraw the attention from the music, nor the music so much of art as to drown out all feeling of the words. The one seemed fitted to the other."

[52] Robin Maconie, *The Concept of Music*, p. 3.

Deryck Cooke, music critic and musicologist, goes even further: for him music is not merely language-like; it is *in fact* a language:[53]

> We may say then that, whatever else the mysterious art known as music may eventually be found to express, it is primarily a language of the emotions, through which we directly experience the fundamental urges that move mankind, without the need of falsifying ideas and images – words or pictures.

With that in mind, Cooke levels this rather biting criticism against the experts who, he believes, should know what music in fact is:[54]

> But we musicians, instead of trying to understand this language, preach the virtues of refusing to consider it a language at all; when we should be attempting, as literary critics do, to expound and interpret the great masterpieces of our art for the benefit of humanity at large, we concern ourselves more and more with parochial affairs – technical analyses and musicological *minutiae* – and pride ourselves on our detached, dehumanized approach.

In order "to bring music back from the intellectual-aesthetic limbo in which it is now lost, and to reclaim it for humanity at large,"[55] Cooke assigned himself the task of "actually deciphering its language." His method was "to isolate the various means of expression available to the composer – the various procedures in the dimension of pitch, time, and volume – and to discover what emotional effects these procedures can produce."[56]

According to Cooke,[57] musical works are built up out of tensions within the three dimensions of pitch, time, and volume. Tempo, he says, expresses different levels of animation, volume expresses various degrees of emphasis given to specific feelings, and intervallic tensions between pitches – these termed by Aristoxenians like Bacchius, *epitasis*

[53] Deryck Cooke, *The Language of Music*, p. 272. Cooke's phrase, "falsifying ideas and images – words or pictures," is strangely ambiguous here. He apparently means that words, images, or ideas need not be imported, since they tend to falsify the emotional content of music.

[54] Cooke, *op. cit.*, *Preface*, p. x.

[55] Cooke, *op. cit.*, *Preface*, p. xi.

[56] Cooke, *ibid*.

[57] Cooke, *op. cit.*, pp. 35ff.

(rise) and *anesis* (fall) – have their equivalents of pleasure-pain within the major-minor modal structures of melodic motifs. Given these three properties, melodic motifs can, in Cooke's view, induce in the listener certain emotional responses that are not only elemental – joy or sorrow – but are also predictable. These elemental responses are triggered, he argues, by sixteen basic terms in music's "vocabulary" to each of which he assigns an objective utility within their respective contexts. Thus, he says, the ascending major triad expresses an assertive emotion of joy, the ascending minor triad, an assertion of sorrow: a descending major sequence from dominant to tonic conveys relief, consolation, and reassurance; a descending minor sequence from dominant to tonic expresses an acceptance of, or yielding to, grief or to the despair connected with death. A rise from the lower dominant over the tonic to the minor third, followed by a resolution to the tonic "conveys the feeling of a passionate outburst of painful emotions"; major sevenths are generally "optimistic" intervals, while minor sixths are laden with anguish.

Assuming that specific meanings inhere in such melodic elements as major and minor intervals and other such distributions, Cooke proceeds to adduce from a wide range of musical literature numerous examples of these types of melodic sequences in action. The climax of Beethoven's *Leonore Overture No. 3* denotes, he says, excited affirmation in an outgoing feeling of pleasure; while the closing pages of Mahler's *First Symphony* arouse an incoming feeling of pleasure.[58] An outgoing feeling of pain is elicited, he says, by "O heilige Schmach" from Act 2 of *The Valkyrie*, whereas an incoming feeling of pain is conveyed in the opening of the finale of Tschaikovsky's *Pathétique Symphony*. The first theme of the third movement of Brahms' *First Piano Concerto* ascends with the notes 5 – 1 – 2 – 3 – 5 – 1 (which can be better heard as A_1 – D – E – F – A – D^1) and conveys courage and heroism, the minor phrase 5 – 1 – 3 awakening this distinctive reaction in the listener. To cite one more example, the progression 1 – 2 – 3 – 2 – 1 in the minor mode is used, Cooke claims, to express gloom, doom, and a sense of inescapable fear, especially when it is repeated over and over. These and many more

[58] Cooke, *op. cit*, pp. 104–5 explains "outgoing" and "incoming" as follows: "The expressive quality of rising pitch is above all an 'outgoing' of emotion. The expressive quality of falling pitch is of an 'incoming' of emotion."

examples allow Cooke to affirm such confidence in his analytic method that he can say:[59]

> Such [examples show] clearly that music has its own method of coherent emotional expression, quite different from that of speech-language. This method is nothing more mysterious than the presentation of some general but clearly-defined attitude towards existence by the disposition of various terms of emotional expression in a significant order.

Cooke's attempt to translate subjective feelings into objective form, indeed, to rationalize what is the ecstatic experience of music, raises an age-old problem: Is there justification for the belief that any number of people can hear the same melodies and react to them in the same way? If there is justification in ascribing the same responses from different listeners to any given melody, then Cooke must be concerned, as indeed he is, with those features that those melodies have in common for all listeners. To be sure, Cooke concedes that one listener's reaction to a melody may differ greatly from that of another, and he even quotes the composer Paul Hindemith to that effect:[60]

> One given piece of music may cause remarkably diverse reactions in different listeners. As an illustration of this statement, I like to mention the second movement of Beethoven's Seventh Symphony, which I have found leads some people into a pseudo feeling of profound melancholy, while another group takes it for a kind of scurrilous scherzo, and a third for a subdued kind of pastorale. Each group is justified in judging as it does.

A clear and unambiguous observation, this, but for Cooke, how utterly wrong! As he sees it, neither Hindemith, nor anyone else in his right musical mind, is entitled to this conclusion, namely, that each listener is justified in judging as he does. In fact, Cooke's whole book may be read as an attempt to refute Hindemith on this critical point. The conclusion to which all his arguments lead is this: listeners' reactions to

[59] Cooke, *op. cit.*, p. 212. However, cf. Jourdain, *Music, the Brain, and Ecstasy*, p. 60; "When it comes to the production of melodies, we are still at the hunting-and-gathering stage."

[60] Cooke, *op. cit.*, p. 21.

music differ in direct proportion to their own understanding of music and their own capacity to feel music's emotional content. As he says:[61]

> The fact is that people can only react to emotions expressed in a work of art according to their own capacity to feel those emotions.

He therefore asks:[62]

> Could it not be that some listeners are incapable of understanding the feeling of music properly?

His answer is unequivocally, "Yes":[63]

> The answer is, of course, yes; and this explains why "tests," in which the reactions of a random collection of individuals are classified and analyzed, prove nothing. Sympathetic understanding is a prerequisite.

In the end, Cooke has to dismiss as "plainly unmusical" all those people who would react to a given piece of music in ways that are at odds with his own feelings:[64]

> Such people, whom one knows to exist, are just plainly unmusical: suppose that such a listener's "memory-image" has no connection with the emotions expressed by the music at all. If someone were to declare the *Eroica* Funeral March to be a sanguine piece, we should unhesitatingly accuse him of being emotionally undeveloped.

As Cooke has it then, those listeners who could even think of the *Eroica* Funeral March as sanguine or could judge the second movement of Beethoven's *Seventh Symphony* to be a "scurrilous scherzo" must be deemed totally unqualified to offer any opinion at all on any piece of music presented to them:[65]

[61] *Ibid.*
[62] *Ibid.*
[63] *Ibid.*, n. 2
[64] Cooke, *op. cit.*, p. 22.
[65] Cooke, *op. cit.*, p. 23.

> One is bound to regard anyone who reacts in this way as either superficial, unmusical, or unsympathetic to Beethoven.

At issue is this: Are there, in fact, natural indications in melodic utterances, or in music generally, that evoke or stimulate the same emotions in the same way in all possible human minds? Cooke would (and, in effect, does) answer "Yes." Hindemith's answer, on the other hand, is an emphatic "No." In truth, however, even if each could find no flaw in the other's argument or reasoning, neither would ever succeed in winning over the other to his own point of view. For the dispute in which they are engaged has been going on for as long as music has existed; it is a dispute in which only the more obstinate participant can presume to have won. When a man of Cooke's critical judgment undertakes to refute the conclusions of composers like Hindemith, he does so in the belief that there is in the human mind an order of absolute truth – truth that is wholly distinct from metaphysical conjectures. Indeed, for Cooke, music presents a case in which exact and precise truth can actually be arrived at. Moreover, his critical attitude toward music is wholly unaffected by the possibility that an exact truth may not be identical with its perceived object; that the known object – music – and the mind's knowledge of it may be two very different realities.

If Cooke tends to be dogmatic in his assertion that there are between musical truths and the knowing mind a perfect parity, Hindemith is no less so in his negation of any such possibility. Hindemith, as it happens, receives strong approbation on this point from the formidable Igor Stravinsky:[66]

[66] Igor Stravinsky, *An Autobiography*, p. 83. Stravinsky's position is an example of musical formalism at its most rigid. As Bowman (n. 51), p. 193, points out, "musical formalism represents a beneficial reaction against the metaphysical excesses of idealistic theory. . . . [but] Its interest in system and structure may devalue personal expression and activity. . . . Its interest in 'objectivity' may devalue parameters and experiences that tend to be more personal and subjective." On p. 94, Bowman cites Stravinsky's pronouncement as an example of the formalist's argument that "states its positive thesis in negative terms." Cooke's thesis is an example of a formalistic approach stated in positive terms, but Bowman does not include it in his discussion.

> I consider that music, by its very nature, is essentially powerless to *express* anything at all, whether a feeling, an attitude of mind, a psychological mood, a phenomenon of nature, etc. . . . Expression has never been an inherent property of music. That is by no means the purpose of its existence. If, as is nearly always the case, music appears to express something, this is only an illusion and not a reality. It is simply an additional attribute which, by tacit and inveterate agreement, we have lent it, thrust upon it, as a label, a convention – in short, an aspect which, unconsciously or by force of habit, we have often come to confuse with its essential being.

The most significant thing in Stravinsky's statement is his refusal to say what music's essential being is. This is not mere musical affectation on his part; it indicates, rather, an abiding philosophical position: that we cannot, or must not, seek to know music's essence as we do other things and their laws. All we can do in the face of music is to cast upon it figurative or metaphoric language, and not, as in Cooke's attempt, the language of literal fact and denotation. Thus, whereas Cooke was intent on assigning aesthetic meanings to specific melodic figures, Hindemith and Stravinsky were bent on divesting music of any sort of representational capacity. In this respect, Stravinsky and Hindemith (as well as most musicians) share the position of Bacchius, and the Aristoxenian school for which he was a keen spokesman. This position derives from the fact that music operates between the hearing ear (*akoz*) and the thing heard (*akoumenon*), without the intervention or mediation of any other element, any *tertium quid*, as it were. It is this fact of sensation that sets music apart from the visual arts, for example. For the eye, as Plato points out, in order to see the visual object in all its colors and forms must have a *tertium quid*: light:[67]

> But there is this need in the case of sight and its objects. You may have the power of vision in your eyes and try to use it, and colour may be there in the objects; but sight will see nothing and the colours will remain invisible in the absence of a third thing peculiarly constituted to serve this purpose.

In taking the position that music does not denote feelings, Hindemith and Stravinsky were expressing the view that music, in its uttermost

[67] Plato *Republic* 507D11–14 (Cornford).

depths, cannot be put into thoughts, cannot be cast into words, and cannot be portrayed in images. For music, as musicians think of it, comprehends no duality; it is not a copy of anything in the sensible universe. As Wittgenstein said in his capacity of a musician, music is one with itself; it is complete in itself; it satisfies itself (see note 5). On these grounds, music can be placed on the same footing and accepted as parallel with that which Plato called Idea.[68] On this construal, music exists beyond the reach of science and beyond the power of representation. It is not a copy of the Platonic idea, but rather, that of the musician's own will, the objectification of which is the music itself. If music is in this sense a copy only of itself, then it must exist outside the world of phenomena and out of time, annulling both but penetrating to the innermost nature of each with a distinctness surpassing that of perception itself. For all these reasons, Schopenhauer set music apart and above all the other arts, ranking it with the Platonic Ideas, as an objectification of the will:[69]

> Hence all of them [the other arts] objectify the will only indirectly, in other words, by means of the Ideas. As our world is nothing but the phenomena or appearance of the Ideas in plurality through entrance into the *principium individuationis* (the form of knowledge possible to the individual as such), music, since it passes over Ideas, is also quite independent of the phenomenal world, positively ignores it, and, to a certain extent, could still exist even if there were no world at all, which cannot be said of the other arts. Thus music is as immediate an objectification and copy of the whole *will* as the world itself is, indeed as the ideas are, the multiplied phenomenon of which constitutes the world of individual things. Therefore music is by no

[68] See note 3 on Bacchius' application of the term *eidēsis* to melody.

[69] Schopenhauer (note 35), p. 257. Commenting on Schopenhauer's insight, Bowman (note 51), p. 72, has this to add: "What art does, in Schopenhauer's view, is give us insights into the nature of the phenomenal world. Music, however, is unlike the other arts. It is a copy of the Will itself, a face-to-face encounter with the innermost nature of existence. Music thus has the power to communicate the incommunicable, to penetrate the rational veil of representation and appearances – to give us insight into truths more profound than reason can ever grasp."

> means like the other arts, namely a copy of the Ideas, but a *copy of the will itself*, the objectivity of which are the Ideas.

It was for such reasons that Felix Mendelssohn left in a letter this celebrated comment for us to ponder:[70]

> People often complain that music is too ambiguous, that what they should be thinking as they hear it is unclear, whereas everyone understands words. With me it is exactly the reverse, and not only with regard to an entire speech, but also with individual words. These, too, seem to me so ambiguous, so vague, so easily misunderstood in comparison to genuine music, which fills the soul with a thousand things better than words. The thoughts which are expressed to me by music that I love are not too indefinite to be put into words, but on the contrary, too definite.

Music thus reveals itself literally *in* itself, not as another object of sense which it resembles. It is not *a* melody that one hears, for example, as that which opens the second movement of Brahms' *Double Concerto*, Op. 102; what one hears is *the* melody that opens the second movement. Because such aesthetic considerations as these were self-evident to experts like Hindemith and Stravinsky, they evinced frustration (as exampled above) and signs of exasperation at having to speak *about* music as though it needed any explanation at all. If the tone of their words is acerbic, it is no less so than that of Philodemus of Gadara

[70] This statement comes from a letter of 1842 written by Mendelssohn in reply to Marc-André Souchay, who had asked Mendelssohn the meaning of certain of his *Songs Without Words*. See *Composers on Music*. ed. Sam Morgenstern, pp. 139–40. It is appropriately cited by Roger Scruton, *The Aesthetics of Music*, p. 165, in connection with the assertion of the Viennese Critic, Eduard Hanslick in his *Vom Musikalisch-Schönen* (*On the Musically Beautiful)* that music cannot express any emotions at all. Cf. the statement to the same effect by Stravinsky (see note 66). In urging this opinion, Hanslick managed to set in motion 150 years of theoretical objections and counterobjections. As Bowman (note 51), p. 141 explains in behalf of Hanslick: "Although musical experience may of course be feelingful, such feeling is neither the source of musical beauty, nor is it music's content."

(first century B.C.),[71] but for an altogether different reason: music had no "essential being" for Philodemus. As he argued, if music had any function at all, it was strictly for amusement: its business was solely to titillate the ear, without ever engaging the mind. That being the case, Philodemus considered music to be an irrational (*alogos*) kind of art without any intellectual content. Thus, the moment he began to consider the question of objective meaning in music, such as Cooke claimed to decipher (see note 57), he was able to settle the case once and for all, and he made a thorough job of it. With two axioms in place, he disposed of the question altogether. His first axiom was: no two (or more) people will be moved in the same way by the same composition; his second axiom: the expressive content of music is an illusion of the mind, and nothing more.

[71] Philodemus, a poet and philosopher living during the time of Cicero (106–43 B.C.), is noted for his criticisms of the theory of musical ethos and its educational value that was first expounded by the teacher of Pericles, Damon (fifth century B.C.) and closely adhered to by Plato in the *Republic* (398E–399C4). This Damonian theory, which Plato so vigorously promulgated, held that music has intrinsic properties to affect the listener for good or ill. The most influential work on this whole subject continues to be that of H. Abert, *Die Lehre vom Ethos in der griechischen Musik* (on Philodemus, see pp. 27–37). Disputing this Damonian theory, Philodemus argued, often with excessive vituperation, that melody has no independent meaning at all. His arguments to this effect are examined by L. P. Wilkinson, "Philodemus on *Ethos* in Music," *CQ* 32 (1938). 174–81. Cf. Bowman (note 51), pp. 138–39. The range of Philodemus' writings, which now are largely lost, was extensive: rhetoric, aesthetics, logic, ethics, and theology. Through his writings, he influenced the most learned men of his day. Several charred rolls of papyri were discovered in the eighteenth century at Herculaneum among the ruins of what had been an elegant villa belonging to Philodemus' friend, Piso Caesonius. Among them was Philodemus' treatise *On Music* in four volumes, of which only fragments of Books I and II remain and some large sections of Book III and almost all of Book IV. The text is provided by Johannes Kemke, *Philodemi de musica librorum quae exstant* (Leipzig, 1884). On the numerous studies on Philodemus, see Warren D. Anderson, *Ethos and Education in Greek Music*, p. 278, n. 13. These studies have been importantly supplemented by Annemarie Jeanette Neubecker, *Die Bewertung der Musik bei Stoikern und Epicureern* and, by the same author, Philodemus' text and German translation and commentary in *Philodemus, über die Musik IV Buch*. His work on music has not as yet been translated into English.

Philodemus thus ridicules the whole idea that music can affect the listener for good or ill, and he does so on grounds that are not unlike the views expressed (see note 54):[72]

> . . . not all people will be moved in the same way by the same melody, any more than a melody can make a soul gentle that is moved by frenzy and set it at rest; nor can a melody alter or turn the soul from one passion to another, or augment or diminish its natural disposition.

Speaking now as if he were a Stravinsky, Philodemus has this to say about melody:[73]

> Some [like Diogenes] allege that one type of melody is stately, noble, sincere, and pure; while another kind is unmanly, vulgar, and illiberal; others term one kind of melody austere and imperious, but another kind gentle and persuasive. Both parties impute to melody things that do not inhere in it at all.

Then, as if in direct response to Cooke who, it will be remembered, urged musicians to treat music not only as a language, but also to work at deciphering its (for him) explicit meaning and content, Philodemus impugns any such enterprise on the following grounds:[74]

[72] *De musica* I. col. IX. 65, frag. 3 (Kemke, 12. 4–14). Commenting on statements such as this by Philodemus, Wilkinson (note 71), 174, says: "As is well established, Philodemus was not an original thinker, though he has a tone in controversy which is all his own. He may be relying here on Epicurus' lost work [*On music*, for which see Diogenes Laertius X, 28]; very probably also on his master, Zeno of Sidon."

[73] *De musica* IV, col. II (Kemke, 64. 19–30). As Wilkinson points out (*ibid.*): "His [*sc*. Philodemus'] chief butt, the Stoic Diogenes of Seleucia, lived a century before him."

[74] *De musica* IV, col. II (Kemke, 64.2–65. 16). Abert (note 71) p. 28, observes of Philodemus' tone: "We have a polemic before us, which is distinguished not by a systematic structure and concise composition of thoughts, but by the acerbity of his manner and, in part, by a plainly vulgar invective. It pleases the author to treat his opponents either in sympathetic tones as comic fools or as plainly insane."

> And, accordingly, the musician who is seeking after the kind of understanding by which he will be able to distinguish exactly what sorts of perceptions are involved, is seeking after an exact knowledge of things that have no reality; and what he teaches on this subject is vacuous; since no melody, *qua* melody, which is irrational (*alogon*), arouses the soul from a state of immobility and stillness and leads it towards its own characteristically natural state, any more than melody calms and sets at rest to its former state a soul that is driven to frenzy.

In making the point that meaning in music is dependent ultimately on the listener's own contributed feelings, Hindemith, it must be admitted, sounds alarmingly like Philodemus. Here is Hindemith:[75]

> If music did not instigate us to supply memories out of our own mental storage rooms, it would remain meaningless; it would merely have a certain tickling effect on our ears. . . . If music we hear is of a certain kind that does not easily lend itself or does not lend itself at all to this connection, we still do our best to find in our memory some feeling that would correspond with the audible impressions we have.

Writing as though he had consulted with Hindemith just yesterday, Philodemus counters with an even sharper distinction between what the ear perceives and what the listener makes of this common aural experience:[76]

[75] See Morgenstern (note 70), p. 482.

[76] *De musica* IV, col. IB (Kemke, 63, 44–64, 3). This is all part of Philodemus' polemic against the view held by Plato that music can affect the listener for good or ill. As Neubecker observes in her commentary on this passage in *Philodemus, über die Musik*, p. 127, while Philodemus will allow that "in the general province of sense-perception, occasionally various predispositions can exert their influence; but in the case of hearing, this holds true: since all people under various predispositions, make the same aural observations, they all feel the same sensations of pleasure or displeasure (Philodemus of course offers no reason for taking these assertions as self-evident); all dissimilar judgments of individual tonal genre, rhythms or melodies depend solely, according to Philodemus, on accustomed beliefs." Bowman (note 51), p. 138 says appropriately: "Epicureans tend to equate perception with sensation, a belief that made them rather dubious about music's potential cognitive or moral significance."

> While people who are similarly endowed in their power of perception agree not in the fact that a certain object is poor, but arrive at the same judgment if the question arises as to whether that object gives them dissatisfaction or pleasure. Indeed, given these matters, it is possible that our perceptions correspond to certain variations in our predispositions; but, on the whole, when it comes to our sense of hearing, there is no difference between us. On the contrary, all ears get the same impression from the same melodies, and they derive virtually the same pleasures from them, so that even when the question concerns the Enharmonic and the Chromatic [genera], the difference between them is not perceived on the basis of aesthetic perception – which is irrational – but on the basis of aural opinions.

To Philodemus, who offered even fewer concessions than did Hindemith, the sense of hearing is itself an act of judgment: noting by ear the differences between the Enharmonic and Chromatic genera, for example.[77] As Philodemus saw it, the ears of all listeners transmit the same information. That being so, no disagreement should arise among any two or more people as to the characteristic features of Enharmonic or Chromatic melodies. Aesthetic feeling, however, because it is in Philodemus' estimation irrational, cannot come into play in this kind of discrimination; it is the ear alone which records melodic information, and it is the ear alone which derives pleasure from, or as Hindemith has it, is "tickled" by certain melodic proceedings. But, even as Philodemus stipulated earlier (see note 72), once these melodic proceedings, or configurations, occur in a living musical context, they will affect no two people in the same way. In sum then, for Philodemus (and apparently to some extent for Stravinsky and Hindemith also) there is not any objective property in music that guarantees the validity and pleasure of the aesthetic experience; rather, the nature of the aesthetic experience is presupposed in the subjective stance of the listener. If, therefore, the supposed character of a major third (joy) or a minor third (sorrow), for

77 The three genera of ancient Greek music expressed in alphabetic notation are: Diatonic: E F G A; Chromatic: E F Gb A; Enharmonic (+ = quarter-tone): E E+ F A. These sequences can be transposed into any pitch range (*topos*) whatever, without alteration of their generic form, e.g.: Diatonic: A B♭ C D; Chromatic: A B♭ C♭ D; Enharmonic: A A+ B♭ D.

example,[78] is wholly and solely a product of our subjective state, there can be nothing that answers to it in the nature of melodies themselves; consequently, to prosecute Philodemus' argument to its conclusion, our knowledge of melodies is without an object. This means that items like major and minor thirds must be taken as mere acoustical data. Whether they be implicated in a melodic context or abstracted for theoretical or acoustical study, they can have no meaning or emotional content in themselves, no more, certainly, than that of a single note or an isolated sound. Thus, music, even when it exhibited laws such as those defining the Enharmonic or the Chromatic genera (with their characteristic major and minor thirds, respectively), constituted for Philodemus a domain whose ordinances are not inscribed in the essence of real or actual things. On his conception, music was a world without intrinsic necessity or intelligibility. His final conclusion had therefore to be: music is a subject unfit for rational knowledge.

Philodemus' observations were not a condemnation; they reflect, rather, a critic's lack of interest in the effect of music, especially if it had for him no apparent didactic purpose. For Philodemus and others like him, music was a kind of learning that cannot be readily grasped by the intellect and, on that basis, is *ipso facto* inferior to purely intellectual pursuits. Its deficiency must therefore consist in the fact that it is a non-scientific mode of discourse, for no object of musical representation can be an object of knowledge in the strict sense of the word. If, therefore, one were to ascribe beauty to the major third, for example, Philodemus would regard this not as a cognitive judgment, but simply an opinion (*doxa*). And this, he argues, is not a logical or rational determination, but

[78] Thus Cooke (note 53), p. 55, on the major third: "It would seem, then, that our major third has established itself naturally as an expression of pleasure or happiness." And on the minor third, he says (p. 57): "Being lower than the major third, it has a 'depressed' sound, and the fact that it does not form part of the basic harmonic series makes it an 'unnatural depression' of the 'naturally happy' state of things (according to Western ideas). Interestingly, it is the Enharmonic Genus (note 77) in which prominence is granted to the major third (or ditone) that Aristoxenus considered *kallistē* (most beautiful). See his *Harmonic Elements* (hereafter, *Harm. El.*) I. 23 (Da Rios, 29. 14–16). Cf. Barker, II, p. 141, n. 89.

an irrationally based aesthetic opinion, signifying that its determining ground cannot be other than subjective.[79]

By contrast, in a theory that gives precedence to the intellect, as is the case with Cooke, it is clearly the objective property of a beautiful construct like the major third, for example, that makes for its peculiar power. This property is actualized in melody only. It has nothing to do with sensuous responses to the beauties of nature, say, or to the ecstasy of love, or to the pain of grief. Indeed, as anyone listening to Beethoven's *Ode to Joy* from the Choral Symphony (itself an ode to the major third), can testify, this is a property that denotes an intellectual type of cognition, one which produces a powerfully disinterested type of pleasure. It is a state of pleasure that demands much more than a sensible intuition of the melodic property in question, however; it is one that calls instead for an aesthetic knowledge that is no less complex than pure intellectual knowledge. If this type of knowledge is therefore on a par with intellectual knowledge, as theorists like Bacchius believed, it must be because it has an object that is identical with that of intellectual knowledge, namely, the substantial reality of something that is animated by its own perfection. This is the perfection or fulfillment – what Aristotle termed *entelecheia* and Wittgenstein called *abgeschlossen* – which guarantees the total completion of what had hitherto existed only potentially in the

[79] As Philodemus argues throughout his polemic, the only conveyor of rational thought is language. Thus Abert (note 71), p. 28: "The basic elements of music, Rhythm and Melody, are, as he [*sc.* Philodemus] says, purely of an external, formal nature, and consequently, in and of themselves indifferent throughout. But since the influence of one psyche on another is possible only through the spoken word as the conveyor of rational thought, it follows that music can never be capable of that." That being the case, music, in Philodemus' view, while a source of pleasure, has as little to do with the human psyche as the art of cooking. Cf. Wilkinson (note 71), 179: "The pleasure is [only of the ear], a direct titillation of the ear in which the mind has no share, analogous to the taste of pleasant food and drink." Philodemus' most extreme positions on music were adopted and amplified by the Skeptic, Sextus Empiricus (c. 200 A.D.), who argued that the only true existent things are feelings. He says, therefore, in his *Against the Musicians* that sound, not being a feeling, but only productive of feeling, is not an existent thing. For additional discussion, see Bowman (n. 51), pp. 139–40; Barker, I, p. 462, n. 19.

mind of the composer. Bacchius identifies this suprasensible reality as melody, the "characteristic property of music."[80]

The question is, if, as Bacchius and Wittgenstein seemed to believe, the subject matter of music – the direct object of a musically aesthetic knowledge – is not in any sense factual, how can music be called a language?[81] Cooke would undoubtedly have objected strenuously to such a question. Indeed, as he insisted, how can music *not* be thought of as a language? Seen from Cooke's point of view, music is not the same as sound; but it communicates something unmistakeably real through sound. And if it makes sense, music is felt to have meaning.[82] Given these types of judgments, music seems to specify itself according to a certain system or logical principle analogous to that of language. For language, like music, is not the same as sound itself; yet it, too, communicates something unmistakably real through sound. And if it makes sense, language is certainly felt to have meaning. Above all, music and language, it is universally agreed, both originate in the human mind. Therefore, both can legitimately be held to belong to the faculty of knowledge. Moreover, both bear witness to an immediate interaction of

[80] The only true knowledge of music must therefore be of its *eidē*, the Forms of melody, and so the cause of musical *epistēmē* or knowledge.

[81] Roger Scruton (n. 70), p. 143, has stated the case in this way: "Aesthetic meaning is real but ineffable. To attempt to make it effable, is to reduce expression to representation, and therefore to lose sight of the essence of art." As for music being a form of language, he has this to say (p. 172): "It is doubtful that music conveys information as language does; but it shares with language another and equally important feature – the fact of inhabiting the human face and voice. We hear music as we hear the voice: it is the very soul of another, a 'coming forth' of the hidden individual."

[82] Bowman (n. 51), p. 168 says on this point: "Absolute expressionism's critical problem is to explain how sound-patterns with no fixed reference achieve 'meaning' and become experienced as feeling. Clearly, not all felt response is musically relevant: a given musical event cannot implicate just any meaning or feeling." Like Cooke, Bowman believes, as he goes on to say: "Meanings that are genuinely musical are the kind found in musically conversant listeners, people who are fluent in the given style and whose feelings arise directly and exclusively from the perception of events and processes objectively 'there,' in the music. But how precisely does this happen?"

this faculty with human feelings of pleasure and delight or, as the case may be, with displeasure, and even abhorrence.

All things considered, however, it is a curious fact that, as comparisons between music and language grow more sophisticated, their claims to the power of proving facts about music in particular seem to diminish proportionately. Cooke was not hampered in the slightest by this paradox; on the contrary, once he delineated his sixteen general terms of music's language, he considered himself eminently justified in inferring from them the emotional meaning of a particular piece of music. As he saw it, the fundamental character or, as the ancients termed it, the *ethos* of a particular composition must inhere in the basic laws of musical expression.[83] After working through whole series of such laws, Cooke arrived at last at results which he felt could be utilized to predict what it is that music is meant to say:[84]

> Perhaps one day, after intensive research into the various aspects of the art – acoustical, physiological, psychological, and simply musical – it may be possible, by correlating many findings, to discover *exactly* what it is that music expresses, and *exactly* how it expresses it; but if the attempt is made, it will have to be guided by the most meticulous regard for absolute truth, especially in the psychological field, where the final answer is likely to be found.

In laying down this challenge, Cooke was in quest of a logic whose common acceptance by musicians would bring forth a new science, one that would be an objective representation of what the world of music really is. The cognitive psychologist, John A. Sloboda, rose to Cooke's

[83] In his identification of tone-color, pitch-function, intervallic tensions, and other such elements as "characterizing agents," Cooke (n. 53), pp. 102ff. moved from a strictly musical aesthetic into the ancient realm of musical *ethos*, an expression for which no modern language possesses an adequate counterpart. For musical *ethos* had to do not solely with musical beauty, but much more so with musical goodness. Its main axiom is: every melody has a certain *ethos* that is able not only to represent the movements of the soul, but also to change these movements by being well-ordered (or the opposite). See Plato *Republic* 401D5-E. As Cooke puts it (p. 212): "*And musical form is simply the means of achieving that order*."

[84] Cooke (n. 53), p. 273.

challenge. In his book, *The Musical Mind: The Cognitive Psychology of Music*, Sloboda assumed the heavy task of giving a cognitive demonstration of music's relation to language. Although he may not have found the final answer that Cooke had sought, he accomplished something that is in itself a sufficient answer to the question: Is music in fact a language? For Sloboda, by examining those properties that are common to both music and language, was able, through a fine process of elimination, to show why analogies between music and language must ultimately fall apart. He begins by saying:[85]

> My view is that the linguistic analogy in music deserves serious attention; although I would make three qualifying remarks. Firstly, it would be foolish to claim that music is simply another natural language. There are many fundamental differences which cannot be overlooked, the most obvious being that we use language to make assertions or ask questions about the real world and the objects and relationships in it. If music has any subject matter at all, then it is certainly not the same as that of normal language. Secondly, this analogy can clearly be exploited in metaphorical ways, of which scientists are right to be wary (e.g. 'music is the language of the emotions'). However, this does not exclude the possibility of a more rigorous use of the analogy. Indeed, it is arguable that the most productive scientific analogies are those which fire the imagination as much as they fuel empirical and theoretical endeavors.

According to Sloboda, it is Cooke's empirical endeavor that best deals with the question: What is it in music, apart from its ability to mimic sounds like bird-calls, and so on, that makes it have meaning for us? He thus recognizes Cooke's contribution:[86]

> One of the most popular suggestions is that musical sequences somehow denote, or stand for, certain emotional states. Deryck Cooke (1959) has presented one of the most fully worked out versions of this thesis. . . . I believe that Cooke has identified a real and important component of musical meaning with his melodic analyses.

[85] John A. Sloboda, *The Musical Mind*, p. 12.
[86] Sloboda (n. 85), pp. 60–61.

Yet, Cooke's thesis does have its flaws, and Sloboda was not slow in pointing them out. To begin with, Cooke can all too easily be charged with selecting those motifs that would best support his theory. More to the point, his pairing of certain motifs with particular emotions cannot be supported by empirical investigations.[87] In addition, Cooke all too often relies so heavily on the words to which a melody is set that his own thesis is undermined by his attempt to correlate the two: the melody and the words that the melody *ought* to mean.[88]

The analogies between music and language that merit our most serious attention are those which focus upon the universal features of each, features that display something fundamental about man's intelligence. As Sloboda sees it, it was Noam Chomsky who, in discovering the supreme intelligibility buried within the manifold processes of language, not only founded the field: the cognitive psychology of language, but also prompted the formation of a new discipline: psycholinguistics.[89] One of the main tenets of Chomsky's theory is that all natural languages have, at the deepest level, the same structure, and that this structure reveals something universal about human intelligence. Sloboda thereupon calls our attention to a striking similarity between Chomsky's revelation about language and that of Heinrich Schenker (1868–1935) about music. For Schenker claimed that all musical masterpieces have, at the deepest level, the same type of structure – a triadic *Ursatz* – and that this structure reveals something universal about man's musical intuition. In Sloboda's words:[90]

[87] Sloboda (note 85), pp. 62–63.

[88] Thus Scruton (note 70), p. 204: "Cooke's interest is aroused primarily by word-settings, since these seem to him to settle unambiguously the question of what the music *ought* to mean. And if the setting is successful, it really does mean what it ought. Cooke's examples are therefore of striking and powerful works, in which a verbal message is conveyed with maximum musical force. This might lead the reader to be sceptical of the claim that Cooke has isolated a genuine vocabulary: a set of phrases and gestures which have a standard meaning for those who are competent to deploy them, regardless of any accompanying text."

[89] Sloboda (note 85), pp. 11ff.

[90] Sloboda (note 85), p. 12.

> There are some striking parallels between Schenker's views on music and Chomsky's on language. For instance, Schenker would wish to claim that, at a deep level, all good musical compositions have the same type of structure, and that this structure reveals to us something about the nature of musical intuition. There is no evidence to suggest that Chomsky knew of Schenker's work at the time of formulating his theories of language. It appears, therefore, that similar ways of viewing language and music arose quite independently in two creative intellects steeped in their own subject matter.

The parallels that Sloboda finds between Schenker's approach to music and that of Chomsky to language are indeed striking; but even more striking and, indeed, more revelatory are the differences between them that emerge from Sloboda's comparisons. These differences arise not so much from the ways in which these two creative intellects framed their respective theories as they do from the different orders of truth by which the two theories are dictated. What Sloboda's comparisons end up showing all too clearly is that music and language are by nature two distinctly different realities, each one in no sense a plurality of many unrelated things, but each a true universality of many related things. As Sloboda explains, on Chomsky's approach the deep structures that lie beneath the surface of a sentence can, by the application of certain rules of transformation, actually generate new sentences or surface structures that bear the same meaning as that embedded in the deep structures. Chomsky's method is in this sense a genuinely generative one. It exposes for talking human beings the most fundamental and universal of human sciences: grammar.

Schenker, by contrast, closely following the clues given him by his musical instincts, burrowed step by analytical step, like a musical archaeologist, deep below the surface of melodies down into their tonal bedrock. There he discovered the primal tonal repositories, the harmonic armatures, the deep structures that underlie all good melodies. These deep structures are what remain after a successive process of reductions through middleground structures lays bare the *Ursatz*. The original melody is now scrubbed bare, as it were, of all its surface features. From what remains of the melody – its tonal bedrock – the ear can be expected to acquire sufficient tonal consciousness to find its way through the complexities of all the surface structures by which melodies

explain themselves. It is at this point that Chomsky's approach and that of Schenker lead to diametrically opposite results, thus demonstrating explicitly why analogies between music and language must break down. For although Chomsky's deep structures can be enlisted to generate new sentences bearing the same meaning, Schenker's *Ursätze* cannot by any amount of transformations ever succeed in generating new melodies bearing the same "meaning." Roger Scruton has put the case succinctly:[91] "The deep grammar proposed by Schenker turns out, in fact, to be the surface grammar of classical music." In other words, the melody is, as Wittgenstein maintained (see note 5), a form of tautology, since its 'meaning' is imparted only and solely through the entirety of its surface details.

Thus, for example, the melody of Beethoven's *Ode to Joy* and that of the opening to the second movement of Brahms' *Double Concerto* are reducible by Schenkerian analysis to the same triadic *Ursatz*: D F♯ A. But no one in his right musical mind would mistake the one melody for the other, despite the fact of their identical deep structures. For these melodies are, like ideas and concepts, to be found not in their deep structures, but in the mind. They start in the mind and are as individual as the individual mind that created them.[92] Feeding as they do on our sensations of pain, pleasure, grief, or joy, melodies such as these manage somehow to represent qualities and sensibilities better than the words to grace them can. Sloboda succeeded in showing, then, that the unmatched power of a melody is but a concrete expression of its supreme mystery: its meaning is conveyed not by its deep structure, but by the details of its prismatic surface.[93] It is in these details only that the mind can surmise or actually experience what it is that melodies "mean." No analytic probe can extract these "meanings"; for this, there is required a deeply intuitive act, a kind of éclat or, as one

[91] Scruton (n. 70), p. 318.

[92] Maconie (n. 52), p. 11.

[93] Thus Sloboda (n. 85), p. 16: "If the *Ursatz* determined the significance of music, then there would be very little for composers to say. Much of the "meaning" of any given piece of music is given to it through the actual surface details. In language it is different. The significance of a sentence, in practical discourse, lies almost entirely in its deep structure."

ancient theorist called it, a *zopyron*, a "flash" of knowledge that defies all analysis.[94] Although Schenker's *Ursätze* are open to the charge of arbitrariness, in postulating them, he did expose for singing human beings the most fundamental and universal of human attributes: musical intuition.

Whereas the "meaning" of a melody inheres in its consummate surface, that of a sentence lies in its deep structure. This sentence, for example, is deeply structured as Noun – Verb – Object: "John phoned Amy." Its meaning remains unambiguously the same even when its surface details change: "John phoned up Amy"; "John phoned Amy up"; "Amy was phoned by John"; "Amy was phoned up by John."[95] In other words, the deep structure of a sentence such as this carries a meaning which can be maintained through transformational changes in the surface structures of actual sentences. But it is ultimately in its deep structure that the meaning of a sentence lies. With a melody, however, the condition is so profoundly reversed that it abrogates all and any analogies between speech and song. The melody which Beethoven set to Schiller's *Ode to Joy*, for example, is an edifice built on the structural pillars: D – F♯ – A. These notes constitute Schenker's *Ursatz*, the deep structure, the distilled essence of the work's tonality. But without the mind of Beethoven to build upon them the surface details of the *Ode to Joy*, they must remain frozen in place, the product of an analysis from which the unique 'meaning' of Beethoven's melody can never be extracted.[96]

[94] The ancient theorist referred to here is Ptolemaïs of Cyrene (for whom, see Chapters 6 and 7), the type of woman whom Juvenal would have characterized as *Rara avis in terris nigroque simillima cygno* (the rarest bird on earth, unique as a black swan). The phrase occurs in Juvenal *Satire* 6. 165. This is the only compliment paid to a woman by Juvenal, but he immediately nullifies it by saying that such a woman would have been so "proud of all her virtues as to be quite intolerable." See Gilbert Highet, *Juvenal the Satirist*, p. 92. Ptolemaïs was, it is most certain, as rare as a Black Swan, but one who showed no consciousness of her own uniqueness.

[95] Cf. Sloboda (note 70), p. 14.

[96] For a comprehensive examination of the distinguishing structural features of melody, see Kivy, *Music Alone*, pp. 124ff., in a chapter aptly titled "Surface and Depth." Here, Kivy shows how resistant Beethoven's "Ode to Joy" theme is to

The predilection to see in music a form of language – to make the objects of musical knowledge conform as closely as possible to those of linguistic knowledge – has produced endless controversy and has engendered an ongoing series of unresolvable questions. All too naturally, these questions aggregate on the border-line that divides speech from song. It is, to be sure, an almost imperceptible line on occasion; yet the moment one crosses it in either direction, a universe is disclosed unlike any other, one that is formed on principles peculiar to itself alone. In failing to distinguish the formative principles upon which music or, conversely, on which linguistic laws are based, we are destined to become immersed in irreconcileable difficulties, the most obvious of which is this: There is nothing in reality to answer to melody. To put it more precisely, if a melody is not something real in the things of the world, how can a likeness be found between it and our words for things of the world? It is a problem as old as language and as immemorial as music.

"deep structure" analysis. What he has to say applies to all other such masterpieces, p. 135: "The only criterion that will do the job – the one indeed which we must assume to have been employed, implicitly, to be sure – is simply that of removing or adding any note whatever until you get what you have already decided is there."

2 We Are All Aristoxenians

> Caught in that sensual music,
> All neglect monuments of
> unageing intellect — W. B. Yeats, *Sailing to Byzantium*

HAD HE BEEN AS LITTLE REGARDED IN HIS OWN DAY AS HE IS today, Bacchius would surely not have had conferred upon him an epithet of good consent, if not affectionate esteem: *Geron*, the "Old Man." In antiquity, certainly, as in some few segments of society now, "The Old Man," like *Der Alter*, was a term reserved for persons of high repute and venerable wisdom.[1] One would like to believe, therefore, that Bacchius was recognized by the musicians of his day for what he represented to them: a repository of ancient musical knowledge. Bacchius was, in fact, a pure logician of music, one who had nothing to do with problems pertaining to mathematics, science, or to any of the other arts. His subject-matter was music and everything pertaining solely to music. This "everything," as he made clear, involved the most vital source of

[1] As Aristotle points out in *Nicomachean Ethics* VI. xi. 6, there is a kind of wisdom that comes with the experience of age: "Consequently, we must pay heed to the unproven assertions and opinions of experienced and elderly people or of prudent people no less than we do to those that are provable; for it is from having experience that the eyes of the elderly see things correctly." In the case of Bacchius, it was apparently his age and experience that gave him the ability to hear things correctly. Bacchius has been translated into English by Otto Steinmayer, "Bacchius Geron's *Introduction to the Art of Music*," *Journal of Music Theory* 29 (1985), 271–98.

musical knowledge: melody.[2] In this very special and restricted respect, Bacchius belonged to a tradition that was founded by Aristoxenus of Tarentum (375/360–after 320 B.C.), the most original and penetrating musical theorist of antiquity. It was to Aristoxenus' method and philosophy that Bacchius paid tribute in his *Introduction to the Art of Music*, down to the last detail, now and then adding material that is not to be found in the extant works of Aristoxenus himself.[3] Following the path charted by Aristoxenus, Bacchius assembled all his musical facts from one source only: the basic truths of musical experience.

The question was, and continues to be, what are the basic truths of musical experience and how is musical knowledge to be built up from them? These are the questions with which Aristoxenus and his followers grappled. They began with the assumption that music is an organic value worthy of study for its own sake, that its existence relates to nothing beyond itself, and that its meaning is self-evident to the musically intuitive mind. This strictly musical approach is a notable example of a potent and vigorous reaction against Pythagorean harmonics, that mathematically grounded doctrine which has dominated musical thinking since the day Pythagoras discovered those musical ratios that hold true to

[2] Melody, as Bacchius explains in his *Introduction I*. 1 (Jan, 292. 9–11), arises from two sources only: (1) the highs and lows of musical pitches and the spaces between them, this constituting its nature (*physis*); (2) the use to which composers put these elements, this constituting the art of composition (*melopoeia*). The point has often been made by scholars that for the ancients melody literally meant music, as their music was strictly homophonic. Understandably then, Pratinas of Phlius, a poet-musician of the 6th–5th century B.C., and composer of dance music (*hyporchēmata*), wrote, with a music-maker's enthusiasm: "It is song that the Pierian muse has enthroned as queen" (Athenaeus *Deipnosophists* xiv. 617D). The study of harmonics had nothing to do with "harmony" in the modern sense of the term, but rather, with the proper "fitting together" (*harmonia*) of those elements detailed by Bacchius and other theorists. The sense of *harmonia* and its etymology are examined fully by Edward Lippman, *Musical Thought in Ancient Greece*, pp. 3ff.

[3] Jon Solomon, one of the few modern scholars who have examined Bacchius' treatise in any depth, discusses its contribution to, and departures from, Aristoxenian theory in his "*EKBOLE* and *EKLUSIS* in the Musical Treatise of Bacchius," *Symbolae Osloenses* 55 (1980), 11–26. Cf. Chapter 1, note 2. As for melody, it is, as Kivy, *Music Alone*, p. 82, remarks, "almost music itself."

the present moment: 6 is to 8 as 9 is to 12.[4] The reaction to Pythagorean doctrine on the part of theorists like Bacchius was not merely a rejection of a mathematically based harmonics but, rather, a positive assertion that the practicing musician's ideas of melody in action are better, freer, and more real, because they correspond more systematically to the actual phenomena of living music than any laws of mathematics can realize. Paradoxically enough, then, music, out of whose rich veins the harmonic ratios had been quarried, was never to reveal its true nature in any of these mathematical extrapolations. This, as Aristoxenus showed, is because the truths of music's nature do not correspond with the mathematical extrapolations by which the phenomena of physical nature can be explained. The truths of music's innermost nature cannot, therefore, take the form of such mathematical extrapolations.

Although Aristoxenus' accomplishments in musical theory and aesthetics were prodigious, his name is almost as unfamiliar today as is that of Bacchius. Yet, he was the first to intuit music's essence, the first to discover the universal laws governing its structure, the first to set out a genuine musical logic and, as is argued in Chapter 5, the first to devise and employ a workable musical temperament. He was led to these discoveries on the strength of two concepts which he derived from the teachings of Aristotle: musical space is homogeneous; musical functions are invariable. To support these fundamental assumptions, he relied on the testimony of the musically educated ear. In showing that the nature of these musical concepts was dissimilar to the spatio-temporal existing things of actual nature, Aristoxenus revealed a fundamental tension between the infinity of musical space and the relative notion of the infinitesimal, as correlative to the musical note that is assumed to be finite. It is these revelations that make his theory so applicable to the concerns of present-day musicians.

Unlike Pythagoras, Aristoxenus had no adherents approaching the stature of a Plato, a Euclid, or even a Nicomachus, to promulgate his doctrine. To be sure, Bacchius transmitted Aristoxenus' teachings with clarity and fidelity to the master's methods. But, like other Aristoxenians, he was not up to the far greater task of relating Aristoxenus' theory to the Aristotelian model on which it was grounded – in particular, to

[4] Cf. Mathieson, Chapter 1, pp. 4ff.

Aristotle's version of infinite divisibility and to Aristotle's method of approximation in dealing with incommensurable magnitudes. This is true also of one Cleonides, who is known to us in name only, and even this has been a matter of scholarly dispute.[5] Cleonides was a thoroughgoing Aristoxenian who, like Bacchius, knew how to formulate the schemata and scale-systems of ancient music in a manner wholly consistent with the teachings of Aristoxenus. He also performed the great service of transmitting material that is not found in the extant writings of the master himself.[6] But Cleonides was no philosopher. His aims having been limited by his own capacities, Cleonides succeeded simply in composing a school-text of an elementary sort; this text has come down to us complete. Cleonides spoke the language of Aristoxenus, but he made no attempt to explain the musical significance of the words he recorded. Thus, Cleonides wrote what he knew, but he seems to have known little of what lay behind the words that he wrote. A case in point is that from his *Introduction*, Chapter 2, in which he speaks of tension (*epitasis*) and resolution (*anesis*), these being the conditions, according

[5] Full references on the question of authorship and the value of Cleonides' treatise are provided by Thomas J. Mathiesen, *Apollo's Lyre*, pp. 366–90. Cleonides' treatise was translated into English by Oliver Strunk, *Source Readings in Music History*, pp. 34–46, who characterized Cleonides as "an abbreviator and popularizer of Aristoxenus" (p. 34). More recently, Cleonides has been translated into English by Jon Solomon, "Cleonides: *EISAGOGE HARMONIKE;* Critical Edition, Translation, and Commentary" (Ph.D. dissertation, University of North Carolina–Chapel Hill, 1980), 162–74. M. L. West, *Ancient Greek Music*, p. 5, sums up the status of Cleonides in these succinct terms: "Cleonides' lucid handbook, formerly misattributed to Euclid, is the most purely Aristoxenian."

[6] Most interesting by far is what Cleonides contributes to the concept of *ethos*, or melodic "character," since most of Aristoxenus' own evidence on the subject is lost. Cleonides' treatment of *ethos* in his *Introduction to Harmonics* 13 (Jan, 206. 3–18) has to do with modulation (*metabolē*) from one *ethos* to another. He refers here to three types of *ethos*: *diastaltic* (dignified, manly, and elevated); *systaltic* (dejected, depressed, and unmanly); *hesychastic* (calm, serene, and peaceful). For full discussion and references, see Jon Solomon, "The Diastaltic Ethos," *Classical Philology* 76 (1981), 93–100; Barker, II, p. 432, n. 150; Mathiesen, *Apollo's Lyre*, pp. 388–89.

to Aristoxenus, that make for the difference in highness and lowness of musical pitch (*tasis*):[7]

[7] Cleonides' *Introduction to Harmonics* 2 (Jan, 181. 7–11). Everything that Aristoxenus had to say on the phenomena of tension and resolution confuted all that Euclid and the Pythagoreans maintained with respect to the behavior of strings under tension. For whereas Euclid refers the height or the depth of pitch to a comparable increase or decrease of vibratory motion (for which, see Chapter 4, pp. 141ff.), Aristoxenus maintained that pitch emerges only on the completion of motion, while it is the process leading to the pitch that is construed by him as motion. In other words, Aristoxenus' analysis of tension and resolution had only to do with the pitches produced by the string under tension or those of the human voice. It was thus that Aristoxenus could conclude that the voice *moves* in producing an interval, but that it *stops* in order to produce a musical note. See C. W. L. Johnson, "The Motion of the Voice, τῆς φωνης κινησις, in the Theory of Ancient Music," 44–45. On the eighteen functional notes to which Cleonides referred, see Fig. 1, which depicts the basic scale system.

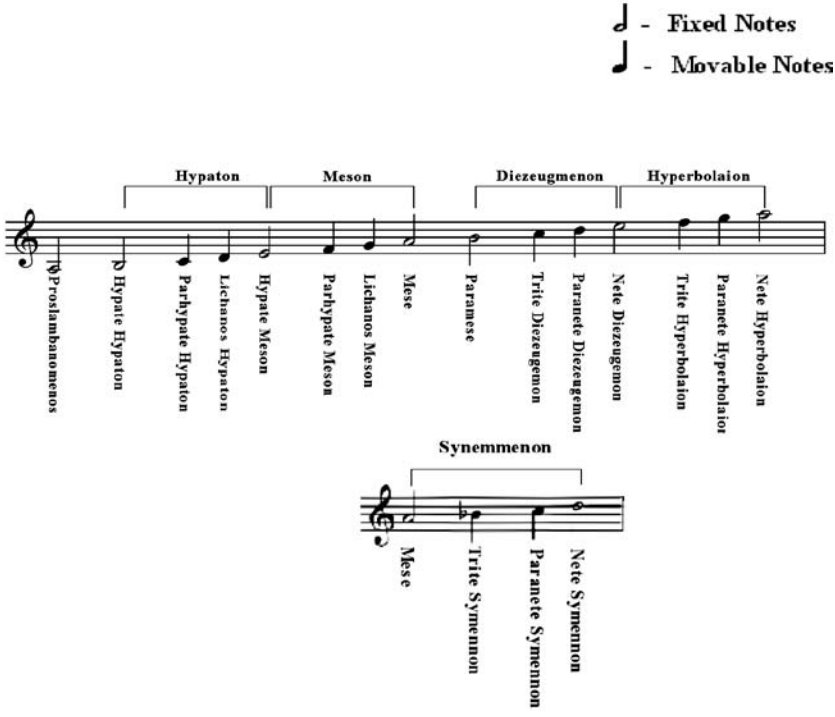


FIGURE 1. The Immutable or Changeless (*Ametabolon*) System

> Pitches are also called notes; we call them pitches because of plucked instruments having their strings put under tension, but we call them notes when they are actualized by the human voice. [For being under tension is a property of both.] Notes are infinite (*apeiroi*) in respect to pitch; but in respect to function (*dynamis*) there are eighteen to each genus.

Cleonides' derivation of pitch (*tasis*) from tension (*teinein*) and his use of an Aristotelian term (*energein*) to mean the "actualization" of notes by the voice raise a number of issues that are pivotal in Aristoxenus' theory of melodic motion: the causal relation between tension and pitch; the criterion for distinguishing a musical note in particular from the infinite possibilities of musical pitch in general; the concept of function (*dynamis*) according to which a note is assigned a musically logical status in melodic utterances.[8] Most important, Cleonides has supplied a word – *energein* – that does not appear in the extant writings of Aristoxenus, but one that explains how Aristoxenus construed the production of musical notes by the human voice, namely, as a process whereby the tensions and resolutions of the moving voice are *actualized* as discrete notes. Thus, even though *energein* (and its related form, *energeia*) does not appear in Aristoxenus' extant writings, everything that he has to say about musical notes and what is realized on their coming into being is commensurate with the meaning of *energeia*.

[8] The concept of *dynamis* is of pivotal importance not only in Aristoxenian theory, but also in all studies that focus on melody; cf. Chapter 1, note 39 on the role of *dynamis* in modulation. To modern theorists, who speak of it as tonal function, it is what distinguishes an abstract scale from a living mode by investing the latter with 'meaning.' Knud Jeppesen, *Counterpoint*, pp. 62–63, refers to it in these terms: "One could define it as the sum of melodic or harmonic motive-impulses attached to certain tones and to a certain extent tending toward the principal tone or final. The way certain tones are emphasized while others are subordinated chiefly determines the mode." And when Walter Piston, *Harmony*, p. 31, speaks of the "tonal function" of each degree of a scale, he has in mind what Aristoxenus calls *dynamis* in *Harm. El.* 33 (Da Rios, 42. 8–13): "Our theory, taken in its entirety, is concerned with melody, whether it is sung by the voice or played by instruments. It is determined according to two things: the ear and the intellect (*dianoia*). We judge the magnitude of intervals by ear; but we observe the functions (*dynameis*) of these intervals with our intellect." This, and other such passages, are central to Aristoxenus' theory of melodic collocation. Cf. Barker, "Aristoxenus' Theorems," 25: "A note's δύναμις determines what other notes it can follow and precede."

For this word, as Aristotle explains it, is associated with the notion of fulfillment and has especially to do with motion:[9] "For actuality (*energeia*) appears in particular to be a movement."

Another Aristoxenian theorist is Gaudentius, a writer of uncertain date and a figure as obscure as Bacchius.[10] Gaudentius not only wrote a treatise entitled *Harmonic Introduction* (*Harmonikē Eisagōgē*), but also won for himself the title "Philosopher" for having done so. He was an important source for Cassiodorus (b.c. 485 A.D.) who, with Boethius (c. 480–524 A.D.) was one of the two great intermediaries between the music of antiquity and that of the Middle Ages. Gaudentius has much to offer that is of more than passing interest.[11] He begins his treatise engagingly enough with an epigraph:[12] "I sing for the educated; you who are profane do close your doors."

Gaudentius introduces the subject by stressing what Aristoxenus considered to be a vital prerequisite for the application of his theory: a trained ear.

Thus Gaudentius:[13]

> One might rightfully begin by touching upon the harmonic proportions. For these are the elements that have to do with notes and intervals and systems, as well as with keys and modulations and melodic composition in all

[9] Aristotle, *Metaphysics*, 1047a30–32.

[10] Mathesien, *Apollo's Lyre*, pp. 498–509, has compiled everything that can be known of Gaudentius, together with complete references to the editions and translations (as by Oliver Strunk) to his treatise.

[11] This is especially the case in Chapter 8 of his treatise (Jan, 337. 5 – 338. 7), where he distinguishes between concordancy (*symphonia*) and discordancy (*diaphonia*), as well as between homophonic intervals, or isotones which are of the class of concords, and paraphonic intervals (*paraphonoi*) which, he says, fall between concords and discords. The examples that he gives of paraphonic intervals are the tritone (as between F and B♮) and the ditone (as between G and B♮). Mathiesen, *Apollo's Lyre*, p. 502, observes of Gaudentius' paraphonic intervals: "This is a remarkable definition on several counts: first, Gaudentius is clearly referring to intervals of a tritone and a ditone; and second, such intervals might have been commonly used in instrumental accompaniments." For, as Gaudentius said, these intervals (Jan, 338. 4–5) seem to be concords when played (literally, plucked) on strings.

[12] In his note to this epigraph, Jan, p. 327, observed that it bears a striking resemblance to an Orphic hymn quoted by the Bishop of Laodicea, Apollinarius (c. 360 A.D.).

[13] Jan, 327. 4–20.

> the genera of attunement. But he who would pay attention to the proportions in all these areas must first train his ear by experience to hear notes accurately and to judge of intervals as to their concordancy and their discordancy, so that he may make use of the proportion that is compatible with his perception of the unique properties of the notes, applying a full-scale scientific knowledge that has now been augmented by trial and rational thought. But he who is to hear the proportions without heeding the note or without training his ear, let him depart, since he has closed the doors on his sense of hearing. For he will stop up his ears, being unable to apprehend by ear those things to which the proportions relate. Let us begin then by speaking of those aspects of sound that are given through empirical training.

From this point on, Gaudentius provides a full-scale and systematic course in Aristoxenian theory, pausing now and then to explain a technical term, or to describe in some detail how the ancient notational characters were formed.[14] In Chapters 10 and 11, however, he departs from this program by introducing into the Aristoxenian context the story of Pythagoras' discovery of the harmonic ratios, adding to this material his own numerical interpretations.[15] Apart from this curious digression, Gaudentius adheres to the Aristoxenian theoretical framework, thus complementing the work of Bacchius and Cleonides.

[14] Thanks to the otherwise unknown Alypius, the Greek musical notation (*parasēmantikē*) has been preserved in his treatise, *Introduction to Music* (*Eisagōgē Mousikē*), the text of which appears in Jan, 367–406. Thanks also to the work of the mid-nineteenth century German scholars, F. Bellermann, C. Fortlage, and R. Westphal, the Greek signs and symbols have been correlated with those of modern notation, so that all the scales and fragments of Greek melodies can be faithfully transcribed. For a full account of Alypius' production, the reader is referred to Mathiesen, *Apollo's Lyre*, pp. 593–607. The place of the Greek system in the history of notation can be found in C. F. Abdy Williams, *The Story of Notation*, pp. 6–42. The fifteen keys (*tonoi*) with their ancient notation are presented by Macran, pp. 46–61. Cf. Fortlage, *Das Musikalische System Der Griechen*, pp. 27–34.

[15] Instead of using the more usual Pythagorean ratios – 6:8 :: 9:12 – to define the consonant intervals, Gaudentius has as his basis 12:16 :: 18:24. Moreover, he computes the octave and a fourth (24:9) as a consonant interval, whereas the more traditional Pythagorean calculation (8:3) confirms the octave and a fourth to be a discord. See Chapter 4, note 24. Cf. Mathiesen, *Apollo's Lyre*, pp. 503–4.

In his monumental *Harmonics*, Claudius Ptolemy (2nd century A.D.),[16] the great astronomer, geographer, mathematician, and musicologist, mentions his predecessors in musical theory but seldom; when he does, it is more often than not to criticize them. This is certainly the case when it comes to "the Aristoxenians," none of whom he speaks of by name. He tells only of "the Aristoxenians" who, he argues, "are wrong in measuring the concords by the intervals and not by the notes"[17] and who "are wrong in assuming that the concord of the fourth consists of two and a half tones."[18] In speaking of these Aristoxenians, Ptolemy suggests that they were both numerous and ubiquitous. But the truth is that, apart from Aristoxenus himself, those Aristoxenians who left enough writings for Ptolemy to criticize are all of a later date than that of Ptolemy. This is certainly true of Aristides Quintilianus, the most eclectic and comprehensive of all the Aristoxenians whose works are extant. As is the case with Bacchius, Cleonides, and Gaudentius, nothing is known of Aristides beyond the fact that he wrote on music. His treatise, *On Music* (*Peri Mousikēs*), consists of three volumes and is one of the three most

[16] Ptolemy's *Harmonics* has been translated into English by Barker, II, pp. 275–391, this preceded by a comprehensive introduction in which Ptolemy's contribution is thoroughly evaluated. A second translation of Ptolemy's treatise has recently appeared, that by Jon Solomon, *Ptolemy, Harmonics: Translation and Commentary*. To these works must be added that of Andrew Barker, *Scientific Method in Ptolemy: "Harmonics."*

[17] Ptolemy *Harm*. I. 9 (Düring, 19. 16–17). Whereas Aristoxenus treated pitches as individual points on a melodic continuum, which he construed to be infinitely divisible, Ptolemy considered the difference between pitches to be intrinsic to the sounds themselves. Cf. Barker, II, p. 293, n. 81.

[18] Ptolemy *Harm*. I. 10 (Düring, 21. 19–20). This was a common complaint leveled by the Pythagoreans against Aristoxenus. For, as they argued, there is, mathematically speaking, no proportional mean between two distances on the line that are in the ratio 4:3, the ratio of the fourth. And this means that the semi-tone or half of a whole tone is not mathematically expressible in whole numbers. According to Barker, II, p. 295, n. 86, Ptolemy's point is well-taken: "Two notes forming, for example, a concord must therefore have distinctive properties (their 'quantities') between which the relation holds. They cannot be featureless points, and the concordance cannot be a property inhering in the empty distance between them." It is on this issue that Aristoxenus and the mathematicians are worlds apart. See Chapter 4, note 39.

influential in the field (the other two being those of Aristoxenus and Ptolemy). According to Thomas J. Mathiesen, the first scholar to have translated Aristides' treatise into English, the style and substance of the writing suggest that "Aristides Quintilianus was active sometime in the late third or early fourth century A.D."[19]

When it came to the promulgation of his doctrine, Aristoxenus was at a most serious disadvantage. For, in addition to the unfortunate fact that his own writings have reached us in an incomplete state,[20] there seems to have been no one among his adherents who was musician enough to understand the significance of his concepts, or philosopher enough to grasp the logic of their content. Yet he was for centuries quoted by some of the most learned and celebrated minds of antiquity, among them: Athenaeus of Naucratis, Plutarch of Chaeronea, Porphyry of Tyre, Vitruvius Pollio (the Roman architect), Cicero, and Boethius, to mention only a few.[21] He was catechized (as by Bacchius

[19] See Thomas J. Mathiesen, *Aristides Quintilianus: On Music in Three Books*, p. 14. This work has been deeply analyzed by Barker, "Aristides Quintilianus and Constructions in Early Music Theory," *CQ*, 32 (i.), (1982), 184–97, and translated by him in Barker, II, pp. 399–535.

[20] The treatise designated in most of the manuscripts as "The Harmonic Elements of Aristoxenus" has come down to us in three books. That it has been compiled from as many as three or even four separate works has been suggested by scholars on the basis of various (perceived) inconsistencies, repetitions, and omissions in its treatment of the subject. The first book defines the scope of harmonics and its subsidiary subjects; the second redefines it, establishing the principles (*archai*) from which its laws are deduced; the third comprises theorems and proofs in the manner of Euclid's *Elements*, breaking off abruptly in the course of examining the species of the fourth. Scholarly opinion on the problem of the work's compilation from a multiplicity of treatises is discussed by Macran, pp. 89–92. The whole question is given penetrating analysis by Da Rios in her *Prolegomena*, pp. cvii–cxvii. For a detailed chapter by chapter programming of the treatise, see Barker II, pp. 120–25 and Mathiesen, *Apollo's Lyre*, pp. 294–344. An argument in favor of a unified work has been offered by Annie Bélis, *Aristoxène de Tarente et Aristote*, pp. 34–36 that is strong enough to make everyone rethink his position on the subject.

[21] Everything that was said about Aristoxenus by the ancients, together with all the fragments from his work that were preserved by them, can be found in the collection assembled by Fritz Wehrli, "Aristoxenus" in *Die Schule des Aristoteles*.

Geron), and compiled (as by Cleonides); he was diagrammed (as by Nicomachus),[22] he was mathematicized (as by Ptolemy);[23] he was stylized (as by Aristides Quintilianus),[24] he was put into one context – the Pythagorean (as by Nicomachus)[25] – and taken out of another – his own

This material has also been compiled by Da Rios in her edition of Aristoxenus under *Testimonia*, pp. 95–136.

22 Nicomachus *Manual*, ch. 12 (Jan, 264. 8–40) and *Excerpta* 9 (Jan, 280. 12–281. 1–17). These diagrams with their ancient notational symbols are reproduced with modern notation by Levin, *Manual*, pp. 175–76; p. 196. Bacchius' treatise is one of three ancient works on music that employ the question and answer format of a catechism. The two others are the nineteenth book of the Aristotelian *Problems* and *The Pythagorean Doctrine of the Elements of Music* by Ptolemaïs of Cyrene, of which only a few fragments remain. Ptolemaïs stands out amid the specialists in music not only for being a woman – a rarity indeed – but, more important, for daring to criticize the Pythagoreans on fundamental grounds, something that no musical writer but Aristoxenus had undertaken to do. To be sure, Ptolemy criticized the Pythagoreans when they seemed too dogmatic to him, but he was at heart a Pythagorean himself. Of the writers mentioned here, Cleonides is the most consistently Aristoxenian. On Ptolemaïs, see Chapter 6.

23 I believe that one of the most important contributions which Ptolemy makes toward our better understanding of Aristoxenus' theory is his demonstration of the Aristoxenian measurement of musical intervals. In *Harm*. I. 13 (Düring, 29, 11–12), he explains Aristoxenus' method as one which treats intervals not as numerical ratios, but as tonal distances on the line of pitch: "He [*sc*. Aristoxenus] divides the tone sometimes into two equal parts, sometimes into three, sometimes into four, and sometimes into eight" . . . (trans. Barker II, p. 303). The implications of Aristoxenus' method are discussed in Chapter 5.

24 Aristides' accounts of notes, intervals, systems, genera, keys, and modulation are derived almost entirely from the teachings of Aristoxenus. When Aristides says in his *De musica* I. 4 (Winnington-Ingram, 4. 18–19): "Music is the study of melody and everything pertaining to melody," he, like Bacchius, places himself squarely in the Aristoxenian tradition. On the importance of this definition, see Mathiesen, *Aristides*, Introduction, note 111; "'Melos' is a technical term that refers to the complete musical complex . . ."

25 In the second chapter of his *Manual* (Jan, 238. 18–240. 26), Nicomachus offers a detailed account of pitch attributes which is based upon a concept of motion radically different from that of the Pythagoreans. Instead of a Pythagorean analysis of motion as consisting in physical vibration, as would be expected here, Nicomachus describes it as a subjective phenomenon residing in sense-perception. This, the method of Aristoxenus, was applied by Nicomachus to

(as by Gaudentius);[26] all this and more, until the dynamic principles underlying his theory were finally oversimplified out of existence. Apart from the incomplete *Harmonic Elements* of Aristoxenus himself, all that remains of his theory can be read in the compilations and abridgements of his followers, the now unknown and long-forgotten "Aristoxenians." For all their efforts, however, they are remembered in the following terms by M. L. West:[27]

> Aristoxenus' harmonic theory was highly influential, and it is *regurgitated* in several works written probably between the second and fifth centuries AD, where lost proportions of Aristoxenus' exposition are also reflected. [italics added]

Given West's infelicitous characterization of these Aristoxenian treatises, it is a wonder that any of them were ever deemed worthy of study. West's opinion notwithstanding, these documents can be effectively mined for the nuggets of Aristoxenian wisdom that they contain. Marcus Meibom, for example, a prodigious scholar of the seventeenth century, thought it important enough to provide the Greek texts of these works together with his own Latin translations and commentaries, a contribution that to this day continues to be of great value.[28] And Charles-Émile Ruelle thought it worth his while to translate them into French between the years 1871 and 1898.[29] To these major contributions to the field must be added that of Andrew Barker, who has

something that was never its specific object – number. His reasons for doing so are discussed by Levin, *Nicomachus*, pp. 56–57.

[26] Whereas Bacchius, following Aristoxenus, refers every aspect of the study of music to melody and all that pertains to melody, Gaudentius seems to have had a different objective, one for which he has received little recognition: to reconcile the mathematically based harmonics of the Pythagoreans with the musical imperatives of Aristoxenian theory. Thus, Mathiesen, *Apollo's Lyre*, p. 503: "It is remarkable that he [*sc.* Gaudentius] presents these intervals [*sc.* octave-and-a-fourth] not only in Aristoxenian terms as the number of tones contained in each interval but also in Pythagorean terms as numerical ratios . . ."

[27] West, *Ancient Greek Music*, p. 5.

[28] Marcus Meibom, *Antiquae musicae auctores septem* (Amsterdam 1652).

[29] Each of Ruelle's translations is listed separately by Mathiesen in *Apollo's Lyre, Bibliography* under items: 1352 (Sextus Empiricus); 1011 (Nicomachus); 329 (Cleonides); 77 (Aristoxenus); 54 (Aristotelian *Problems*); 26 (Alypius and Gaudentius; Bacchius).

translated more ancient musical treatises into English than anyone else since Meibom first cast them into Latin. Barker offers this reason for his failure to include among his translations the writings of Bacchius and Gaudentius:[30]

> In mitigation I can plead that the bulk of the information they offer can be found either in Aristoxenus' *El. Harm*. itself, or in the first book of Aristides Quintilianus. ... I might add that they make dreary reading: Aristides Quintilianus, for all his faults, at least has some fire in his belly.

To make matters worse for the followers of Aristoxenus, Giovanni Comotti sees no reason for studying them at all:[31]

> It is evident that the authors of these treatises were little concerned with music as it was performed, and only wished to define the theoretical underpinnings of music in the abstract. The motives behind such attitudes can be explained by the traditions of ancient thought. But in any case a lengthy consideration of their works would not contribute appreciably to our knowledge of the history of Greek and Roman musical culture.

On the contrary. The *De Musica* of Aristides Quintilianus, to take just one example, not only contains evidence of a high antiquity, but also offers a considerable mélange of Aristoxenian material, together with some critical insights into Aristoxenus' originality of thought.[32]

[30] Barker, II, p. 3.

[31] Comotti, *Music in Greek and Roman Culture*, p. 3.

[32] In his *De mus.* I. 7 (Winnington-Ingram, 12–13), Aristides lists a set of nine ancient modes in a notation, the first line of which appears to be a set of numbers, while the rest is not to be found anywhere else. Cf. Barker, II, p. 412, n. 80. Even more provocative is what Aristides has to offer in *De. Mus.* I. 9 (Winnington-Ingram, 19. 2ff.); for here Aristides lists together with ancient notation the six *harmoniai* or modes which "the divine Plato mentions in the *Republic*" [399A]; these, the Lydian, Phrygian, Mixolydian, Dorian, Iastian, and Syntonolydian (Intense Lydian), so graphically described by Aristides, have provoked almost endless discussion and controversy among scholars. For a vigorous argument against the authenticity of Aristides' evidence, see Monro, *The Modes of Ancient Greek Music*, pp. 95–100. The evidence is treated most seriously by Landels, *Music in Ancient Greece and Rome*, pp. 103–06 and by Anderson, *Music and Musicians in Ancient Greece*, pp. 154–57, who queries the translation and commentary of Mathiesen on this passage (p. 154, n. 9).

Much of this evidence was passed on by Martianus Capella between 410 and 439 A.D. in an idiosyncratic treatise addressed to his son and entitled *On the Nuptials of Mercury and Philology* (*De Nuptiis Mercurii et Philologiae*).[33] Aristoxenian musical formulations are preserved also in the fragment known as Bellermann's *Anonymus*;[34] they show up where least expected, in a Pythagorean context, by way of Nicomachus of Gerasa;[35] they reappear intact in the treatise on harmonics by the Byzantine theorists, Manuel Bryennius and George Pachymeres.[36] Apart from the valuable anecdotal evidence of such writers as Plutarch, Strabo, Athenaeus, Vitruvius Pollio, Boethius, Porphyry, and many others of sufficient stature to guarantee Aristoxenus' fame by the mere mention of his name, Aristoxenus remains today either unknown or, where known, unappreciated for his remarkable accomplishments in music. The technical manuals, however important they may be, do not do justice to Aristoxenus' originality of thought. They are devoted for the most part to the applied aspects of Aristoxenus' theory, as distinct from the *a priori* principles of unity in which they are grounded. In sum

[33] Thus, Mathiesen, *Apollo's Lyre*, p. 523: "Aristides Quintilianus remains unmentioned by name in any datable source earlier then Martianus Capella." Capella has been translated into English by W. H. Stahl, *Martianus Capella and the Seven Liberal Arts*.

[34] These are the remains of three treatises containing vital information on rhythm and rhythmic notation, musical theory, and melodic composition, much of which is derived from Aristoxenian sources. The set has been edited, translated, and provided with a commentary by Dietmar Najock, *Drei anonyme griechische Traktate über die Musik*.

[35] See note 25. Nicomachus' motive, as suggested, seems to have been to advocate the primacy of Pythagoras and his followers in all matters concerning music.

[36] Manuel Bryennius, the tutor of the astronomer Theodorus Metochites (c. 1260–1332 A.D.), wrote a three-volume *Harmonics* sometime around 1300 A.D., and his older contemporary, George Pachymeres (1242–after 1308) wrote a *Quadrivium* (comprising arithmetic, music, geometry, and astronomy) sometime earlier. Both works are fascinating, not only for their appreciation of Aristoxenus, but also for their abundant transmissions from the works of Aristides, Ptolemy, Nicomachus, *et al.* For Bryennius, see G. H. Jonker, *The Harmonics of Manuel Bryennius*; for Pachymeres, see P. Tannery, *Quadrivium de Georges Pachymère*. These and other Byzantine works are discussed by Lukas Richter, *Momente der Musikgeschichte Antike und Byzanz*, pp. 188ff.

then, among all the Aristoxenian specialists from Aristoxenus' own day until the Byzantine era, there was no one of Aristoxenus' own intellectual capacities to argue for his aims or for his methods. Andrew Barker states the case in these words:[37]

> By contrast [with Aristoxenus' own approach], many later 'Aristoxenian' writers sought only to give a scholastic exposition of the master's 'doctrines,' and to reduce them to an academic system, neglecting the need for harmonic understanding to be grounded in real musical experience, and ironing out many of the penetrating ideas that Aristoxenus had derived from that source himself.

However dull, dreary, and derivative the writings of his followers may be thought by scholars, Aristoxenus himself, while not an especially graceful writer, is always an arresting one, particularly for musicians. For that, he deserves a fate far better than the one allotted to him by intellectual history. He should, in fact, be numbered, as he once was, among those most eminent in their field. Indeed, like "The Geometer," as Euclid was known, "The Philosopher," as Aristotle was called, and "The Astronomer," as Ptolemy was titled, Aristoxenus was for centuries almost always distinguished by the epithet, "The Musician." If today he is scarcely known at all by musicians, the cause cannot be assigned solely to the dullness or apishness of his followers' writings. Nor can Aristoxenus' loss of standing in the history of musical thought be ascribed to the "confusion with which subsequent theorists interpreted his central thesis," as Richard Norton postulates:[38]

> Hack theorists disseminated a mixture of Aristoxenian theory, casually mixed with their own speculations, until Aristoxenus was made to support the very idea he sought to defeat – the principles of pitch measurement according to atomic microtones and the authoritative basis of Pythagorean number proportions.

As matters stand today, whenever and wherever the subject of music is raised, it is, for the most part, not Aristoxenus, but Pythagoras, who is held to be the universal well-spring of musical knowledge.[39] But

[37] Barker, II, p. 5.

[38] Richard Norton, *Tonality in Western Culture*, p. 101.

[39] Lukas Richter has recently summed up the power and the dimensions of Pythagorean thought in his *Momente der Musikgeschichte Antike und Byzanz*,

this was not always the case. To Cicero, for example, who lived almost three-hundred years after him, Aristoxenus was still the ultimate and unchallenged authority in all things musical:[40]

> Quantum Aristoxeni ingenium consumptum videmus in musicis?
>
> Do we realize how great the talent of Aristoxenus was that he lavished on musical subjects?

Moreover, as Cicero saw it, Aristoxenus, like Damon before him, was to music even as Hippocrates was to medicine and as Euclid and Archimedes were to geometry.[41] And as if Cicero's words were not praise enough, Alexander of Aphrodisias (*fl.* early third century A.D.), the great commentator on Aristotle, had this to say of Aristoxenus:[42]

> θείη γὰρ ἄν τις ὡς ἔνδοξον τὸ ὑπὸ
> Ἱπποκράτους λεγόμενον ἐν ἰατρικῇ
> καὶ τὸ ὑπὸ Ἀρχιμήδους ἐν γεομετρίᾳ
> καὶ τὸ ὑπὸ Ἀριστοξένου ἐν μουσικῇ
>
> For anyone might hold as a matter of the highest importance what is said by Hippocrates in medicine, and what is said by Archimedes in geometry, and what is said by Aristoxenus in music.

Writing some two hundred years after Alexander of Aphrodisias, Martianus Capella ranked Aristoxenus with no less a performing musician than Orpheus himself, the very personification of music:[43]

> Linum, Homerum Mantuanumque vatem reditos canentesque conspiceres, Orpheum atque Aristoxenum fidibus personantes, Platonem Archimedemquem sphaeras aureas devolventes.

p. 196: "The Pythagorean philosophers conceived of number as the very essence of things (*arithmetica universalis*) and saw *harmonia*, that is, the union of opposites, realized in the whole world, in the soul of humankind and in the lawful ordering of intervals in the tonal system. The particular sequence of intervals of the individual modes determined the characteristic effect of the music which we account for as a repatriation of the soul to the cosmic *harmonia*."

[40] Cicero *De finibus* V. 18. 49 = *Fr.* 69b Wehrli.

[41] Cicero, *De oratore* III, 33, 132 = *Fr.* 69a Wehrli.

[42] Alexander Aphrodisias *Comment. In Aristot. Topica 105a34* = *Fr.* 69e Wehrli.

[43] Martianus Capella, II. 212 = *Fr.* 69f Wehrli.

> You might contemplate Linus, Homer and the Mantuan bard,
> and the wreathed singers, Orpheus and Aristoxenus, playing on their strings; Plato and Archimedes spinning their golden spheres.

Orpheus, Hippocrates, Euclid, and Archimedes, not to mention Plato, Homer, and Virgil, stand today with the most towering figures in the history of human knowledge: no one needs to be told who they are or why their fame has persisted undiminished through the ages. Aristoxenus, however, whose work in music was once considered one of the high-water marks of ancient Greek musical culture, entitling him to be ranked with the likes of Orpheus, Plato, and Euclid, et al., is today all but forgotten, even by his own countrymen and by sharers in the art of music. For, ironically enough, despite all the evidence of his once supreme eminence in music, Aristoxenus continues to be eclipsed by the figure of Pythagoras. It is a modern Greek composer then, who offers this maxim for present-day musicians to contemplate:[44]

> We are all Pythagoreans.
> Xenakis might have said more rightfully:
> We are all Aristoxenians.

For the inescapable truth is that if we claim to be musicians, then we are, at heart, all Aristoxenians.[45]

Once as well-known a name as Euclid's and Archimedes', that of Aristoxenus has been almost completely obliterated from the pages of intellectual history, and no single reason can be assigned for this penalty against musical thought and aesthetics. According to Barker (note 37), and Norton (note 38), the cause must lie with the incapacity of his followers to transmit his doctrine adequately. By contrast, Comotti,

[44] This statement by Iannis Xenakis stands as an epigraph in Jamie James, *The Music of the Spheres.*

[45] There are two things that give Aristoxenus' theory a universal significance; the first is explicit, the second must be inferred: (1) melody admits of sizeless points between which there is an alignment and among which there are centers of gravity. Though they have no size, these points have position, so sharp and so apparent that they coincide perfectly with idealized instants of time. (2) the time which music inhabits becomes by virtue of this habitation an eternal present; it is this that makes all music contemporary.

by managing to confuse the doctrine of Aristoxenus with that of the Pythagoreans – something that Aristoxenus' followers never did – contributes greatly to Aristoxenus' loss of standing in the field:[46]

> The method, formulations, and goals of Pythagorean acoustical research had a decisive influence on the direction of all subsequent speculative work in the field of music. Damon and, after him, Plato and Aristotle especially, advanced the study of the effects of music on the human soul. Aristoxenus, on the other hand, and all the scholars of the Hellenistic and Roman periods – with the exception of the Epicurean theorists – *took as the basis for their research the physical and mathematical principles embodied in Pythagorean doctrine*. [italics added]

Perhaps the most interesting reason for Aristoxenus' fall from grace is that offered by Meibom (see note 48). For by the time Meibom completed translating Aristoxenus' *Harmonic Elements* into Latin (1652), Aristoxenus had already been long since forgotten. In rediscovering Aristoxenus' writings for himself, Meibom realized that he had before him the work of the most illustrious expert in music and the chief authority on the subject (de hac disciplina scriptis celeberrimum, Musicorum principem, Aristoxenum). As Meibom explains, the cause of Aristoxenus' hard fate was twofold: the near-limitless difficulty of his theory; the ignorance of those who attempted to fathom it. He says:[47]

> Quamvis enim semel atque iterum, sua lingua ac Romana loquens, in publicum fit productus, semper tamen, ob scientiae sublimitatem, neglectus ab omnibus ad bibliothecarum angulos redire fuit coactus. Tantum potuit mali creare viro maximo fatalis nostro aevo ac superiore Musicarum litterarum ignoratio.
>
> For although he has now and again been published, speaking in his own and in the roman tongue, he has nevertheless always been forced to return to the narrow corners of libraries, neglected by all because of his having touched upon almost the highest limit of his science.[48] Only ignorance of

[46] G. Comotti, *Music in Greek and Roman Culture*, pp. 27–28.

[47] Meibom, *Antiquae Musicae*, Vol. I, Preface to Aristoxenus, entitled *Lectori Erudito*, p. 1 [pagination supplied].

[48] Meibom thought of Aristoxenus' musical theory as *sublimitas*, literally, "beneath the very topmost limit" of a science, a theory which was a genuine

> musical studies in our own and in earlier eras could have contrived for our most eminent man such a fate.

Aristoxenus' *scientia* of music is indeed as difficult as Meibom's word *sublimitatem* implies, and critics from antiquity to the present day have differed as to its proper interpretation. When Meibom charged Aristoxenus' interpreters with ignorance, however, he did not have in mind critics like Ptolemy or Theon of Smyrna or Porphyry of Tyre, for example;[49] on the contrary, he reserved this judgment for the likes of Antonius Gogavinus, whose Latin translation of 1542 he found to be completely inept, and of Johannes Meursius, whose Greek edition of 1616 he considered to evince a gross ignorance of musical theory.[50] In contrast to these Renaissance copyists and translators, whose views can be dismissed on Meibom's warrant, those of Ptolemy are substantive enough to have influenced the course of Aristoxenian studies for centuries to come. As Ptolemy saw it, the Aristoxenians were arbitrary in placing so much value on the evidence of

science insofar as it contained truths that are independently verifiable. It was in this sense that medieval humanists regarded poetry as a science to which revelatory functions could be assigned. To Meibom, the value of Aristoxenus' theory consisted in its handing down to posterity a fine grasp of the nature of music, and a full consciousness of music's technology and technical elements.

49 On Aristoxenus' ancient critics, see note 18. Theon of Smyrna (*fl.* 115–140 A.D.) wrote a work in three parts (Arithmetic, The Numerical Laws of Music, Astronomy) entitled *Mathematics Useful for Understanding Plato* (*Expositio Rerum Mathematicarum Ad Legendum Platonem Utilium*). His criticism, like that of other mathematicians, focuses on Aristoxenus' violation of the norms of mathematics by insisting that the fourth "comprises two and a half perfect tones" (Hiller 67. 10–12). Porphyry of Tyre (232/233–304/305 A.D.), originally named Malchus (Arabic Malik = King), was called Porphyrius (the regal "Purple") by his teacher, the Neoplatonic philosopher, Longinus. Porphyry's *Commentary on the Harmonics of Ptolemy* is of special interest for transmitting fragments from the work of one of Aristoxenus' rare champions, Ptolemaïs of Cyrene, for whom, see Chapter 6. Only portions of Porphyry's *Commentary* have been translated into English, by Barker, II, 229–44. A full English translation is forthcoming by Jon Solomon.

50 Cf. Macran, pp. 91–92. Da Rios, *Prolegomena*, pp. xii–xiii concurs in characterizing the edition of Gogavinus as "abounding in serious errors (cum gravissimis mendis redundet) and that of Meursis as afflicted by a "total ignorance of Aristoxenus' doctrine" (maxime Aristoxeni doctrinae omnino ignarum se praebuit).

perception (*aisthēsis*) and unscientific in treating reason (*logos*) as secondary in importance.[51] Porphyry, the commentator on Ptolemy's *Harmonics*, agreed

[51] It is on these fundamental grounds that Ptolemy *Harm*. I. 9 (Düring, 20. 11–14) attacks the Aristoxenian method. For, as he argues, the Aristoxenians [namely, Aristoxenus himself] allege that a whole-tone is perceived by the ear to be the difference between the fourth and the fifth; yet they ignore the fact that the whole-tone is in reality the difference between two notes standing in an epogdoic (9:8) ratio. In pursuing this argument against Aristoxenus, Ptolemy himself ends up in a logical cul-de-sac, because he cannot abide the fact that Aristoxenus'method of tuning by consonances reveals the consequence of an infinite number of pitches within each ratio. As Barker, II, p. 294, note 85, so aptly puts it, Ptolemy's argument here "is a fantastic muddle." On the various kinds of ratios under discussion, see Fig. 2, in which ratios are listed by name.

RATIOS

A. ἐπιμόριοι	superparticulares	superparticulare	
1. ἡμιόλιος	hemiolius	hemiolic	$1+\frac{1}{2}$
2. ἐπίτριτος	sesquitertius	epitritic	$1+\frac{1}{3}$
3. ἐπιτέταρτος	sesquiquartus	sesquiquartan	$1+\frac{1}{4}$
4. ἐπίπεμπτος	sesquiquintus	sesquiquintan	$1+\frac{1}{3}$
5. ἐφεκτος	sesquisextus	sesquisextan	$1+\frac{1}{6}$
6. ἑφέβδομος	sesquiseptimus	sesquiseptimal	$1+\frac{1}{7}$
7. ἐπόγδοος	sesquioctavus	sesquioctave	$1+\frac{1}{8}$
B. ἐπιμερής	superpartientes	superpartients	
1. ἐπιδίτριτος	superbitertius	epiditritic	$1+\frac{2}{3}$
2. ἐπιτριτέταρτος	supertriquartus		$1+\frac{3}{4}$
3. ἐπιτετράπεμπτος	superquadriquintus		$1+\frac{4}{3}$
4. τρισεπίπεμπτος	supertriquitus		$1+\frac{2}{3}$
5. τετρακισεφέβδομος	superquadriseptimus		$1+\frac{4}{7}$
6. πεντακισεπένατος	superquintinonus		$1+\frac{5}{9}$
C. πολλαπλάσιοι	multiplices	multiples	
1. διπλάσιος	duplex	duple (double)	2 : 1
2. τριπλάσιος	triplex	triple	3 : 1
3. τετραπλάσιος	quadruplex	quadruple	4 : 1
4. πενταπλάσιος	quintuplex	quintuple	5 : 1
5. ἑξαπλάσιος		sextuple	6 : 1
6. οκταπλάσιος	octuplus	octuple	8 : 1
7. δεκαπλάσιος		decuple	10 : 1

FIGURE 2. Names of Ratios

with Ptolemy and suggested further that whatever else one might say of the Pythagorean method, one thing was certain: the Pythagoreans were altogether right in supposing that nature's logic is discoverable only through the instrumentality of the intellect. Porphyry's implication is that any theory that accords too little value to the activity of reason risks admitting random properties. And because random properties cannot be supposed to exist in nature, they must derive from the vagaries of perception. It was in treating reason as secondary, therefore, that the Aristoxenians ended up contradicting the norms of scientific inquiry as Ptolemy and Porphyry understood them.[52]

Ptolemy argued, accordingly, that the Aristoxenians contradicted the norms of scientific inquiry first, by failing to accept the Pythagorean ratios as clearly established by mathematical truth; second, by placing more emphasis on the spaces between the notes of melody than on the notes themselves; third, by defining the musical elements in a circular fashion.[53] To such errors as these Ptolemy added that concerning the measurement of the fourth:[54]

> They [the Aristoxenians] are mistaken, furthermore, about the measurement of the first and smallest concord, composing it as they do from two tones and a half, so that the fifth is put together from three and a half tones, the octave from six tones, and each of the other concords in the way that follows from this one. For reason [*logos*], being more worthy of trust than perception [*aisthēsis*] in the case of differences as small as these, proves that this is not so, as will be clear.

[52] As Aristoxenus saw it, the nature of music is so unlike that of the world at large that the laws of traditional mathematics, by which the Pythagoreans explained the natural phenomena, could never succeed in explaining the phenomena of music. Barker, in "Music and Perception: A Study in Aristoxenus," *JHS* xcviii (1978), 12, states the case succinctly: "Perceived similarities simply do not correspond to mathematical ones, and it is the *perceived* similarities which constitute properly musical groupings or categories." Barker's insight here lies at the heart of the Aristoxenian method.

[53] The circularity (*peritrapein*) of which Ptolemy complains in *Harm*. I. 9 (Düring, 20. 18–22) is unavoidable according to Aristoxenus quite simply because all the elements in Aristoxenus' harmonic science must, in the final analysis, be defined by reference to one another, and not by reference to anything outside music's own domain. Cf. Barker, II, p. 294, n. 83.

[54] *Harm*. I. 10 (Düring, 21. 21–22. 1); trans. Barker.

And this criticism is enlarged upon by Theon of Smyrna with the added authority of Plato:[55]

> It is agreed by everybody that the fourth is greater than two whole-tones, but less than three whole-tones. But Aristoxenus says that it consists of two complete whole-tones and a semitone, while Plato says it consists of two whole-tones and the so-called *leimma* (remainder). He says that this remainder (*leimma*) does not have a name, but that it stands in a number to number relation as 256 to 243; and that this is an interval the difference between whose terms is 13.

To Meibom, Aristoxenus was engaged in something that he thought worthy enough to be characterized as *scientiae sublimitatem* (see note 48); to his critics, however, Aristoxenus' theory could not be called a science at all, for, as they saw it, the object of his musical representation could not be an object of knowledge in a strict sense. In other words, musical notes, as they construed them, do not have any significative force, but are merely the sound of a voice or an instrument. In short, Aristoxenus' critics could not consider the possibility that music might reveal the nature of things with an intensity and depth unmatched by rational thought. Aristoxenus was engaged in something, to be sure. If, as his critics alleged, it was not worthy of the name, science, it might more properly be called logic – musical logic. Indeed, its limits are even as Aristoxenus' modern critics have judged them to be. But they were self-imposed limits. For they arise from Aristoxenus' having debarred his field of inquiry from all other branches of knowledge, leaving himself with nothing to deal with save music itself and its forms. This was his intention, but it won him the same disapprobation in modern times as befell him in antiquity, so much so that he might with justice have said (*pace* Peter Abelard):[56] "Omnibus me supra modum ignotum atque obscurum effeci." [I have made myself opprobrious beyond measure as well as unintelligible to everyone.]

Aristoxenus cannot rightfully claim the credit for making himself *ignotum* and *obscurum* to all, nor can his followers be charged with

[55] *De limmate* (Hiller, 67, 8–16). Theon's treatise has been translated by R. and D. Lawlor.

[56] Cf. Peter Abelard, *Epistola* I: *Historia Calamitatum Abaelardi* (Cousin) I, p. 17.

having effaced him from the literature on music. Modern musicologists have managed to do this without any assistance from the followers of Aristoxenus or from the master himself. Examples of these successful efforts abound. To J. F. Mountford, an especially acrid critic, who could conceive of harmonic theory in no other terms than those of Pythagorean science, Aristoxenus was scarcely worthy of study:[57]

> Sharply distinguished from this metaphysical theory [the Pythagorean doctrine] is the system which was first enunciated by Aristoxenus. For him pure mathematics and physics had no attraction. He postulated that in music the ear is the sole and final arbiter and that a mathematical formula has little or nothing to do with music. In this he was absolutely wrong, so far as theory goes; and so far as the art of music is concerned, he was only partially right. Ears differ in sensitivity and one naturally asks what kind of ear is to be the sole criterion; is it to be the ear of a highly critical musician or the average listener? To rely only upon the ear for the data of a system of musical theory is to use a rough-and-ready method.

Having said this much, Mountford went on to provide a concise and very accurate outline of Aristoxenus' theory, stressing with admirable clarity its two pivotal axioms, both of which contradict at the deepest level the Pythagorean doctrine of harmonics: (1) the progression of melodic sound from the low pitch to the high ones and back again is as a continuous line; (2) this continuous line of pitch, or *continuum*, is infinitely divisible into equal parts. Given these axioms, Aristoxenus could arrive at the logical conclusion that the difference between a perfect fourth and a perfect fifth is a whole-tone, which is itself capable of infinite divisions. This Aristoxenian anti-Pythagorean doctrine provoked Mountford to say grudgingly that although it "has a superficial lucidity," it cannot be taken too seriously:[58]

> This linear conception of intervals, however, lies at the very root of the Aristoxenian theory and proves to be a quite impenetrable barrier to a proper knowledge of the nature of Greek scales. It would be grotesque to suggest

[57] In K. Schlesinger, *The Greek Aulos*, Introduction, p. xxi.

[58] *Ibid.* Aristoxenus knew, as do all musicians, that "the subtle and changing relationships of tones and semitones within the structure of a scale produce continuous variations to the discerning ear." Cf. Blum, *Casals and the Art of Interpretation*, p. 35.

> that this theory can be entirely neglected or to deny that from it we can infer much that is worth knowing; but for the fundamental question about the size of the intervals of the Greek scales it is too unscientific to be of real service.

Another critic of Aristoxenus' method had this to say:[59]

> Aristoxenus was a prolific writer who has been extensively quoted by later authors. He scorned the application of numbers to music. He preferred his own slipshod method of guesswork.

Thus, when Aristoxenus estimated the enharmonic diesis (quarter-tone) to amount "to one fourth of the difference between the fifth and fourth," or, more easily said, one-fourth of a whole-tone, the same critic, E. Clements, professing, as it seems, to know more about ancient Greek scales than Aristoxenus himself, observed:[60] "The diesis of Aristoxenus was a conception of no practical value."

More important by far than the criticism of Aristoxenus voiced by Mountford and Clemens, is that of R. P. Winnington-Ingram. For in a single article dating from 1932 and entitled "Aristoxenus and the Intervals of Greek Music," Winnington-Ingram succeeded in influencing the course of Aristoxenian studies for the rest of the twentieth century.[61] Winnington-Ingram is not only historically important in his own right; he represents, better than any other scholar in the field of ancient Greek music, an important type of musical theory: the mathematical. Equally important, in his criticisms of Aristoxenus, he has, in many respects, clarified Aristoxenus' own teachings, for he has revealed, with as much consistency as possible, the type of theory advocated by Aristoxenus and the critical points at which it conflicts with the mathematical. His arguments are always compelling, and his statements of Aristoxenian philosophy are far clearer than those of Aristoxenus' own followers. His sympathies are with Ptolemy, Archytas, the Pythagorean of the fourth century B.C., Eratosthenes, the astronomer and mathematician of the third century B.C., and Didymus, a first century B.C.

59 E. Clements, "The Interpretation of Greek Music," *JHS* xlii (1922), 139.
60 *Op. cit.*, 140, n. 7.
61 R. P. Winnington-Ingram, "Aristoxenus and the Intervals of Greek Music," *CQ* 26 (1932), 195–208.

source for Ptolemy and for Ptolemy's commentator, Porphyry.[62] At the same time, however, his respect for Aristoxenus is very great, save for one instance, when he could not contain his annoyance at Aristoxenus' having disregarded the enharmonic computations of Archytas. But even here, he confined his displeasure to a parenthesis: "(he [*sc.* Aristoxenus] was, by all accounts, a maliciously-minded person)."[63]

As Winnington-Ingram saw it, Aristoxenus' theory is very hard to understand not because of its *sublimitas*, but because of its lack of scientific rigor. Thus, whereas Aristoxenus' predecessors and contemporaries realized that musical intervals can only be expressed as ratios of string-lengths and that the addition and subtraction of intervals requires mathematical processes of squaring and finding the square roots of the numerical terms in the ratios, Aristoxenus ignored these laws of mathematics entirely. Instead, he consulted his own musical consciousness and relied on the data given him by the ear. He postulated therefore that intervals must be treated spatially, that musical space is symmetrical, and that musical intervals, being capable of infinite division, can be added together and subtracted from one another by the most convenient methods of addition and subtraction of whole numbers. Because of what Winnington-Ingram considered the unscientific nature of these postulates, he could not accept the results to which they led. Whereas he conceded that the mathematical theorists may be under "suspicion of letting irrelevant factors intrude into their calculations, he [*sc.* Aristoxenus] must equally be suspected of yielding to the attractions of symmetry and convenience."[64] Winnington-Ingram would accept

[62] Ptolemy *Harm.* II. 14 (Düring, 70. 5–74. 3) compares his own division of the canon in all three genera with those of Archytas, Eratosthenes, and Didymus, using the ratios of mathematical theory. In addition, he attempted to convert Aristoxenus' measurement of the same intervals into the language of mathematical ratios. Barker, II, p. 346, n. 117, says rightfully of his effort: "Since the arithmetical differences between terms in Pythagorean ratios are quite different forms of quantity from the 'distances' between Aristoxenian pitches, the attempt is quite incoherent."

[63] Winnington-Ingram (note 61), 201, took Aristoxenus' failure to distinguish properly, that is, mathematically, between the major and minor whole-tones as a deliberate snub against Archytas.

[64] Winnington-Ingram (note 61), 195.

Aristoxenus' computations on one condition only: if they happened to agree with those of Archytas.[65]

When, therefore, Aristoxenus added and subtracted musical intervals as though they were so many equal units or quanta, and did so, moreover, according to the dictates of an arithmetic system which he himself devised, Winnington-Ingram could not but think of such an approach as productive of factitious nuances. In addition, Aristoxenus' method of tuning by consonances, whereby he revealed how in the circle of fifths the consonances end up varying within a minute locus, is rejected out of hand by Winnington-Ingram on the basis that the human ear cannot be trusted to judge correctly of ten successive consonances.[66] Finally, where other scholars have been

[65] As Winnington-Ingram (note 61), 208, puts it: "The chief service of Aristoxenus' account of the genera seems, on examination, to be the confirmation of Archytas."

[66] See Winnington-Ingram (note 61), 199. On the circle of fifths, see Fig. 3, in which the tuning discrepancies are shown.

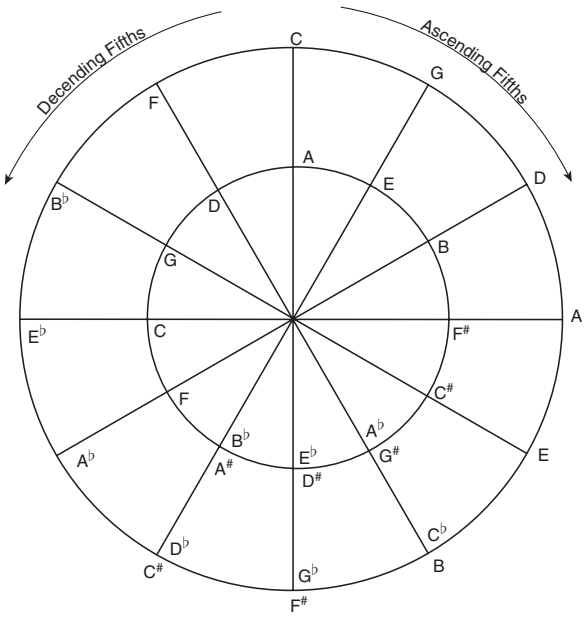


FIGURE 3. Circle of Fifths

willing to credit Aristoxenus with having discovered a method for tempering the scale, Winnington-Ingram has this to say:[67]

> The term "equal temperament" is often used in connection with Aristoxenus; and in a sense by dividing the octave into six and the tone into two he has produced "equal temperament." But the difference between his procedure and the temperament of modern theory is more important than their resemblance. Our equal temperament is dictated by practical convenience in the matter of modulation. The modern theorist knows that the intervals are distorted upon a tempered instrument and by how much. But Aristoxenus did not live in an age when temperament in the modern sense was either necessary or desirable.

In sum then, Winnington-Ingram judged Aristoxenus' theory to be, if "not wholly nonsensical," then seriously flawed for being wholly unscientific.[68] All things considered, it would appear on Winnington-Ingram's reading that Aristoxenus' chief service to the field consisted not so much in his aiming at strict equality in the division of musical space, but, more important, in his computations of melodic intervals that confirm those of Archytas.

Aristoxenus was struggling to do something that he said had never been attempted before: to offer composers a system that would allow for a flexibility that is matched only by the workings of the human voice and that is sanctioned only by the perception of the human ear. In addition, Aristoxenus aimed to codify the laws of melodic consecution so as to satisfy the requirements of true melodiousness (*emmelēs*). To accomplish this task, he fashioned a theoretical panoply of fixed and moveable notes, linked and unlinked tetrachords, genera and nuances, in order to systematize and account for everything that he recognized to be melodious in the music he had been hearing from childhood on. His theory was designed to incorporate, therefore, everything that made for what Plato had characterized as the "rightness" (*orthōtēs*) of a melody.[69] But Aristoxenus' critics found more to censure than to praise

[67] Winnington-Ingram (note 61), 198.
[68] Cf. Winnington-Ingram (note 61), 208.
[69] Plato *Laws* 670A6–B6.

in this, his codification of melody's laws. For one thing, as Winnington-Ingram demonstrated, the size of the *diesis*, or quarter-tone, seemed to be unsettled in Aristoxenus' schemata, but varied with the nuances of melody beyond what theory could capture. Even worse, the very symmetricality of his systems seemed to level away all the remarkable subtleties and nuances of the ancient modes known as *harmoniai* to Aristoxenus' predecessors.[70]

Warren Anderson argued, accordingly, that the ancient and distinctive modes of diverse ethnic origin and called *harmoniai* (tunings) – Dorian, Iastian (Ionian), Phrygian, Aeolian, and Lydian – were all but lost in the theoretical symmetries of Aristoxenus:[71]

> All this [Aristoxenus' system] has a certain complex majesty, but it takes us into a realm of theoretic perfection which Harmoniai of the earlier Hellenic period can hardly hope to have known.

As Anderson explains, by locating these independent modal tunings within the framework of a fifteen-note diatonic system – the Greater Perfect System, so-called – a sequence found in the white keys of the piano ranging from A to A^2, Aristoxenus transformed what had hitherto been a series of differentiated modes into segments or species (*eidē*) of the octaves contained in this larger system. In so doing, he effectively dissolved the ancient *Harmoniai* into a homogenized system in which more was apparently lost than gained. Anderson's exposition of Aristoxenus' standardized system is clear and concise; but for reasons unstated by him, he omits what may be vital where the issue of modality is concerned: the *chroai*. Thus, Anderson:[72]

> A final principle of differentiation is the distinction according to genus (*kata genos*), determined by the type of interval sequence within the fixed

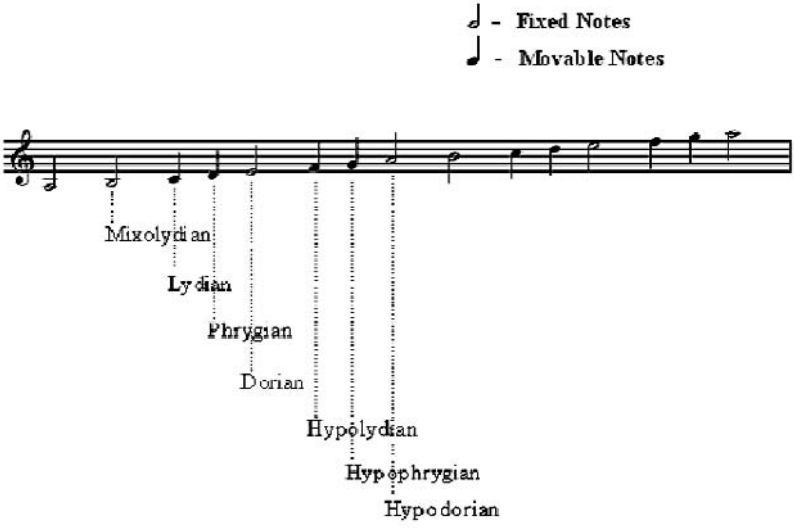
[70] Aristoxenus *Harm. El.* II. 36 (Da Rios, 46. 9–10) speaks of his predecessors as being occupied "only with the seven octochords which they called *harmoniai.*" As Henderson, "Ancient Greek Music," 349 justly points out, Aristoxenus himself identified these tropes of the classical idiom as *melē* – "melodies" – and not merely octave scales.

[71] Anderson, *Ethos and Education in Greek Music*, p. 18.

[72] Anderson (note 71), pp. 17–18. Cf. Fig. 4, showing the octors species and the tonoi.

boundary notes of each tetrachord. While Aristoxenus lists a variety of diatonic and chromatic genera, we may disregard the nuanced varieties known as "shadings" (*chroai*) and cite the following sequences as fundamental.

Criticism of Aristoxenus by some of the leading specialists in the field has continued to the present day. Reading it, one is given the decided impression that Aristoxenus is a writer whose evidence on ancient Greek music can either be dismissed as completely unreliable or, at best, taken *cum grano salis*. M. L. West, for example, suggests that



Octave Species

Mixolydian	B	-	B
Lydian	C	-	C
Phrygian	D	-	D
Dorian	E	-	E
Hypolydian	F	-	F
Hypophrygian	G	-	G
Hypodorian	A	-	A

Tonos

Hypodorian	F	-	F
Hypophrygian	G	-	G
Hypolydia	A	-	A
Dorian	B♭	-	B♭
Phrygian	C	-	C
Lydian	D	-	D
Mixolydian	E♭	-	E♭

FIGURE 4. Paradigmatic System with Octave Species

Aristoxenus did not even understand what he himself was saying as, for example, in the case of the whole-tone interval:[73]

> Apart from that [*sc.* shades of intonation], there is a problem about what exactly is meant by a "tone." The Greek writers define it as the interval by which a fifth is greater than a fourth. Strictly speaking, that is the interval given by the ratio 9:8, or 204 cents. But Aristoxenus regards it as being at the same time a unit of which a fourth (properly 498 cents) contains exactly two and a half. In effect he is operating with a tempered tone of 200 cents and a tempered fourth of 500 cents. He does not understand that that is what he is doing; he is simply working by ear. . . . Sometimes he speaks of intervals such as a third of a tone or three eighths of a tone. We must take these with a little pinch of salt, not as mathematically precise measurements but as approximations gauged by the ear.

Making matters even worse for Aristoxenus, there is discord among his critics. This is especially true when the question of modulation arises. As Winnington-Ingram stated authoritatively (see note 67), equal temperament is most necessary for the purpose of modulation, but in Aristoxenus' day temperament, he said, was neither desirable nor even necessary for this purpose. But West insists, as did Anderson before him (see note 71), that the ancient modal scales were submerged in Aristoxenus' system of keys precisely because he was more interested in providing for the possibilities of modulation than in preserving modal distinctions:[74]

> In working out his system of keys, Aristoxenus was not primarily concerned with the placement of modal scales interpreted as octave species. . . . but allowed the concept of mode to be submerged in that of key. What he was more concerned about was to provide for and account for every kind of modulation.

Nor is that all. Aristoxenus was not only condemned to inferiority as a musical theorist, but also to unreliability as a historian of his own musical heritage. To be sure, Aristoxenus' arresting personality, which he never attempted to mitigate, will probably always provide historians

[73] West, *Ancient Greek Music*, p. 167.
[74] West (note 73), p. 229.

of music with ample material about which to quarrel.[75] Yet one thing is certain: Aristoxenus must have known far more about his own musical tradition than we can hope to fathom completely at this distance in time. When, therefore, he tells us about the enharmonic genus, for example, and the great difficulty that it presented to musicians of his own antiquity, his words bespeak the intimate knowledge of the art that is given only to a skilled practitioner. He says this:[76]

> There appear to be three genera: for every melody that is consistent with the same tuning throughout is either diatonic or chromatic or enharmonic. We must consider the first and oldest of these to be the diatonic, for human nature happens upon it first; the second is the chromatic and the third and most recondite (*anōtaton*)[77] is the enharmonic, for the ear becomes accustomed to it at the very last and only after great effort and practice.

That is firsthand evidence from Aristoxenus himself. But there is also some thirdhand evidence on the enharmonic genus and its discovery that appears in the Plutarchian treatise *On Music*, which reports what Aristoxenus had learned from certain unnamed musicians who apparently lived long before him:[78]

[75] Thus, Barker, II, p. 119: "He [*sc.* Aristoxenus] was notoriously humourless, acerbic, and opinionated, outspoken and unscrupulous in speech as in writing. . . . To judge by what he says [in his *Harmonic Elements*], all previous writers on harmonics were incompetents or charlatans. He and he alone has understood how the subject should be pursued, and has grasped the truths it contains."

[76] *Harm. El.* I. 19 (Da Rios, 24. 17–25. 4).

[77] *Anōtaton*, a superlative formed from *anō*, is, in its literal meaning, "topmost," virtually unintelligible, as numerous scholars have pointed out. A history of the various solutions offered is reviewed by Da Rios, *Aristosseno*, p. 29, note 1, who prefers "piú elevato degli altri" ("the most elevated of the others"). Barker, II, p. 139, n. 73, allows her reading as possible, but suggests instead, "Most sophisticated," which would aptly describe the enharmonic genus. Macran's "most recondite" seems most fitting, however, in the light of Artistoxenus' words that follow, these detailing the musical difficulties associated with the enharmonic genus.

[78] Ps.-Plutarch *De Musica*, Ch. 11; 1134f. Because the treatise that has come down under his name is judged not to have come from the hand of Plutarch himself, it is designated by some scholars as Pseudo-Plutarch or Plutarchian. On its importance as a source for the classical period, see West, *Ancient Greek Music*, pp. 5–6.

> Olympus, as Aristoxenus says, is accepted by the musicians as having been the inventor of the enharmonic genus; for before him all melodic genera had been diatonic and chromatic. The musicians conjecture that the invention came about in this way: that Olympus was turning a melody around upside down (*anastrephomenon*) in the diatonic genus and time after time he had it leap over to the diatonic *parhypatē* (F), sometimes starting from the *paramesē* (B), and sometimes from the *mesē* (A): and when he omitted the diatonic *lichanos* (G), he became aware of the beauty of the *ethos*. He was so struck with wonder at the system composed from this tonal distribution that he adopted it and in this system he composed music in the Dorian key.

What has been set out here with archaic simplicity is the discovery of the most primitive form of the enharmonic genus, that which exposes the interval most characteristic of this genus: the ditone or Major third. According to this account, Olympus was composing music using five notes from the diatonic genus: E (which is inferred to be *hypatē*), F (*parhypatē*), G (*lichanos*), A (*mesē*), and B (*paramesē*). Any number of fine melodies could (and have been) composed from even so few notes as these. But in writing down these bare and lifeless pitch-relations shorn of all melodic motion, rhythm, and timbre, we cannot begin to capture the essence of the melodies that Olympus was composing, much less the genus he was in the act of discovering. We can only imagine the oboelike tones of his *aulos*[79] descending sometimes from B to F and sometimes from A to F, filling in the notes between (B A G F) and (A G F E). It was

[79] The *aulos*, the chief wind-instrument of the ancient Greek musicians, has often been mistranslated as "flute," an error noted by West (note 78) in words that bear repeating (Introduction, p. 1): "... and now the only excuse for calling an aulos a flute is that given by Dr. Johnson when asked why he defined 'pastern' as the knee of a horse: 'Ignorance, madam, pure ignorance.'" The most important work on the *aulos* remains that of Kathleen Schlesinger, *The Greek Aulos*, who argued vigorously for the *aulos* as a true conveyor of the ancient *harmoniai*, or modes. Her work, having generated more criticism than praise, is, as a consequence, sadly underappreciated. Thus West (above, note 78), p. 96: "Kathleen Schlesinger wrote a massive, terrifying book, *The Greek Aulos*, based on the belief that the Greek pipes too had equi-distant finger-holes." Schlesinger's theory deserves more attention than it has received. There were two types of *auloi*: the double-reed, like the modern oboe; the single-reed, like the modern clarinet. Virtually everything that is known about the *auloi* is summarized by West (note 78), pp. 81–109.

only when Olympus left out the note G (*lichanos*) that he came upon the beauty of the ditone, F – A, and conversely, A – F. And he was seized with wonder at the sound, the same wonder that Aristoxenus expressed when he too encountered the indescribable beauty of the Major third in its enharmonic context.[80] But this was only at a later stage, when the full complement of the enharmonic genus was established. With Olympus' invention, we are given a glimpse of the most primitive form of the scale (descending): E – C – B – A – F – E.

Holding Aristoxenus to be the primary source for this account, West faults him on the details, thus discrediting him as a reliable theorist. He allows that Aristoxenus was right, however, in inferring that the enharmonic developed from a "single infix" into the semi-tone, an inference that is not even voiced by Aristoxenus (or his sources) in this account. Taking no notice of *paramesē* (B) and its role in Olympus' melodic experiments, West goes on to say:[81]

> Aristoxenus went wrong, however, in supposing that Olympus arrived at his e f a trichord by leaving out a note. On the contrary, the diatonic system represents the filling in of the wide interval in the trichord by means of a second infix.

West's analysis of this passage most effectively challenges and depreciates Aristoxenus' evidence on the history of the genera. For, as Aristoxenus had stated explicitly, the diatonic in the distribution E F G A was the oldest of the genera. That being the case, its "second infix," so-called by West, must have been present and used by musicians long before the invention of the enharmonic genus. In other words, no second infix was required.[82]

[80] *Harm. El.* II, 39 (Da Rios, 49. 12–14).

[81] West (note 78), p. 164.

[82] On the reading of Aristoxenus' evidence by *Fr.* Aug. Gevaert, *La musique de l'Antiquité*, I, pp. 298–300, the earliest form of the enharmonic genus was that of a transilient scale resulting from the omission of the diatonic *lichanos* (G): "As for the enharmonic, its beginnings fall in an epoch that was already claimed by history: Aristoxenus expressly confirms that it came last of all. His testimony must be taken as veracious and must prevail over the contrary opinions which were already current in antiquity." As Gevaert goes on to explain, the primitive enharmonic was founded upon a rudimentary perfect system of tetrachords (descending: E – C – B and A – F – E). These, when converted to a trichordal

In sum then, however willing Winnington-Ingram, Anderson, West, and others have been to grant Aristoxenus importance for being the earliest writer on music whose work has reached us at least in part, they have, more often than not, seen more to censure than to praise in his approach to the subject. At best, his theory has been judged to be overly symmetrical and unrealistically homogeneous in its forms and structures; at worst, it is considered to be too unscientific to offer any real assistance to our understanding of the ancient art. The most recent scholar to express these views is John Landels, who says:[83]

> This [the acoustic theory of the Pythagoreans] is in contrast to the 'musical' school of thought, best represented by Aristoxenus, which held that the notes were, so to speak, points with no magnitude, and that the intervals between them were the measurable quantities. Members of this school pretended, by the use of fractions and additions and subtractions, that they were making scientific measurements, but in fact they were merely judging the "quantities" of the intervals by ear and by guesswork.

As is all too evident then, Aristoxenus has always provided his critics with a battle-ground for opposing and contradictory views. On the one side, that, for example, of Comotti, Aristoxenus is considered to have been too involved with Pythagorean mathematics and mathematical principles to notice, as did Plato and Aristotle before him, the effects of music on the soul. On the other side, that of Mountford, Winnington-Ingram, Landels, et al., Aristoxenus is seen to be fundamentally unscientific and unreliable precisely because he relied solely on the perception of the ear (*aisthēsis*) while ignoring the Pythagorean mathematical principles secured by reason (*logos*). With the possible exception of Comotti, Aristoxenus' critics seem to have agreed on one thing: mathematics has the final word in all things musical, because,

conjunctive system in the Dorian Mode (D C B♭ A F E), "were decomposed into two intervals only: a major third [A – F] and a semitone [B♭ – A], the whole-tone [A – B] now lost in the conjunction [B♭ – A]" (p. 300). The resulting scale was (descending): D B♭ A F E. At this stage in the formation of the enharmonic genus, the "infix" into the semi-tone had not as yet been made. This innovation was probably introduced later by the *aulos*-virtuoso, Polymnastos (p. 300).

[83] Landels, *Music in Ancient Greece and Rome*, p. 132.

as the Pythagoreans had so successfully demonstrated, quantity exists in music, and where there is quantity, there is number. To his critics, Aristoxenus' hostility toward the views of the Pythagoreans, and his insistence on the irrelevance of mathematics to music ultimately led him into difficulties from which he could never extricate himself.[84] But it was not for this alone that he was condemned to the near-total obscurity in which we find him today. It was because his theory, in contrast to that of the Pythagoreans, had applications to one subject only: the nature, or *physis*, of music.[85] In comparison to Aristoxenus, who never looked beyond music and its laws, Pythagoras and the Pythagoreans, universalized the laws that they had derived from the lengths of the lyre-strings so as to include the whole cosmos and all of human life. Aristoxenus' theory, derived as it was from the ear's knowledge and from musical intelligence (*dianoia*), could never extend beyond music and its own laws.

Meibom's assessment in 1652 of Aristoxenus' status in the history of Western culture is, therefore, almost as true today as it was then. For if Aristoxenus is to be found anywhere at all, it will not be in bookstores or in the more-frequented sections of public libraries; instead, he can be discovered only in the narrow confines of academic libraries, "neglected by all because of his having touched upon almost the highest limit of his science."[86] Pythagoras, however, will be found almost everywhere. This is a strange and ironic fate for "The Musician" of antiquity, the author of 453 separate works, some twenty or more of which were devoted to every aspect

[84] Barker (note 52), 10, finds Aristoxenus' understanding of the nature of music and the principles to which it leads to be innovative and fruitful. Attractive as it may be, however, Aristoxenus' approach leads to problems; as Barker says, "... it generated difficulties from which I am not sure that he can disentangle himself." The difficulties to which Barker calls our attention in this important article are examined in Chapter 5.

[85] The nature, or *physis*, of music, as treated by Aristoxenus, has nothing to do with the physics or acoustical theory of sound. It has reference, rather, to the natural movement and the functional principles according to which musical pitches are ordered in melody. On Aristoxenus' conception, melody has within it both a principle of movement and an initiator of movement (the human voice), the final form of which is governed by musical intelligence (*dianoia*).

[86] See note 7.

of music.[87] But despite the penetrating and innovative treatment of music that emerges from his extant writings, Aristoxenus' name is seldom to be found in even the most comprehensive and sophisticated studies of the subject written in this century. On the rare occasions in which he is mentioned at all, he is usually given only a passing reference or, at times, only a nodding acknowledgment from a secondary source. Jamie James, for example, mentions Aristoxenus twice in *The Music of the Spheres*, his first citation coming by way of Leibniz, his second from Oliver Strunk.[88] Although he is not mentioned even to this extent in the writings of numerous specialists in music, Aristoxenus' principles and analyses are mirrored, almost uncannily so at times, in their treatment of the subject. He comes vividly to life, for example, in the words of Ernest Ansermet, a musician whose credentials cannot be challenged. Thus, where Aristoxenus was roundly criticized for his method of tuning by consonances,[89] Ansermet not only details that same method of tuning, together with the musical notation, but even sees in it the very foundation of tonality in Western music. He credits the method simply to "die griechischen Musiker" and says of it:[90]

[87] We know from numerous sources, such as Plutarch, Philodemus, Athenaeus, Porphyry, and others, that Aristoxenus wrote on the technical, the practical, and the performing aspects of music, the dance, and instrumental practices. Of all the works he wrote, only portions of his *Harmonic Elements* (note 20) and fragments from the second book of his *Rhythmic Elements* are extant. The latter have been translated by Barker, II, pp. 185–89. The texts have been edited, with a translation and commentary, by Lionel Pearson, *Aristoxenus: Elementa Rhythmica*. Among the lost works of Aristoxenus to which our authorities refer are: *On Composition*, *On Hearing Music*, *On the Unit of Time*, *On Instruments*, *On Auloi*, *On the Boring of Auloi*, *On Choruses*, *On Dance in Tragedy*, *On Auloi Performers*, for which, see Louis Laloy, *Aristoxène de Tarente*, pp. 16–17; Wehrli, *Frs*. 69–94. Of all his works on music, the loss of two in particular is cruelly felt: *On Music*, which, judging from the fragments that have come down, must have contained much valuable information on the practices of Aristoxenus' own musical tradition; *On Keys* (*Tonoi*), this being one of the most disputed of all the topics discussed by scholars in the field. The definitive work is that of Otto J. Gombosi, *Tonarten und Stimmungen der Antiken Musik*. The subject is reviewed by André Barbera, "Greece," *New Harvard Dictionary of Music*, 346–57.

[88] James, *The Music of the Spheres*, p. 78; p. 90.

[89] See note 66.

[90] Ernst Ansermet, *Die Grundlagen der Musik in menschlichen Bewusstsein*, pp. 676–77.

> Without knowing it, the Greek musicians had already prepared the way through their fifth-fourth tuning of the lyre to the beginning of a world of key positions, which is based on a fifth-fourth relationship, that is, of a world of key positions stemming from their inner relations to one another (B stands in an inner relation to A through the middle member E; the relation of the ascending fourth and the descending fifth is the inner, genetic foundation of the outer relation B–A).

Aristoxenian thought permeates the works of countless musicians and musicologists in places and in situations too numerous to mention. On the question of notation, for example, Robin Maconie might well have been consulting Aristoxenus, for his estimation of the role of notation in music is identical to that of Aristoxenus:[91]

> And yet, despite its [notation's] remarkable intellectual virtues, the inadequacies of notation are obvious enough. Ask any ethnomusicologist about the problems of transcribing recorded folk music, and among the first to spring to mind is the inherent impossibilities of accommodating standard notation to the more elusive qualities of musical expression among oral cultures, including ambiguous pitches and characteristic instabilities of intonation.

One has only to ask Aristoxenus, who states in the second book of his *Harmonic Elements* that notation, like the study of meter, should be treated as a separate science for the very reasons mentioned by Maconie: notation cannot capture the essence of a melody. As Aristoxenus puts it:[92]

> It is not necessary that one who has written down a Phrygian melody know exactly what a Phrygian melody is. Clearly then, notation should not be the end-all of our aforementioned science.

And when Robert Jourdain speaks of the demands made by music on the listener, he, too, is of a mind with Aristoxenus. Thus, Jourdain:[93]

[91] Maconie, *The Concept of Music*, p. 114. Cf. E. Pöhlmann and M. L. West, who, in their Preface to *Documents of Ancient Greek Music*, speak of Aristoxenus' condemnation of notation as a "banausic skill, too defective to form part of serious musicology."

[92] *Harm. El*, II. 39 (Da Rios, 49, 12–16).

[93] Jourdain, *Music, the Brain, and Ecstasy*, p. 135.

> Form makes itself known through a kind of intellectual discovery that demands a well-developed musical memory for sustaining musical fragments over long periods.

Aristoxenus adds importantly to what Jourdain says, with these details:[94]

> It is clear that understanding melodies as they are being played is to follow with both the ear and the intellect things that are happening in their every distinction; for melody, just as the other parts of music, consists in a coming to be (*genesis*). The understanding of music comes from these two faculties: perception and memory. For we must perceive what is happening, and remember what has happened. There is no other way to follow the events in music.

To make the interesting point that composers themselves often cannot account for the source of their own inspiration, Peter Kivy cites Socrates' insight in the *Ion* to this effect:[95]

> For I think Socrates' insight in the *Ion* still stands. The creator may draw from he knows not where – the gods, if you like, or the depths of the unconscious, if you prefer . . .

Kivy might have cited Aristoxenus on this point to even greater effect. For whereas Socrates in the *Ion* sees poets as seers or Bacchanals who compose in a state of inspiration in which they are "possessed" by a higher power, Aristoxenus places the composition of music on a much higher plane. For him, the composition of music is an intellectual activity (*synesis*) on a par with *phua* or native genius that "is hidden deep within the soul."[96]

Once prominent in antiquity as "The Musician," Aristoxenus' claim to a minor place in intellectual history derives today primarily from his nonmusical writings. These include works on history, education, mathematics, and Pythagorean studies. Of his historical works, those that

[94] *Harm. El.* II. 38 (Da Rios, 48. 11–18).

[95] Kivy, *Music Alone*, p. 119.

[96] *Harm. El.* II. 41 (Da Rios, 51. 16). Aristoxenus may have been making a conscious reference here to Heraclitus' observation on the unfathomable depths of the soul (*Vors.* B22; *Fr.* 45): "Though you were to travel every road, you would not discover the limits of the soul; so deep is the logos it contains." Cf. Albert Cook, *Myth and Language*, pp. 86–87 on the link between the logos and the soul.

have won him some little celebrity are biographies, these constituting a new literary genre that he and his colleague at Aristotle's Lyceum, Dicaearchus, inaugurated. Of the five *Lives* whose titles have come down to us, four were devoted to Pythagoras, Archytas, Socrates, and Plato, the fifth being that of Telestes, presumably the dithyrambic poet-musician of Selinus. Thus today, where he is cited at all, as by Hellenists, it is more for the evidence that he brings to Pythagorean studies than for his contributions to philosophy and aesthetics, let alone to musical lore and theory. Most recently, he is mentioned as an authority on the Pythagorean principles of procreation, the testimony offered in his name never even once hinting at his greater authority in music.[97]

Yet, with Aristoxenus, we are not merely in the world of those once-renowned musicians whom he notices – Lasus of Hermione, Epigonus of Ambracia, Eratocles, or Agenor of Mitylene – all masters of the art of music, the teachers and theorists of ancient Greek music;[98] we are also among the musicians of the world – performers and composers – for whom the modes of music represent all the states of the human soul. The transmutation of these spiritual states into melodic expression is the work of musicians. Aristoxenus, as will be argued here, by his orderly research into this work of musicians, was the first to reveal the logical bases and the permanent elements that underlie the apparently limitless diversity and heterogeneity of the musical phenomena. The instruments of his research were his ear (*akoē*), his musical rationality (*dianoia*), and his musical intuition (*synesis*).

Aristoxenus did not begin his task by making hypotheses or inventing principles according to which he sought to explain everything musical. Instead, he based his research on the concepts of practicing musicians, organic concepts that have been controlling the whole range of musical utterance from the very beginning of our knowledge. On the strength of these concepts, he was able to reveal those subtle links that bind the music of all ages and cultures together and that make music intelligible

[97] See Kathy L. Gaca, "The Reproductive Technology of the Pythagoreans, *Classical Philology* 95 (2000), 113–32.

[98] Aristoxenus mentions these, his predecessors, primarily to criticize them, at times most scathingly. His reasons are analyzed instructively by Andrew Barker, "`OI KALOUMENOI `ARMONIKOI: The Predecessors of Aristoxenus," *PCPS* 24 (1978), 1–21.

on its own terms. These links stand out in his theory as the universals of music. They are mind-made – invented by the musically intuitive mind but at the same time classifiable by the logic of musical rationality. They possess the unchanging and eternal characteristics of all universals but, as Aristoxenus was concerned to show, they are not incompatible with the changing interests of musicians or the changing forms of music. In consideration of these factors in Aristoxenus' orientation to music, his theory shows him to have been musically conscious to a degree attained by no other ancient authority and by few of their modern counterparts.

Aristoxenus' aim was to trace the active principle in music – that strange, mysterious faculty which draws strength, color, and character from musical pitches – to disengage it and to mark the degree to which it penetrates all utterances that are instinct with melody. He began with a knowledge that consists of these factors: music moves in its own space and time; its energy derives from the smallest of units; its range is not simply one of a vertical pitch dimension but also one of depth, the perspective of which may hinge on the function, or *dynamis*, as he called it, of a single note. With all that in mind, he framed an adaptive system, a collection of simple, germinal elements that interact to generate subsequent bits of melody, these to be molded into a complex whole. Throughout this endeavor, Aristoxenus showed himself to have been gifted enough to take the prodigious path of a true disciple of Aristotle, the better to raise in the name of philosophy an immense and opulently appointed edifice of musical knowledge. There he housed those few principles into which all the forms of music could be accommodated, and all the particular sciences of music for which they set the conditions: harmonic, rhythmic and metrical sciences, compositional, instrumental, and vocal sciences – in short, all those branches of the theoretical, practical, and performing disciplines that are, he said, "embraced by the general science that concerns itself with melody."[99]

[99] *Harm. El.* I. 1 (Da Rios, 5. 4–6).

3 The Discrete and the Continuous

At the recurring end
of the unending — T. S.Eliot, *Little Gidding*

ONE OF THE MOST PERPLEXING MUSICAL QUESTIONS FAMILIAR TO everyone was raised early on by Aristoxenus in his *Harmonic Elements*: What is it that makes one melody musical and another unmusical?[1] The

[1] From the time of Pythagoras to that of Aristoxenus, the ancient theorists saw the particularity of this problem, but they approached it in terms of different questions: What is there in the phenomena of music that makes for their objective coherence or intelligibility? What is there in the phenomena of music that makes for their affective power? It is essentially the latter question that is posed in Ps.-Aristotle *Problems*: "How is it that rhythms and melodies, which are only sound, resemble moral characters, while flavors, colors, and scents do not?" And Plato was in effect asking the former question when he observed of melody in *Laws* 657B2: "If one could grasp what sort of thing constitutes the correctness of melodies, one could confidently submit them to law and order." On this law-like inevitability in musical melody, Roger Scruton, *The Aesthetics of Music*, p. 79, speaks in these Platonic terms: "This virtual causality is sometimes perceived as physical relations are perceived: namely, as law-like and inevitable." Aristoxenus goes even further; as he has it, the difference between a musical melody (*emmelēs*) and an unmusical one (*ekmelēs*) is not simply a matter of law and order; it involves something far more fundamental (*Harm. El.* I. 19; Da Rios. 24. 5–10): "For now, let me say in a general way that although a well-attuned melody admits of many differences in its collocation of intervals, nevertheless, there is a kind of attribute that will be asserted of every well-attuned melody, an attribute that is one and the same for all, and is possessed of so great a power that with its removal, the attunement is destroyed." This crucial power is understood by Aristoxenus to be continuity (συνέχεια). It assumes not only the proper or lawful succession of intervals, but also the proper or musically rational analysis of intervals. For intervals, as Aristoxenus defines them, are not simply the sizes of the spaces between notes; they are

full implications of his answer to this central question should become clear by considering the way in which he dealt with more specific problems, in particular, with the more controversial ones. Of these problems, one of the most important concerns the proper analysis of the concept of the *continuum* – the topology, as it were, of melody, and the notion closely allied to it – that of infinity. The problems connected with the *continuum* and with that of infinity arise in an early stage in Aristoxenus' reflections on two issues that pertain to melody: the apparently unlimited possibilities available to the human voice of placing notes in continuing sequences on the line of pitch; the apparently limitless possibilities granted to the human voice of subdividing the distance between any two of these sequential pitches. These problems reemerge at later and more subtle stages in Aristoxenus' analysis about discrete and continuous quantities and their relation to the *continuum* of melody.

In the earliest stages of his education, Aristoxenus would have probably concerned himself with infinity, not on its own account, but as it related to Pythagorean mathematical theory – a relation that, especially in Pythagorean harmonics, was very close. Indeed, Aristoxenus' early training in Pythagorean mathematical theory would have brought him into direct contact with two of the greatest problems related to infinity that had occupied Greek mathematicians for centuries: the irrationality in the length of the hypotenuse of the right-angled triangle and the irrationality in the magnitude of the whole-tone musical interval. For on the division of the hypotenuse of the right-angled triangle and on that of the whole-tone musical interval in the ratio 9:8, there is yielded the same irrationality in the form of $\sqrt{2}$ – a number as productive of infinity as π, the ratio of the diameter of the circle to its circumference.[2] Archimedes

more properly determined by the functions (*dynameis*) of the notes delimiting them. Implicated in this crucial power is the *continuum* against which all melodic changes occur.

[2] See above, Ch. I, pp. 5–6. As the Pythagoreans discovered to their dismay, there is no number which, when multiplied by itself, will yield $\sqrt{2}$. It is an irrational number, because it cannot be expressed as a ratio of whole numbers, as is the case with the octave (2:1), the fourth (4:3), and the fifth (3:2). At the same time, if converted into decimals, $\sqrt{2}$ is productive of infinity: 1.4142135. But whereas $\sqrt{2}$ is a solution of the algebraic equation , $x^2 - 2 = 0$, π, which can

had attacked the problem of π by inscribing and circumscribing in the circle a regular polygon of ninety-six sides, thereby demonstrating how his method of reconciling the irreconcilable could be carried to any degree of approximation. As Archimedes showed all too clearly, this is all that any method of approximation can accomplish with the problem of π.[3] Aristoxenus' goal was much the same as that of Archimedes: to arrive at a similarly effective kind of approximation for dealing with the irrationality of the whole-tone musical interval. Succeeding in this endeavor meant providing a degree of approximation that would answer the practical needs of singers and instrumentalists. In order to prosecute this goal, however, Aristoxenus had to find a way to deal with infinity, not simply as it related to mathematical theory, but on its own account.

Aristoxenus was singularly well qualified for the undertaking. Born about 360 B.C. in Tarentum in southern Italy, his life was almost

be rendered in decimals as 3.14159265358 to infinity, differs from $\sqrt{2}$ in that it cannot be the root of an algebraic equation. This means that π is not only irrational, it is also transcendental. On the transcendance of π, see Beckmann, *A History of* π (Pi), pp. 166ff. In brief, π transcends infinity; $\sqrt{2}$ does not. As Lavine, *Understanding the Infinite*, pp. 246–47, interestingly remarks: "We have already seen that the infinite is nowhere to be found in reality, no matter what experiences, observations, and knowledge are appealed to." Perhaps it is in music that we are experiencing what is not available to us in any other kind of reality: namely, infinity. That is, the space of a whole-tone can be thought of as infinite, since every point within its limits is as much a "center" as any other point.

[3] Archimedes' method, sometimes referred to as "the method of exhaustion," is ascribed to Eudoxus (c. 408–c. 355 B.C.), but its foundation was laid by earlier mathematicians of the Athenian school, in particular, by Hippocrates of Chios (in Athens during the second half of the fifth century B.C.) and by Menaechmus (?375–25 B.C.), a pupil of Eudoxus and a somewhat older contemporary of Aristoxenus. The method of exhaustion led Archimedes to an exact result in squaring the parabola; but in the case of the circle, it led to successive approximations. By inscribing and circumscribing a regular polygon of 96 sides, Archimedes proved that π is less than $3\frac{1}{7}$ and greater than $3\frac{10}{71}$. This means that $(3+\frac{10}{71})$ Diameter < Circumference <$3 + \frac{1}{7}$ Diameter. The upper limit is the Archimedean approximation $\pi \sim \frac{22}{7}$, or 3.1428571. For a detailed account, see Dijksterhuis, *Archimedes*, pp. 222ff. See also Heath, *A History of Greek Mathematics*, I, pp. 221–23, who traces the method back to Antiphon, the Sophist and contemporary of Socrates.

coextensive with one of the most transforming periods in the history of the ancient world – that of the campaigns and conquests of Alexander the Great.[4] Around the time that he was to begin his studies at the Lyceum with Aristotle, Alexander was defeating the Persians in the battle of Gaugamela (331 B.C.) and Darius, king of the Persians, was fleeing for his life. Of all these events there is no mention in the extant works of Aristoxenus. He was turning his mind far away from the spectacle of war and human misery to contemplate instead the mysteries of music and the eternal world of ideas and beauty. In this, he was in total harmony with the most learned men associated with the city of his birth: Philolaus (b. c. 470 B.C.), Lysis (fourth century B.C.), and Archytas (first half of the fourth century B.C.), Pythagoreans all. For Tarentum was a leading center of Pythagorean studies and activities, the city in Magna Graecia where Pythagoras himself was said to have lived for a time long before.[5] Philolaus and Lysis had of course died long before Aristoxenus

[4] Most of what we know about Aristoxenus' life comes from the historical and literary encyclopedia compiled at the end of the tenth century A.D. and known today as *Suda* or *Suidas* (Fortress or Stronghold). The article on Aristoxenus, consisting of about a dozen lines, is reproduced by Laloy, *Aristoxène*, pp. 1–2 and translated also, p. 2, n. 1. It is offered by Wehrli as *Fr.* 1 and also by Da Rios, p. 95. It states the following: "Aristoxenus, the son of Mnesias, also called Spintharus, a musician from Tarentum in Italy; after studying in Mantinea, he became a philosopher. Having also devoted himself to music, he achieved no small success. He was a student of his father and of Lamprus of Erythrae; thereafter, he studied under Xenophilus the Pythagorean and, finally, under Aristotle. After Aristotle died, he indulged in disrespectful language toward him, because Aristotle chose Theophrastus to succeed him as head of the school, even though he, Aristoxenus, had won a great reputation among the students of Aristotle. He lived during the time of Alexander and his successors, so that he is of the one hundred and eleventh Olympiad [c. 333 B.C.] and a contemporary of Dicaearchus of Messina. His works on music, philosophy, history and every aspect of culture come to 453 books." Additional facts about Aristoxenus come from the writings of Athenaeus, Porphyry, Ptolemy, Vitruvius, Boethius, and many others. Cf. Da Rios *Testimonia*, pp. 95ff.

[5] We owe a charming story about Pythagoras in Tarentum to the Neoplatonist philosopher from Syria, Iamblichus (c. 250–c. 325 A.D.) For it was here in Tarentum that Pythagoras, as Iamblichus has it, demonstrated his Orpheus-like dominion over animals. One day, Pythagoras encountered a bean-eating cow in the pastures around Tarentum. When he suggested to the cowherd that he instruct his cow to

was born, but Archytas may quite possibly have been known personally to Aristoxenus, as he was a close friend of Aristoxenus' father.

Aristoxenus' first teacher in philosophy, mathematics, and music was in fact his father, Spintharus, a renowned musician, whose circle of friends included the celebrated musicians Damon (*fl.* c. 430 B.C.) and Philoxenus (c. 435–c. 380–79 B.C.), as well as the cultivated Boeotian general Epaminondas (d. 362 B.C.) and even Socrates (469–399 B.C.). In due course, Aristoxenus was sent for advanced study to a certain Lamprus of Erythrae, whose name has come down to us only because he had once taught the son of Spintharus. Sometime thereafter, Aristoxenus progressed to Mantinea, an outstanding center for the serious cultivation of music, a city where his welcome reception would have been guaranteed by Spintharus' association with Epaminondas, its Theban liberator.

In Mantinea, everyone was, it seems, either a professional musician or, at the very least, a cultivated and knowledgeable listener. The city actually required that every young man up to the age of twenty be well trained in all aspects of music. Indeed, no city could have been more congenial to the tastes and inclinations of the young Aristoxenus, for it was here that the artistic excellence of ages past was being preserved with a diligence matched only by that of Aristoxenus himself. The Mantineans' humility toward the composers of the past – toward Pindar, Terpander, Alcman, and Tyrtaeus – was acknowledged throughout the world of music. Their conservatism was vigorously upheld by Aristoxenus, whose own devotion toward time-honored traditions and techniques had been instilled in him since early childhood. In Tarentum, he had had Spintharus to form his tastes and set his standards of excellence; here in Mantinea, he was to perfect his skills under the guidance of music's master-teachers.[6] A city of the plain, Mantinea was the Curtis Institute of its day.

give up eating beans, the cowherd laughed aloud, observing that he did not speak cow language. Pythagoras thereupon took the cow aside and whispered into its ear for a while. The cow responded by giving up beans forever after, and the citizens erected a statue of a cow in the town square to commemorate the miracle. See Iamblichus *De vita Pyth.* 13. 61–62 (Deubner, 33. 8–21).

[6] Philodemus (for whom, see Chapter 1, note 72) thus speaks of the people of Mantinea, Lacedaemon, and Pellana as being leaders in the field of music and known for their diligent and intense practice of the arts (Kemke, 10. 19). And Ps-Plutarch confirms him in speaking of the conservative practices of

When Aristoxenus marveled at the wondrous ordering in music, he had one thing in mind: melody. He had no patience, therefore, with composers like Timotheus, Philoxenus, and Krexus – the avant-garde of the fifth century B.C. – who, in his view, had introduced their own idiosyncratic ideas into the ancient art solely for the sake of newness. To Aristoxenus, their technical innovations were more for self-proclamation than for genuine musical expression.[7] When melody was at issue, he bound himself to the practices of such composers, now ancient in his own day, as Alcman (7th century B.C.) and Stesichorus (c. 632–c. 556 B.C.). In his words:[8]

> There is an innovation of the Alcman and Stesichorean type; indeed, these practices do not depart from the beautiful. But Krexus and Timotheus and

"the Lacedaemonians and Mantineans and Pellanians, who selected only one mode or, at most, only a few modes which they believed would contribute to the harmonization of upright characters, this being the music which they practiced" (*De Musica*, ch. 32; Ziegler-Pohlenz, 26. 24–27). Cf. Anderson, *Ethos*, p. 153. Aristoxenus' community of spirit with the Mantineans is noted by Lasserre, *Plutarque*, p. 32: "The excellent reputation won by Aristoxenus among the exemplary practices of the Mantineans, to whom he devoted a book, is certainly inseparable from their attachment to musical institutions reputed to be as ancient as those of the Lacedaemonians."

[7] Timotheus is the most famous representative of this revolutionary school of music. His dates – c. 450–360 B.C. – make him a contemporary of Aristophanes. He is known for having increased the number of strings on the cithara to eleven and for having been a virtuoso as well as a prolific composer. According to Ps.-Plutarch *De Musica*, ch. 30 (Ziegler-Pohlenz, 25. 3), the comic poet, Pherecrates, in his now lost play, *Cheiron*, speaks of Timotheus' melodic innovations as "deviations in the form of ant-tracks," an opinion no doubt shared by Aristoxenus himself. Cf. Comotti, pp. 35–37. Philoxenus (c. 435 – c. 380–379 B.C.), a famed composer of dithyrambs, is cited frequently in Ps.-Plutarch *De Musica* for his originality and ornamental style. And Athenaus XIV. 643D–E has the comic poet, Antiphanes, speaking of Philoxenus in these none-too-measured terms: "He was a god among men, since he truly knew music." Krexus (c. 450–400 B.C.), also famous as a composer of dithyrambs, introduced something remarkable for his time: an accompaniment on the cithara using notes that differed from those of the song itself. Traditionally, the accompaniment on the cithara doubled in unison the notes of the vocal melody. In sum, then, Aristoxenus was deploring musical practices that were being introduced roughly a hundred years before his time.

[8] Ps.-Plutarch *De Musica*, ch. 12 (Ziegler–Pohlenz, 10.19–11.2).

> Philoxenus and the composers of that era have become more vulgar and lovers of novelty in their pursuit of what is now called a prize-winning and crowd-pleasing style. For it turns out that the simple and noble type of music based upon a small number of strings has become completely antiquated.

In this approach to the classical art of music. Aristoxenus, like the conservative Mantineans, had a special kind of moral purity and loftiness of purpose, which is most impressive. He was consistently sincere, even though he was all too often shrill, censorious, and impatient; but, above all, he was invariably concerned to tell the reader as simply as he could what he felt to be more important than all else: melody. When, therefore, he asked his like-minded colleagues, few in number as they might be, "to go off by ourselves and remember what sort of art music used to be," he can easily be imagined discussing the melodies of old with the musicians of Mantinea.[9]

One of these, a certain Telesias of Thebes, who had received an exemplary training in music, tried to break away from established melodic norms and compose in the style of Philoxenus and Timotheus, with this result:[10]

> [Aristoxenus] says that it befell Telesias of Thebes to have been trained when he was still a child in the finest kind of music and to have studied the works of the most highly-esteemed composers, among whom were Pindar, Dionysius of Thebes, Lamprus and Pratinas and other masters of lyric and instrumental composition.

Aristoxenus goes on to relate that, in the course of time, Telesias became so captivated by the theatrical music being written by such composers as Philoxenus and Timotheus that he made every effort to master its techniques himself. But when he actually tried to compose

[9] Athenaeus Deipnosophists xiv a–b = *Fr.* 124 Wehrli.

[10] Ps.-Plutarch *De Musica*, ch. 31 (Ziegler–Pohlenz, 25. 16–21) = *Fr.* 76 Wehrli. Commenting on this passage, Wehrli points out, p. 71, that the Telesias mentioned in this citation is otherwise unknown; but Dionysius of Thebes was the teacher of that most cultivated military man, Epaminondas. Cf. West, *Ancient Greek Music*, p. 371.

in the Philoxenian style, he failed utterly. The reason for this lay in his early training: he had been so firmly grounded in the classical art as practiced by Pindar, Lamprus, Pratinas, and others that he could not compose vulgar and theatrical melodies even when he tried.

The metaphysics of Aristoxenus began in Mantinea, where no one seems ever to have feared such a thing as musical obsolescence. His success there as a practicing musician was considerable, but from his earliest days he felt himself equally drawn to the study of philosophy. And, as it came fruitfully to pass, it was philosophy that would eventually provide him with a way to deal with the nonmathematical properties of music's nature.[11] He left Mantea, then, for Athens, his original intention being to master Pythagorean mathematical theory down to its most refined details; to this end, he sought out the leading Pythagorean expert of the day: Xenophilus the Chalcidean of Thrace, who, like many other celebrated Pythagoreans, had made his home in Athens.[12] Traveling from Mantinea to Athens, Aristoxenus made an important stop in Corinth, where he became acquainted with the exiled tyrant

[11] Among those properties that mathematics cannot represent are the musically logical ordering of melodic intervals, those rules of continuity that are, as it were, the counterpart of a linguistic grammar. In her *Language, Music, and Mind*, Diana Raffman quotes F. Lehrdahl and R. Jackendoff, *A Generative Grammar of Tonal Music*, to this effect (p. 18): "Like its linguistic counterpart, the musical grammar (the "M-grammar") models an underlying *competence*. That is, it models "the largely unconscious knowledge (the 'musical intuition') that the listener brings to his hearing – a knowledge that enables him to organize and make coherent the surface patterns of pitch, attack, duration, intensity, timbre, and so forth." It is this knowledge, this innate competence, that Aristoxenus calls "the *synesis* of music," or musical intuition. Mathematics is defeated by its range of understanding. Cf. Levin, "*Synesis* in Aristoxenian Theory," 213–14.

[12] Xenophilus is so obscure a figure today that his name is not to be found in even the most specialized works on Pythagoreanism. Citing Aristoxenus, Lucian (c. 120 A.D.), the satirist, wrote some five hundred years after Xenophilus in his work on old age (*De longaevis* 18 = *Fr.* 20a Wehrli): "Xenophilus, the musician, as Aristoxenus says, devoted himself to the philosophy of Pythagoras and lived past one hundred and five years." According to Diogenes Laertius VIII. 46, Xenophilus was among the last of the Pythagoreans whom Aristoxenus knew. They were themselves pupils of Philolaus and Eurytus of Tarentum, Aristoxenus' hometown.

of Sicily, Dionysius the Younger, who had now become a teacher of grammar. This would have been some time around 340 B.C., some three years or so after Dionysius had surrendered his dominion to Timoleon and was granted asylum in Corinth. To judge from Aristoxenus' account of this meeting, Dionysius, now given to recalling incidents from his former days of power, found in Aristoxenus an avid listener, one who used the occasion to record whatever bore especially upon his interest in Pythagoreanism. Just as he had absorbed many anecdotes as a boy from his father, Spintharus, so now he took down from Dionysius the celebrated story of the model Pythagorean friends, Damon and Phintias, which he later incorporated into his *Life of Pythagoras*.[13]

At this point in Aristoxenus' career, there could have been no better master living than Xenophilus who, in addition to being a most erudite Pythagorean scholar, was also a musician. Aristoxenus tells us this of Xenophilus:[14] "Xenophilus, the musician, lived in Athens until past the age of 105, having devoted himself to the philosophy of Pythagoras."

It was from Xenophilus, therefore, that Aristoxenus, already initiated into the fundamentals of Pythagorean harmonics, would have learned how inextricable were the bonds between harmonics and arithmetic, geometry, cosmology, and astronomy. For the Pythagoreans believed that harmonics was the medium through which the laws of number become applicable to the whole physical universe. That Xenophilus also exemplified the best to which the human spirit can aspire induced Aristoxenus to say in remembrance of him:[15] "He lived exempt from all human disadvantage and died in the glorious supremacy of his consummate erudition."

His mind invincible to the end, Xenophilus had become a friend as well as a master to Aristoxenus, despite his being more than sixty years older than Aristoxenus. To his credit, Aristoxenus evidently treated a man's age as a mediator rather than as a barrier between himself and the

[13] Much of what Aristoxenus wrote concerning his visit to Corinth has found its way into Iamblichus *De vita Pyth.* 233–37 (Deubner, 125. 18–127.11). Cf. *Frs*. 26–32 Wehrli.

[14] See note 12.

[15] *Fr*. 20b Wehrli = Valerius Maximus VIII. 13. Ext. 3: "ut ait Aristoxenus musicus, omnis humani incommodi expers in summo perfectissimae doctrinae splendore exstinctus est."

accumulated knowledge he was seeking. And Xenophilus could not but bring him all the closer by his years to the source of this knowledge: Pythagoras. The repository of wisdom that the years of Xenophilus represented to Aristoxenus must have rewarded both of them well with an age-proof friendship.

Aristoxenus' last teacher was Aristotle. He entered Aristotle's school, the Lyceum, when he was probably in his late thirties and remained there until Aristotle was forced by his political enemies to withdraw to Chalcis, where he died in 322 B.C.[16] Here at the Lyceum, where lay the grove sacred to Apollo and the Muses, Aristoxenus walked up and down every morning in the *loggie* with Aristotle and his other pupils discussing the most profound questions of philosophy and logic. Afternoons and evenings were devoted to lectures on less esoteric subjects to which the general public was invited. Aristoxenus offers us a rare glimpse into one of these lectures – the exoteric or popular type – in which he tells us what Aristotle had to say about a lecture on the Good given by his master, Plato, at the Academy:[17]

> It is perhaps better to go through what kind of study this is beforehand, so that, understanding in advance the road, as it were, which we must travel, we may proceed more easily by recognizing what part of it we are at and by not getting unawaredly a misconception of the plan of study. As Aristotle used always to relate, such was what befell most of those who listened to Plato's lecture on the Good. For, he said, they came, each expecting to find out some one of those things that people think good, such as wealth, health, strength – in general, some kind of wonderful happiness. But when the discourse was manifestly concerned with mathematics and numbers and

[16] The biography of Aristotle (384–22 B.C.) by Diogenes Laertius V is our main authority for his life. According to Diogenes, a charge of impiety was brought against Aristotle on the basis of a hymn and an epitaph that he wrote on Hermeias, a former fellow-student of his at the Academy. Both the hymn and the epitaph are reproduced by Diogenes; neither of them betrays any sign of impiety, but only admiration for the courage of Hermeias who had fought against the Persians. More likely, it was the surge of anti-Macedonian feeling after the death of Alexander that provided the Athenians with the real reason for forcing Aristotle to leave.

[17] *Harm. El.* II. 30–31 (Da Rios, 39.4–40.4).

> geometry and astronomy and the end result was that the Good is One, it seemed to them, I think, to be quite contrary to their expectations; some of them either derided the subject matter, while others found fault with it.

Aristotle's point in telling this story was not lost on Aristoxenus: in order to discover the principles and natural divisions of any science – one has first to define its province. In applying Aristotle's dictum to his own concerns – the science of Harmonics – Aristoxenus defined its province as Melody. From the facts of melody and melody alone, Aristoxenus was able, therefore, to discover the principles and natural divisions of harmonic science.[18]

Listening to Aristotle's advanced discourses – the acroamatic – as well as the more popular ones – the exoteric – Aristoxenus found the master-influence of his life. In the years he spent at the Lyceum, he was numbered among a group of gifted men who were united by a common view – that articulated by Aristotle: the goal of the state must be to produce men of the highest cultivation, men who would combine a love of learning and of the arts with a respect for law, "equality according to proportion," and virtue. At the same time, each member of this select company was encouraged to pursue his own individual goals with what must have been a splendid independence. It was during these years, then, that Aristoxenus developed his theory of music to a point far beyond what any musician had theretofore conceived of. He seems also to have lectured occasionally – as Aristotle's most outstanding pupils were invited to do – on the aspects of music that were to form the basis of his most important works. One such lecture was apparently of the advanced, or acroamatic, type, for like Aristotle's course of lectures on Physics called *Physikē Akroasis*, it was called *Mousikē Akroasis*.

[18] Melody, as Aristoxenus says in the opening lines of his treatise, comprehends numerous types of study, Harmonics being only one of them; the other types that pertain to melody are not defined by him but must be inferred from the treatise itself. In his words (*Harm. El.* I . 1; Da Rios, 5. 4–7): "The study of melody is a multifarious one and is divided into numerous types, of which the one called Harmonic must be considered first in rank and having an elemental function." The question raised by scholars is: What are the other studies of melody besides Harmonic? These, the other studies pertaining to melody, are the focus of the chapters to follow.

In time, Aristoxenus won a great reputation for being a thinker among thinkers, prodigious scholars all. They came to be called the Peripatetics from the fact that they would walk with Aristotle, listening and learning, discoursing and speculating, whether in he covered *stoa*, the *peripatos*, or among the Lyceum's groves.[19] To a man, these Peripatetics were polymaths of dazzling accomplishments, men to whom philosophy was a way of life. These were some of Aristoxenus' colleagues: Clearchus of Cyprus, who wrote on ways of living (*bioi*), on zoology and mysticism, as well as an encomium to Plato, erotica and paradoxes; Eudemus of Rhodes, who wrote on logic and rhetoric, as well as on theology, astronomy, geometry, and on Aristotle's *Physics*; Dicaearchus of Messana, who was much admired by Eratosthenes, Plutarch, Josephus, and Cicero, who wrote a history of culture, numerous biographies, and a geography of the known world. He also wrote on the constitution of various cities, a dialogue on the soul, works on Homer, and on competitions in music and poetry; Demetrius of Phalerum, who wrote on history, literary criticism, and rhetoric. He also wrote fables and proverbs; Meno, who wrote compendia of ancient medicine; Phaenius of Eresus, a valuable source for Plutarch and author of works on tyranny. And then there was Theophrastus, also of Eresus, the most renowned of all the Peripatetic pupils of Aristotle. Unlike Aristoxenus, whose father was a distinguished musician and scholar, Theophrastus was of humble descent, his father having been a fuller, or what we would call today a dry cleaner. Despite what disadvantages he may have suffered as a youth on that account, Theophrastus' abilities as a writer, scholar, speaker, and teacher came to be so valued, not only by his colleagues at the Lyceum but also by the Athenian public at large, that as many as two thousand people were said to have attended his lectures. His name was originally Tyrtamus, but in recognition of the "divine gracefulness of his style,"

[19] See Ross, *Aristotle*, p. 5. The image of Aristotle discoursing with his prodigious students in the groves of the Lyceum brings to mind what Victor Hugo said in his *Oration on Voltaire*: "... great men rarely come alone: large trees seem larger when they dominate a forest; there they are at home. There was a forest of minds around Voltaire; that forest was the eighteenth century. Among those minds were summits." Among his students, Aristotle may have been the summit; but he was not the only tall tree in the Lyceum.

Aristotle renamed him Theophrastus. When it came to selecting his successor to head the Lyceum, it was Theophrastus whom Aristotle designated in his will for this signal honor.[20]

Like Aristoxenus, Theophrastus wrote an enormous number of works, all of which were said to have abounded in excellence of every kind. Judging from the titles that have come down to us, Theophrastus' intellectual range seems to have had no limits. He was an authority on plants, animals, human nature, the law, politics, mathematics, physics, precious stones, meteorology, and much more. Nor was that all. Theophrastus was the only Peripatetic, aside from Aristoxenus, to have written on music. Of his works in this area – *On Music* in two volumes, and *On the Musicians* – only a few pages from the first-named work have survived.[21] Enough remains, however, to show that he and his colleague, Aristoxenus, though born of Aristotle, as two branches of a common trunk, disagreed so fundamentally from each other that they must have spent their days at the Lyceum in perpetual discord. On one thing only were they in full agreement: Pythagorean mathematical theory could never succeed in accounting for the perceived properties of music.[22] When it came, however, to defining the precise nature of these

[20] Aristotle's relationship with Theophrastus was long-lived and profound. Theophrastus' influence on Aristotle in scientific matters especially was so great that some scholars, such as Werner Jaeger and Joseph Zürcher, have gone so far as to attribute to Theophrastus many of the writings of Aristotle himself. See Grene, *A Portrait of Aristotle*, pp. 28–29. As reported by Diogenes Laertius, Theophrastus was a man of remarkable intelligence; industrious and diligent, he was always fond of discussion and always ready to do a kindness. His disciples seemed to have adored him and kept alive many of his pithy sayings, one of the nicest being: "The re-reading of one's writings makes for revisions. The present generation no longer puts up with delay and is altogether negligent."

[21] These have been preserved by Porphyry in his *Commentary on Ptolemy's Harmonics* (Düring, 61.16–65.15). They have been translated by Barker, II, pp. 111–118.

[22] Thus Lippman, *Musical Thought in Ancient Greece*, p. 161: "Common to the theories of vibration and velocity is the explanation of pitch as basically numerical, and in taking issue with this conception, Theophrastus joined Aristoxenus in a rejection of Pythagoreanism that is much more radical than that of their teacher, for in spite of his revision of the status of number, Aristotle made use of it in his own way to account for the phenomena of music."

properties, Theophrastus and Aristoxenus stood worlds apart from each other. Theophrastus approached the problem as a philosopher of nature or, in the ancient sense, as a physicist; Aristoxenus' approach was solely that of a musician.

According to Theophrastus, "The nature of music is one."[23] This singleness or oneness of music seems, on Theophrastus' analysis, to be a kind of circumambient stuff wherein the high and low pitches of music are present, but not limited or made distinct by any intervening intervals. Musical pitches seem simply to exist as a plurality of distinct things in a single mixture in which they are all together and from which they are somehow separated out. As Theophrastus has it then, pitch is all in all; it is in fact that without which sound itself cannot be said to exist. At the same time, he is at pains to argue that sound and, hence, pitch, has nothing to do with quantity, since, as far as the ear is concerned, quantity is a wholly irrelevant factor:[24]

> But if it is as notes that high and low ones differ from one another, we shall no longer have any need of quantity: for their own intrinsic difference will be sufficient by itself for the generation of melodies, and knowledge of the difference will be possible. For the differences will no longer exist in accordance with the quantities, but in accordance with the quality peculiar to the sounds, as is the case with colors.

Theophrastus apparently meant by this statement that the ear does not hear the quantitative causes of pitch differences – frequency of vibration or plurality of impacts on the air; the ear registers only the effects of such causes – the pitch differences themselves. And these pitch differences are sufficient in themselves for the generation of

[23] Porphyry *Commentary on the Harmonica of Ptolemy* (Düring, 65. 14–15).

[24] Porphyry *Commentary on the Harmonica of Ptolemy* (Düring, 62.21–25, translated by Barker, II, p. 113. As Barker II, p. 113, n. 12 explains, "His [*sc.* Theophrastus'] claim, summarily, is that nothing can be a note, or even a sound, without having a pitch. Hence, if the quantitative aspect were taken away, then on this hypothesis what was left would not be a sound at all, no matter what other attributes it had. … Sound is not a pitchless material on which pitch, in the guise of some 'quantity', can be imposed – a sound must have some pitch in virtue of being a sound."

melodies. The next question would be: What is the nature of that special class of musical pitches which the mind intuits to be melodious? Theophrastus did not address this problem; instead, he continued trying to prove by one argument after another that quantity is an irrelevant factor in the production of melody. Paradoxically, the harder he tried to prove this point, the more deeply and inextricably he involved himself in quantitative analysis. He began by interpreting pitch in terms of human physiology; then he proceeded to interpret pitch in terms of musical instruments; and when he failed to find a positive basis for the irrelevance of quantity, we see him reduced ultimately to a psychological explanation for the production of melody. His physiological analysis, as applied to the wind-pipe of the singing voice, almost immediately demanded certain terms of quantity: longer, shorter, wider, narrower:[25]

> This is clear from the force exerted when people sing. For just as they need a certain power in order to give out a high sound, so do they also in order to utter a low one. In the one case they draw in the ribs and stretch out the windpipe, narrowing them by force; in the other they widen the wind-pipe, which is why they make the throat shorter, since the width contracts the length.

Turning to musical instruments, Theophrastus was led into the same cul-de-sac: the relevance of quantity. The quantitative terms in this case are "shorter," "longer," "thicker," "thinner":[26]

> In *auloi*, in fact, the case is even clearer; for a high note requires less labor, since it arises from the holes that are higher up, while a low note demands greater force, if the breath is impelled through the whole [pipe], so that however much length is added, there is added the same amount of strength in the breath. In strings it is clear that there is equality in the two cases: for by whatever amount the tension of the thinner is tighter, by the same amount the one that seems slacker is thicker.

[25] Porphyry *Commentary on the Harmonica of Ptolemy* (Düring, 63.1–6), trans. Barker.

[26] Porphyry *Commentary on the Harmonica of Ptolemy* (Düring, 63.6–14), trans. Barker, II , p. 114.

Whereas he insisted that the apprehension of pitch differences is based on qualitative factors, Theophrastus ended up framing his explanation of these factors on the laws of quantity in the production of pitch. It is this that makes his inquiry so hard to follow. Lippman, *Musical Thought in Ancient Greece*, has put the case in this way:[27]

> Theophrastus does recognize special laws of quantity in tone, but the discussion of these, based as it is on a sensitive but qualitative science, constitutes a section of his inquiry that is quite difficult to comprehend.

Theophrastus' psychological explanation of melodic production is somewhat easier to follow, since it accords so well with the manifest conclusions of many modern musicologists, especially those who assert that music is the language of the soul:[28]

> For the movement productive of melody, when it occurs in the soul, is very accurate, when it [the soul] wishes to express it [the movement] with the voice. It [the soul] turns it [the voice], and turns it just as it wishes, to the extent that it is able to turn that which is non-rational.

Up to a certain point, then, Theophrastus is plain enough. The singing voice and that of musical instruments produce various pitches that can be organized into melodies. Such melodies derive, he argues, not from mathematical factors, but from the intrinsic qualities of musical pitches themselves. The musical form that such melodies eventually take is actuated by one agency only: the human soul. He says of music's nature, therefore:[29] "It is the movement of the human soul that occurs on its release from the evils arising from the passions."

It is when he turns to the question of the musical pitches themselves and their nature that Theophrastus is at his most original and maverick-best. The question that he addresses is: What makes the pitches differ from one another? In answering this question, he seems almost

[27] Lippman (note 22), p. 158.

[28] Porphyry (*Commentary on the Harmonics of Ptolemy* (Düring, 61.22–24), trans. Barker. See Bowman, *Philosophical Perspectives on Music*, pp. 103ff. on the "Sounding Inwardness" of music.

[29] Porphyry *Commentary on the Harmonics of Ptolemy* (Düring, 65.14–15).

to endow pitches with an actual existence, up to their very shape and form; that is to say, they are not for him lifeless components of melody that resemble nothing in actual nature. He insists then that musical pitches differ in highness and lowness from one another not by virtue of such quantitative factors as frequency of vibration, but solely because of differences in their intrinsic characters. A low note, for example, tends to spread out more in that it "travels everywhere all around," whereas a high note is more directional in that it moves "in the direction in which the utterer compels it to go." Theophrastus' argument to this effect is based on an assumption that is fundamental to understanding his theory: pitch differences are not constituted by the distances of intervals between them. Indeed, for him, intervals as such do not exist independently. In his words:[30]

> Again, it is not the intervals, as some people say, that are the causes of the differences [between pitches] and hence their principles, since if these are left out the differences still remain. For when something comes into being if certain things are left out, these are not the causes of its existence, not as productive causes, but [only] as things that do not prevent it.

It is not easy to give precision to Theophrastus' point regarding musical intervals, but, according to Barker, this is what he seemed to mean: a sound's pitch cannot be understood simply in terms of its distance – or interval – from another pitch, but must be an intrinsic feature of the sound itself.[31] That being the case, an interval cannot be thought of as the cause of a difference between any two pitches; rather, the difference between any two pitches must exist as an independent phenomenon. As Barker says:[32]

> The distance on the continuum between two given notes is conceived as constituted by a range of intervening pitches. Theophrastus is arguing that these cannot be the 'cause' of the difference between the two given notes, since that exists whether the intervening pitches are sounded or not.

[30] Porphyry *Commentary on the Harmonics of Ptolemy* (Düring, 64.25–27), trans. Barker.
[31] Barker, II, p. 117, n. 39.
[32] Barker, II, p. 117, n. 40.

The critical word in Barker's exegesis is *continuum*, this being one of the three elementary phenomena of music. The two others are time and motion. In this, the extant portion of his treatise *On Music*, Theophrastus was not only arguing that intervals on the *continuum* cannot be causes of pitch differences; he was doing much more: he was looking for a way to divide the *continuum* of melody without destroying it.

Zuckerkandl, *Sound and Symbol*, spoke of the melodic *continuum* in this way:[33]

> What do we hear – a progress advancing in uninterrupted continuity or an alternation of skips and halts, a discontinuous progress? There can be no doubt about the answer: we could not hear the melody as motion if we did not hear it as continuous. . . . Where is the continuously progressing line, the symbol of continuity of motion? Stasis-gap-stasis-gap; our graph is the perfect image of discontinuity. One is at a loss to understand how *this* can be heard as a continuous process.

The gapped motion heard by Zuckerkandl is thoroughly paradoxical: a melody which is perceived by the ear to be moving by leaps along an unbroken line, is in reality stopping at every change of pitch. How then can there be such a thing as tonal motion if a thing in motion from one place to another does not skip any of the intervening spaces, much less stop in any of them? This is the paradox for which Theophrastus found a thoroughly inventive solution. To begin with, he knew that pitches could not constitute spatially distinguishable parts of what is continuous, because what is truly continuous cannot be constituted of distinguishable or indivisible parts.[34] To deal with the paradox presented by tonal motion, Theophrastus followed the teachings of his master, Aristotle: that is, he thought of musical space as Aristotle thought of homogeneous natural substances – air or water. Aristotle thought of such substances as *continuous* in that their extremities are of a oneness and wherein there is no intermediate boundary or point. It is in this

[33] Zuckerkandl, *Sound and Symbol*, pp. 118–19.

[34] Thus Aristotle *Physics* 231a24–26: "Something that is continuous (*synechēs*) cannot be composed of indivisibles, such as a line composed of points, if indeed the line is continuous, but the point is indivisible." On Aristotle's claims to back up this argument, see White, *The Continuous and the Discrete*, pp. 23–24.

sense that Theophrastus speaks of music as something whose nature is "one." He believed in the reality of the melodic *continuum*, as he had no difficulty in conceiving of it as a real substance: pitch, pure and simple, a substance that is common to all melody and, hence, to all music. But if all such substance were to be sounded at the same time, there would be no melody and, certainly, no music. There would only be stasis. Rothstein has described the case in these terms:[35]

> Our sense of musical movement is continuous while pitches change in a melody by discrete steps. The movement from A to E is a leap. Yet somehow we experience it as a continuous movement. If we attempt literally to connect these tones, to fill the spaces between them, we get no more than a slide or a siren – a sound which is almost musically irrelevant and can also seem oddly static.

Theophrastus' plan was to give a logical explanation of the melodic *continuum*; but he met with difficulty at the very beginning of his undertaking. How could one speak of a *continuum* while allowing at the same time for the simultaneous presence of the distinct and paradoxically separate pitches of melody? Was the melodic *continuum* in fact a continuous magnitude constituted of nothing but a set of densely ordered discrete elements? Was there a fundamental continuity by virtue of the fact that the limits of these elements touched one another, thereby becoming one? Theophrastus' answers to these questions was apparently "Yes" – this because he believed in the existence of an actually subsisting *continuum* of melody. To him, it was something concrete, and not simply a confused perception of reality. Instead of trying to prove its existence, however, he left it to the composer of music to deal with as an actually existing musical substance. In this, its raw and unworked state, he saw it to be something unmelodic (*ekmeleia*), something as ill-defined as a slab of unpolished stone. In order for the composer to transform it into a work of art – something melodious (*emmeleia*) – he had to extract each note from its surroundings and reject all of the underlying

[35] Rothstein, *Emblems of Mind*, pp. 100–1. So, too, Theophrastus in Porphyry *Commentary on the Harmonics of Ptolemy* (Düring, 64.32–65.2): "For if someone were to sing simultaneously the continuous series of intervening positions as well, the sound produced would certainly be unmelodic" (trans. Barker).

continuum that he judged to be irrelevant or unfit for his melody. The finished product is understood by Theophrastus to be a copy of the human soul being released from its passions' ills. It is like a sculpted work – the Milan Pieta of Michelangelo Buonarroti, say, seen emerging from its stony medium. Unlike the effort of Michelangelo, however, in which part of the unsculpted background appears in the finished product, nothing appears in melody but the notes themselves. What has been discarded and rejected are the intervals between the notes of the finished composition. In Theophrastus' words:[36]

> It is therefore a great help that melody revolves around these [the melodic and the unmelodic], enabling us to find the notes that are attuned to one another. But it is these notes that are the causes of melody, while if the rejected intervals are made apparent, they are the causes of the unmelodic, whose principles they might be said to be, not those of melodic sound. Thus neither are the intervals the causes of the melodic, but damage it, at least when they are made apparent.

The ultimate conclusion of Theophrastus' theory is that there can be no empty space in the domain of music: there are only *continua* filled to capacity with densely ordered sonorous potentialities, some of which are pressed into service as melodic notes on the release of the soul's emotions. Whether this conclusion was faithful to Aristotle's conception of one-dimensional continuous magnitudes is open to argument. But one thing is certain: Theophrastus was trying his utmost to produce a theory that would satisfy Aristotle's criteria for the formal, structural properties of continuous magnitudes. That being the case, he felt confident that his was a defensible theory. Otherwise, he would not have treated intervals as he did, namely, as the principal cause (*aitia*) of *ekmeleia*: all unmelodic utterance. Intervals, on Theophrastus' conception, were by their very nature cacophonous; this was because they admitted an infinite number of discrete elements which, when sounded simultaneously, produced the very opposite of melody. Those people, therefore, who, he says, treated intervals as "the causes of pitch differences and hence

[36] Porphyry *Commentary on the Harmonics of Ptolemy* (Düring, 65.4–9), trans. Barker.

their principles," were in his estimation altogether misguided. Indeed, such people seem to him to be trivializing the significance of something real – pitch itself – by focusing their attention on something completely unproductive – intervals. Theophrastus' criticism on this point is a none-too-thinly-veiled attack on his colleague, Aristoxenus. For it was none other than Aristoxenus who was arguing vigorously (and probably constantly) that the miraculous order he heard in the constitution of melody had as much to do with intervals of various types of composition as it did with notes of variously distributed pitches.

When it came to dealing with his critics, Aristoxenus was never at a loss. Quite the contrary: he could be downright scathing. Unfortunately, he was not in a position to refute his most challenging critics: Ptolemy, in the second century A.D. and R. P. Winnington-Ingram in the twentieth. But he was on hand to reply to Theophrastus, and in this instance he was simple, direct and, it must be said, typically withering. To be sure, he did not mention Theophrastus by name; instead, he allowed the otherwise unknown harmonician, Eratocles (and his school), to bear the full brunt of his scorn:[37]

> Most of the harmonicians did not even perceive that a treatment of the subject (intervals) was necessary; but we discussed it in an earlier work. Eratocles and his school said only this much: that melody splits in two in either direction from the interval of a fourth; but they do not distinguish whether this melodic split derives from every fourth, nor do they say what the cause of this is; neither do they inquire into what way other intervals are put together with one another, or even whether there is a principle of synthesis for the constitution of one interval with another. . . . And though there is a miraculous ordering in the composition of melody, music is charged by some people with the height of disorder, because of how some of them have meddled with the subject under discussion.

To be sure, Theophrastus never charged music with the height of disorder, but his treatment of intervals may easily have been regarded by Aristoxenus as "meddling" in the extreme. For, as Aristoxenus saw

[37] *Harm. El.* I.5 (Da Rios, 9.15–10.7). On Eratocles, see Barker, II, p. 129, n. 22.

it – quite in opposition to Theophrastus' stated position – the intervals between the notes of melody are as vital as the notes themselves. His reason for this was a strictly musical one: it is within the intervals between the notes of melody that the motion of melody takes place.[38] To Theophrastus, in his attempt to give a scientific account of music's nature, two sets of phenomena seemed important: the intrinsic character of musical notes themselves, in isolation from their context; the gaps or intervals between notes, the contents of which are left out in the formation of melody. Aristoxenus, however, being the complete musician he indeed was, evidently had little patience with the subtleties of arguments such as that of Theophrastus. He regarded only one thing as supremely important: the knowledge of his own ear; and he valued theories and hypotheses only insofar as they were consistent with this knowledge. Theophrastus' theory had to have been regarded by Aristoxenus as inconsistent with the ear's knowledge in the most fundamental respects, the most obvious being that single notes taken out of context do not make for melody. Equally important, the context of melody is dependent upon the intervals between the notes of melody. Most important of all, Theophrastus' theory took no account of those phenomena that Aristoxenus considered axiomatic in the perception of melody: time, motion, and the melodic *continuum*.

Aristoxenus began by asserting the existence of motion by the singing voice as it progresses from note to melodic note in the production of melody. "Does the voice really move?" Theophrastus might have asked with a certain degree of scepticism. There is nothing conciliatory or accommodating in Aristoxenus' reply to such a question:[39]

[38] As Aristoxenus explains in *Harm. El.* I. 13 (Da Rios, 18. 1ff.), Pitch (*tasis*) must be distinguished from tension (*epitasis*) and relaxation (*anesis*) on the grounds that pitch is where the voice comes to rest, while the risings and fallings of the voice occur between the pitches. Barker, II, p. 133, note 42, says accordingly: "he [*sc.* Aristoxenus] applies the terms primarily to vocal sound itself, not to the physical means of its production: it is the vocal sound, not the vocal organ, that is tensed or relaxed . . ."

[39] *Harm. El.* I. 9 (Da Rios, 13.23–14. 4), trans. Barker. In his note on this passage, Barker, II, p. 132, n. 38, points out that the "different enquiry" deals with the physics of sound.

> Whether it is actually possible or impossible for the voice to move and then come to rest upon a single point of pitch, is a question belonging to a different enquiry, and for the purposes of the present science an account of the motion involved in each of these is unnecessary.

The implication in Aristoxenus' reply is clear: no musician would think of asking such a question. The reason is incontestable to a musician: motion is what the ear apprehends in melody. Zuckerkandl has stated the case in terms that Aristoxenus would have been pleased to acknowledge:[40]

> Musical contexts are *motion* contexts, kinetic contexts. Tones are elements of a musical context because and insofar as they are conveyors of a motion that goes through them and beyond them. When we hear music, what we hear is above all motions.

The skeptic might have pursued the subject: "If, as you insist, the motion of the singing voice is not merely an illusion, that it is, in fact, real to the ear, and that the focal point in music must be sought in melodic motion, tell me, then, where does this motion actually take place and how specifically does it manifest itself?" Aristoxenus says, "The voice must in singing a melody pass imperceptibly through the space (*topos*) of the interval." "Well, then, if, as you say, the motion of the voice through the spaces or intervals between the notes of melody is imperceptible, what is it that the ear apprehends as genuine motion?" Aristoxenus explains that the two key elements that promote the motion of the singing voice are tension (*tasis*) and resolution, or relaxation (*anesis*):[41]

> It is evident that in singing the voice must make its tensions and relaxations imperceptibly, and when uttering the pitches themselves must make them apparent; for its progress across the interval which it traverses, whether relaxing or increasing tension, must not be detected, whereas it must give out the notes that bound the intervals clearly and without movement.

In these profound utterances on the nature of melody, Aristoxenus applies the concept of motion not to the vibration of vocal chords, or to

[40] Zuckerkandl, p. 76.
[41] *Harm. El.* I. 10 (Da Rios, 15.6–12), trans. Barker, II, p. 133.

that of strings or air-columns, not, that is, to the physical production of musical sound; he applies the concept of motion uniquely to musical sound itself. It is here that Aristoxenus reaches across the millenia to modern musicians. Thus Cooke who, in his exasperation with many writers on music, sounds much like Aristoxenus himself:[42]

> The simple but amazing fact is that, although certain directional movements of pitch have occasionally been analysed as 'symbols' (Schweitzer on Bach, for example), no one has seriously got down to the business of discovering, in each particular context, *exactly what the notes of the scale are and what tensions exist between them*.

And Rothstein might have been consulting Aristoxenus himself when he wrote:[43]

> Continuity [what Aristoxenus termed *synecheia*] comes from something beyond literal pitch, found in the very notion of relation. Just as a mathematician can see a number in terms of its nexus of relations to others – its properties granted by nature, and its place in a particular organization of concepts and rules established by the art – so does a musician experience tone. Melody is a musical state in which those relations are harnessed, creating a field of tensions and relaxations, anticipations and surprises.

The main doctrine to which Aristoxenus remained constant throughout his writings on music concerned melodic determinism and the composer's own musical intuition. He believed that there is no such thing as chance in melody, but that the production of a melody that will move the soul is determined by natural laws.[44] He insisted, therefore, that the composer must understand the properties of his material – the notes

[42] Cooke, *The Language of Music*, p. 40.

[43] Rothstein, *Emblems of Mind*, p. 102.

[44] To Aristoxenus, the natural laws governing the ordering of the melodic elements are comparable to those productive of intelligible language. As he explains in *Harm. El.* I. 27 (Da Rios, 35.10–17), the placement of notes and intervals in melody is not at all random, but is as carefully determined as the placement of letters and syllables in language. Analogies between speech and melody did not originate with Aristoxenus, but seem to have their roots in Plato *Philebus* 17Aff. Cf. Barker, II, p. 213, n. 12.

and intervals of the singing voice in its progress (by moving and stopping) through musical space. This motion and stopping peculiar to the singing voice he termed "intervallic" (*diasystēmatikē*) as opposed to the motion of the speaking voice, which he called "continuous" (*synechēs*). As he says:[45]

> We say, therefore, that the speaking voice is continuous, since when we speak the voice moves through space in such a way that it seems never to stand still [on pitch].

Without a careful discrimination between the properties of speech and song, he felt that it would be very difficult, if not impossible, to understand the properties of notes and intervals, the elemental components of the material of music. For just as bricks can be reduced to their components – earth and water, let us say – so the material of music – the sounds produced by the singing voice – are reducible to single units in the form of individual notes and the intervals between them. When, however, these single units or notes are compounded into scales and modes in which each is assigned a particular function (*dynamis*), something miraculous happens, something that no pile of bricks or stones is capable of achieving: a gravitational field of motion is released within the scale, the distinctive characteristics of which are transferred to any melody that is composed according to the laws of that particular scale.[46] Such a melody was for Aristoxenus the embodiment of true *harmonia*, and he regarded it as a miraculous ordering of the musical phenomena.

The concept of *harmonia* under which Aristoxenus assembled the components of melody had as its ruling principle, not number, but,

[45] *Harm. El.* I. 9 (Da Rios. 14. 12–14).

[46] Zuckerkandl, *Sound and Symbol*, p. 100, speaks of this phenomenon in melody as a dynamic field: "... a rise and fall not in tonal space but in the tonal dynamic field, in relation to a given audible center of force." And Rothstein, *Emblems of Mind*, p. 103, observes of such systems of organized notes and relations: "It is a kind of musical geometry, articulating relations of closeness and distance, curved lines and intersections. And it is found in this most elementary form of relation – two notes defining a kind of force field in musical space." Thus Barker, "Aristoxenus' Theorems," 62: "Not all our tunes obey all Aristoxenian laws, but *his system has been modified and elaborated rather than buried*" [italics supplied].

rather, the disposition or natural *synthesis* of magnitudes or intervals that admit motion (*kinēsis*) and the discrete points of pitch that admit stoppage (*stasis*). The figures, or *schēmata*, of this type of synthesis were as formal as those of an Aristotelian syllogism; they admitted of no extraneous elements. As Aristoxenus explained it, a melody that accords with the laws of *harmonia* owes its melodiousness not simply to the notes and the intervals of which it is composed, but above all, to a definite principle of *synthesis* whereby its intervals (possessed of motion) and its notes (possessed of stoppage) are collocated. Aristoxenus' fundamental conviction, which lies at the basis of his doctrine, is this: the ear, whether educated by habit or ordained by nature, will find continuity wherever it truly exists. And he firmly believed that continuity exists in its purest form in melody.

There is, however, a serious logical difficulty about this doctrine. It begins with Aristoxenus' description of the singing voice and its motion through the intervals or spaces between the notes of melody. It is a motion that is peculiarly "intervallic":[47]

> The voice seems to the ear to stop on a point of pitch, and having done so, to come to a stop on a second point of pitch, and to repeat this alternating process continuously.

The question is, if the voice is moving from note to note, where in fact is it in the interval between notes? And what, in reality, is the nature of an interval between notes? In Aristoxenus' theory, intervals are spatial magnitudes that appear to have the formal, structural properties of infinite divisibility. Indeed, he even characterizes the locus of a particular note – the *lichanos* – as *apeiron*, or infinite. At the same time, he treats all notes as discrete entities that are themselves indivisible. But these facts, derived as they are from the ear's perception, contradict the fundamental ontological principle laid down by Aristotle: what is infinitely divisible and continuous cannot be constituted of discrete points. Theophrastus had seen the dimensions of this problem in music and had attempted to deal with it just as Aristotle did with the universe: he approached the domain of music as if it were a self-contained

[47] *Harm. El.* I. 9 (Da Rios. 14.8–12).

universe, one that was finite in extent and filled entirely with matter (see note 20).

As Aristoxenus saw it, the most obvious way of avoiding the logical difficulties of the *continuum* was first to accept motion of the voice as a fact of experience; and following that, to assume that a thing can move only in an empty space. In this way, he could proceed to distinguish between matter – the material elements of music – and space – the *topos* wherein the voice moves. According to this view, musical space cannot be construed as nothing, but is of the nature of a receptacle which may or may not have any given part filled with matter.[48] And where there is *not* matter, there is still *something*: those miraculous tensions and relaxations occurring between the notes of melody. Aristoxenus believed further that musical space, if considered solely in the abstract, is infinite in extension and, as seen in the case of the locus of a *lichanos*, infinitesimal in diminution.[49] Thus, he says of both concords (octaves, fifths, and fourths) and discords (all intervals smaller than a fourth) that they are theoretically capable of infinite extension; for if one adds to an octave, for example, any concord, "whether greater than, equal to, or less than an octave, the sum is a concord. From this point of view, there is no maximum concord."[50] One can also say that, theoretically, there are micro-intervals without number. But since Aristoxenus was determined to compose a theory of music based wholly on human capabilities and not on abstract theory, he did not consider it necessary to deal with anything such as a maximum interval that lies beyond the ear's

[48] Within this *topos* or *continuum*, there are only two directions in which the voice can move: up and down; but there are countless ways in which the voice can effect changes of quality and aspect. To an Aristotelian, all such changes are construed as motions. At the same time, the *topos* in which these motions transpire is defined in Aristotelian terms solely by the things that move within it. The moving thing in this case is the moving voice, and the *topos* in which it moves is a *continuum* which is, by definition, homogeneous.

[49] As Aristoxenus puts it in *Harm. El.* I. 26 (Da Rios. 34.3–4): "The number of *lichanoi* must be thought of as infinite." The identity of any *lichanos* depends upon its melodic function (*dynamis*), as Aristoxenus goes on to explain in more elaborate detail in *Harm. El.* II. 47–48 (Da Rios, 58. 10–60. 3). On Aristoxenus' concept of *dynamis*, see Barker, "Aristoxenus' Theorems," 52ff.

[50] *Harm. El.* I. 20 (Da Rios, 25.17–26.2).

ability to judge, or to admit of any interval too small for the voice to negotiate. He therefore limited his theory to two things only: that space within which the ear can discriminate every type of melodic motion that is musically intelligible; those intervallic motions in musical space which the voice is capable of executing. In his words:[51]

> Whether the constitution of melody, if considered in the abstract, will turn out to extend to infinity, should perhaps be the subject of another inquiry and not necessary to the matter at hand; this is a subject that must be undertaken at a later time.

For the present then, Aristoxenus was concerned only to establish the most natural limits of melody: the minimum concordant interval – the fourth – within whose bounds the voice can place its pitches and those placements of pitch that the ear can readily distinguish; the maximum concordant interval that the ear can readily identify and the voice can securely produce. This maximum interval, he concluded, was at best two octaves and a fifth. He says accordingly:[52]

> It is evident from what has been said, therefore, that when we progress toward the smaller limit, it is the nature of melody itself that determines the smallest concord, the fourth; but when we progress toward the greater limit, the greatest of the concords is determined somehow by our own natural capacities.

To make his theory a fully realized system of logically interrelated propositions, Aristoxenus had first to link it to three things of major import: what is thought, what is experienced, and what is. What is thought belongs in Aristoxenus' view to the activity of the musically intelligent mind (*dianoia*) and, because this activity determines what is embodied in the synthesis of all melodic consecution, it must be consulted as the final authority on the subject. This activity is powerful enough to transcend the ear's perception and, as such, is so mysterious that Aristoxenus cannot attribute any predicates to it; he can only speak

[51] *Harm. El.* I. 15 (Da Rios, 20. 11–14). This theoretical point is not discussed subsequently, however.
[52] *Harm. El.* I. 21 (Da Rios, 27. 8–11).

of it as something that is all-knowing and all-determining. His word for this activity of the musical mind is *synesis*. Because it is capable of determining *a priori* to there being given in particular instances of melody those conditions that make for all musical discourse, it is rendered here as "musical intuition":[53]

> And if musical intuition (*synesis*) is hidden somewhere deep within the soul, and is not palpable or visible to the average man, as is the working of the hands and other such operations, we must not suppose our statements to be inconsistent on that account. For unless we regard that which does the determining as our absolute and ultimate authority as opposed to that which is determined, we shall end up missing the truth altogether.

It is as hard to say exactly what one experiences on hearing music as it is to define time. Time – the paradigmatic *continuum* – was defined by Plato as "the moving image of eternity," because time, he felt, could not be thought of apart from the eternally moving planets in their heavenly orbits.[54] So, too, music, as it is experienced, may be thought of as the moving image of time, since it cannot transpire apart from the time it charters for itself. As it is experienced, music makes for us a world of ideal conditions; it appears to the ear to admit of sizeless points between which there is an alignment and among which there are centers of gravity. Although these points have no size, they do have position, and they seem so sharp and apparent that they coincide perfectly with idealized instants of time. They seem to be alive, but at the same time they are as evanescent and ungraspable as points of energy or bolts of lightning. Combined together into melodies, they erupt from emotional depths whose source can never be adequately plumbed because, as Aristoxenus

[53] *Harm. El.* II. 41 (Da Rios, 51.16–52.4); Cf. Levin, "*Synesis* in Aristoxenian Theory," pp. 212–14.

[54] Plato *Timaeus* 37D5. Thus Taylor, *Commentary*, p. 187: "The sensible world is a thing of passage, but it never passes *away*; its passage fills all time, and of course, the formal laws of its structure remain the same throughout. So it really is a moving or passing 'image' of the truly abiding. . . . i. e. time, which is *measured* duration, may be said to be, in virtue of its character as measurable, an image of eternity."

says, "It is hidden somewhere deep in the soul." Musical notes in a melodic context are not experienced as sound waves; if anything, they seem to be more akin to brain waves. At the very moment they are present to us, they have already begun to pass away – like time itself. They are felt to be as untrappable as visual photons.

In attempting to describe the experience of music, Aristoxenus invoked the phenomena of tension and resolution in the motion of melody; and these provided him with a model of gravitation and attraction, a model of magnetism in action.[55] This in turn rewarded him with an awareness of the energy, or *dynamis*, operating between the notes of melody. He argued therefore that wherever one perceives tension and resolution in melody – and one perceives it everywhere – one can assume that a mobile order of attunement has brought a diversity of musical functions (*dynameis*) into play. Zuckerkandl says much the same thing, for like Aristoxenus, he has consulted his own ear of musical reason. He begins by asking if on hearing two notes in succession – E and A – whether we hear only the succession of two notes of different pitch, and answers his question in this way:[56]

> It is music with which we are dealing – and we have found that in the entire range of music no such thing as "the tone e" or "the tone a" occurs; what occurs is always and only the tone e *with a particular dynamic quality*, the tone a *with a different dynamic quality*. The dynamic quality, not the pitch, makes the tone a musical fact. Hence, whenever we have a succession of two notes, an interval, as a piece of tonal motion – as an element, that is, in a musical context – we must necessarily hear something in it besides different pitches, namely, different dynamic qualities.

[55] The phenomena of tension and resolution that are so keenly felt between the notes of melody were described for the first time by Aristoxenus. Since then, almost every practicing musician has had occasion to remark on their mysterious vitality in melodic expression. And this has in turn provoked many a musician to insist that the spaces between the notes of melody are as important in the interpretation of melody as are the notes themselves. Thus Cooke, *The Language of Music*, p. 40: "The expressive basis of the musical language of Western Europe consists of the intricate system of tensional relationships between notes which we call the tonal system." See note 43.

[56] Zuckerkandl, *Sound and Symbol*, p. 91.

These are just some of the phenomena that are experienced in melody and which, if changed even minimally, cause the melody to lose its essential identity. The problem is that these phenomena have no correlation in the real world, namely, in the world of what is. For what is consists of those things that have actual existence outside of the mind, but which cause in us a mental sign of their existence. The second part of the problem is this: everything that exists in the real world contradicts what is experienced by the ear in melody. On hearing a melody, the ear experiences a continuity of intervallic motion in a symmetrical and homogeneous space; here the singing voice moves unimpeded by anything save its own limitations and those constraints imposed on it by the laws of melody. In reality, however, melody is composed of distinct parts – discrete notes that cohere closely together while leaving spaces between them. The question is: How can what is truly continuous be constituted of distinguishable or indivisible parts? It is a conundrum similar to that involving time: measuring time and finding discrete instances of time is, as Rothstein observed, "like considering an arrow to be at rest at every moment of its flight because we can specify its location."[57] To resolve this difficulty, Aristoxenus had to do much more, then, than to distinguish the material elements of melody from the space or *topos* within which the singing voice moves in its distinctly intervallic way. He had to find a way of reconciling what he believed to be the continuity of musical space with the physical fact of its discontinuity. In other words, he had to do something virtually impossible: he had to create continuity out of discontinuity.

This discontinuity can be discovered in two ways: tuning an instrument by ear; dividing an octave in the ratio 2:1 mathematically. The first method, which is described by Aristoxenus in explicit detail, is still being used today by harpsichordists and players of stringed instruments. It entails tuning by the concords, fourths and fifths.[58] If one

[57] Rothstein, *Emblems of Mind*, p. 49, thus points out: "These paradoxes succeed in baffling us because they present an idea of space and time that is utterly at odds with our everyday experience. They proclaim an endless procession of detached points instead of a seamless, continuous motion, in which there is no such thing as a 'next instant' in time or a 'next point' in space.

[58] In *Harm. El.* II. 55, 56 (Da Rios, 68.17ff), Aristoxenus explains how the loci of all the pitches of a scale are determined solely by tuning through fourths and fifths; the final arbiter in this process is the musically educated ear. Tuning

tunes by taking successive true fourths, that is, fourths that conform to the Pythagorean ratio 4:3, and true fifths, that is, fifths that conform to the Pythagorean ratio 3:2, all the resulting octaves will be slightly out of tune. The reason lies in the physical discontinuity of musical space. To put it in mathematical terms, the expansion of the octave ratio, 2:1, is incommensurable with the expansions of the internal concords, fourths and fifths. This means that one can never hope to arrive at a true octave by tuning with true fourths and fifths. This rupture in the musical *topos* – a kind of shifting of its planes – is commonly represented in the circle of fifths, so-called, within which any starting note will become a launching pad into an infinity of discontinuities. The result is an endless series of notes differing in pitch from one another, such as: E♯ and F, B♯, C and D𝄫, F+, G and A𝄫, and so on into infinity.[59]

Richard Wagner said somewhere that music is "the inarticulate speech of the heart, which cannot be compressed into words, because it is infinite." This "inarticulate speech of the heart" is in fact confirmed by mathematics to be *alogos*, literally, "inexpressible" or "unspeakable." It is embedded in the square root of 2 (or $\sqrt{2}$), which appears on the division of the whole-tone in the ratio of 9:8. It is considered to be *alogos*, an irrational number, for when it is converted into decimals it yields the infinite series 1. 4142135... . Infinity thus pours forth from the discontinuity of the octave with its own parts,

by consonances from a pitch E, for example, will fix the pitches in a diatonic scale:

E – A, A – D, D – G, G – C, C – F, F – B♭, B♭ – E♭, E♭ – A♭

Tuning in the reverse order from the same pitch E, will fix these pitches:
This yields the pitches of the sequence

B♭ C D E♭ F G A B♭

E – B, B – F♯, F♯ – C♯, C♯ – G♯, G♯ – D♯

This yields the pitches of the sequence:

E F♯ G♯ A B C♯ D♯ E

For a diagram of the process, see Barker, II, p. 169, note 114. Cf. Hubbard, *Harpsichord Regulating and Repairing*, pp. 29–31.

59 The problem is explained in detail by Sir James Jeans, *Science and Music*, pp. 165–68. See Fig. 3, in which ascending and descending Perfect Fifths lead to an infinity of atonalities.

so that the octave is to its internal twelve semi-tones as .666666666 to infinity.[60]

[60] Infinity has not always been greeted with enthusiasm. For example, Felix Holt (in George Eliot's novel of the same name) observed: "Your dunce who can't do his sums always has a taste for the infinite." The most alarming occurrence of the infinite series 666 is to be found in *Revelations* 13.18: "Here is wisdom. Let him that hath understanding count the number of the beast: for it is the number of a man; and his number is Six hundred threescore and six." Perhaps this is another way of saying that music and nature are alike in that they are each inexhaustible, that theories about both will always end up being "better and better approximations, and that this process will never come to an end." See Richard Morris, *Achilles in the Quantum Universe*, p. 117.

4 Magnitudes and Multitudes

> A quantity is a multitude
> if it can be numbered;
> but it is a magnitude if it can be measured.
>
> Aristotle, *Metaphysics* 1020a8–10

IN THE OPENING LINES TO BOOK II OF WAR AND PEACE, TOLSTOY offered the following insight on the continuity of motion:

> By adopting smaller and smaller elements of motion we only approach a solution to the problem, but never reach it. Only when we have admitted the conception of the infinitely small, and the resulting geometrical progression with a common ratio of one tenth, and have found the sum of this progression to infinity, do we reach a conclusion to the problem.

This is almost exactly what Aristoxenus did. As he conceived the problem of melodic motion or, as he put it, the motion of the singing voice, it was only by taking infinitesimally small units for measurement and attaining to the art of integrating them – that is, finding the sum of these infinitesimals – that he was he able to arrive at the laws of melodic consecution. The common ratio commanding the infinitely small basic elements of his progression was one-twelfth. In working toward this goal, Aristoxenus contrived to break down the mathematically logical opposition between magnitude (*megethos*) and multitude (*plēthos)*, in effect, between geometry and arithmetic.[1] The very nature

[1] Aristoxenus' method, which will be treated in the chapter to follow, is one of approximation. It allows as legitimate the notion of complete divisibility,

of his method invited him to believe that he could achieve his goal without altering the nature of scientific reasoning. Directly inspired by his knowledge of Pythagorean theory, he discovered a completely new way of dealing with musical intervals: he substituted arithmetic for geometry. It was necessary for him to do this if he wished to extend mathematically rigorous methods to problems where no quantity was involved.

Magnitudes, as Aristoxenus had learned, were measurable distances that could be expressed only as ratios of string-lengths or air-columns,[2] the results of these computations proving that the octave in the ratio of 2:1 was in truth something less than the sum of six whole-tones and that the whole-tones in the ratio of 9:8 could not be divided into equal halves. Yet, Aristoxenus maintained in the face of these mathematical proofs that the octave consisted of six whole-tones and twelve semitones, that the fourth was equivalent to two whole-tones and a semitone, and that the fifth was equivalent to three whole-tones and a semitone. What is more, he insisted that the *diesis* or enharmonic quarter-tone was obtained on the division of the semitone. This meant that such micro-intervals as the *eklysis* consisted of three quarter-tones

according to which a musical interval or magnitude can be divided potentially at any point whatever. In formulating his method, Aristoxenus relies on the concept that in such an interval there is always at every point of division a remaining interval left over, an interval that is itself subject to being further divided. This amounts to saying that the conceivable divisions of an interval are infinite. Accordingly, Aristoxenus saw his task as one of delimiting the genuinely melodious divisions of any intervals while, at the same time, *saving* the infinite from all efforts of geometers to abandon that concept.

[2] A column of air when enclosed by some rigid material like wood or metal is, in some respects, comparable to a stretched string in its mode of vibration. There are also some marked differences between them: for example, the sound-producing waves traveling along a stretched string are transverse, being caused by displacements of the string at right angles to the direction in which the wave is traveling. In the case of air-columns, the traveling waves are longitudinal, the compressions and rarefactions of air traveling in the same line as the waves themselves. In both cases, the same thing holds true: the period of each vibration is exactly proportional to the length of the column of air which is vibrating and to the length of string that is vibrating. Cf. Jeans, *Science and Music*, p. 64; p. 113.

and the *ekbolē* of five quarter-tones.[3] He went so far as to verify these calculations by a process of adding and subtracting rational numbers.

To an orthodox Pythagorean, this was mathematical heresy. In truth, however, Aristoxenus was doing something highly original and daring in the extreme, something that no one had ever attempted before: he was arithmetizing continuous magnitudes.[4] His goal was to facilitate the treatment of these magnitudes as *continua* constituted of individual elements or sets of points that are intuited to be discrete. His method – a radical departure from that of the Pythagoreans – consisted in the ordering and disposition of the melodic elements – notes and intervals – as they are apprehended by the ear. From his perspective, the Pythagoreans, through their use of pure mathematics, had divorced melodic knowledge from its perceived objects, thereby making of it an abstract science. Aristoxenus arrived at a different method, one that was determined by the peculiar nature of these same objects. In brief, to Aristoxenus melody was distinct from mathematics as a science, because its proper object was not quantity, but the motion of the singing voice. And this could not be expressed in mathematical ratios as if

[3] Aristoxenus' method of verifying these empirical facts involved tuning by consonance, the details of which are discussed in Chapter 3, note 58. But because this method was no more reliable than that of mathematics for determining the sizes of the various micro-intervals in melodic use, Aristoxenus invented a strategy for mapping out these intervals, the originality of which is alluded to in note 1 of this chapter. It was, in fact, so original that Aristoxenus had every reason to assert in *Harm. El.* II. 35 (Da Rios, 44. 10–13) that no one before him had ever attempted to do what he managed to accomplish against all mathematical odds: that is, establish the loci (or *topoi*) of all the moveable notes in the tetrachord, those notes which made for all the generic notes in melody. In making this claim, Aristoxenus was implicitly rejecting the method of the Pythagoreans, for this method proved that there was in fact no way to represent the micro-intervals. On that basis, they could be said to exist only in the imagination of the mind's ear.

[4] The standard Pythagorean model for dealing with musical intervals was a geometrical extension, or magnitude (*megethos*). In opposition to this frame of reference, Aristoxenus was bent on reducing the mathematical model to an arithmetic concept of multitude (*plēthos*), or collection of discrete quanta. On the Pythagorean model and its origins in the Academy, see B. L. van der Waerden, "*Die Harmonielehre der Pythagoreer*," 164–65.

it were nothing other than quantity. In the end, however, Aristoxenus was to controvert all the harmonic truths that had accumulated over the centuries and that were eventually comprehended in the work of the Pythagoreans' best exemplar: the *Sectio Canonis* of Euclid. In the process, he won for himself many critics and few, if any, champions. As matters now stand, therefore, direct study of Aristoxenus' theory is matched only by endless disputes about his treatment of the subject – harmonics. Admittedly, his writings, like those of his teacher, Aristotle, go far to sustain the disputes. At the same time, however, the teachings of Aristotle do much to support his position. It could scarcely be otherwise: Aristotle, to contemplate nature, took his evidence from the same source as did Aristoxenus – the direct testimony of the senses – and Aristoxenus approached music from the same position and for the same purpose as Aristotle approached nature: to account for the way in which the phenomena in question present themselves to the senses.

The phenomena in question here are pitch, loudness, and timbre, all of which can be represented mathematically on the basis of the unifying principle – motion. To their eternal credit, the Pythagoreans had found a way to describe pitch in terms of number and numerical ratios. They saw the cause of pitch in the vibratory motion of a stretched string; they heard the effect of such motion in the corresponding rise and fall of the pitch produced by the stretched string. Thus, they could conclude, as did Archytas, that high-pitched notes move faster than low-pitched ones.[5] When it came, however, to vouch for the truth of their observations, they used their eyes. To be sure, they used their hands to pluck the strings whose behavior they had under observation. But it was the additional use to which they put their eyes in the interest of such observations that led them to relate the elements of pitch and interval to

[5] Musical pitch was regarded by the Pythagoreans as inseparable from the motion that is its cause. Archytas says accordingly (*Vors*. 47B1; DK, I. 435. 13–14): "That the high-pitched notes move more quickly, while the low-pitched notes move more slowly, has been made obvious to us from numerous examples." The assumption of motion as the principle, or *archē* of audible phenomena, and the adduction of number as the primary element or *stoicheion* of musical pitch supplied the *Urstoff* for the Pythagorean doctrine of harmonics. Cf. Alan C. Bowman, "The Foundations of Early Pythagorean Harmonic Science: Archytas, Fragment 1," *Ancient Philosophy* 2 (1982), 79–104.

number. This use was twofold: to make accesssible to science what the eye cannot see; to verify and, if necessary, to correct what the ear hears or, as the case may be, what the ear seems to hear. What the eye *can* see is that the shorter the string under tension, the faster it vibrates, and conversely, the longer the string, the slower its rate of vibration. To these facts of observation, the ear can add its own: the lower pitches emanate from the longer string; the higher pitches are emitted by the shorter string. Because, however, the eye cannot determine with any exactitude how fast a string may actually be moving, and the ear cannot be relied on, according to the Pythagoreans, to recognize the exact degree by which one pitch may differ from another, the evidence of measure had to be enlisted. For, touched at visually measurable lengths, a stretched string can thereby be made to reveal its mathematical properties to a certainty – to the same certainty, that is, as those of a straight line. Observing that between every pair of points on the line – or on the stretched string – there was a relation having certain properties in virtue of which such a relation was measurable, the Pythagoreans developed a theory of harmonics based upon the metrical division of the canon. This process of division or segmentation of a string stretched above a canon, or ruler, was pursued by the Pythagoreans as a geometry of the straight line or, what Bertrand Russell in his *Principles of Mathematics,* calls metrical geometry.[6] The mathematical truths derived over the centuries from this course of study were eventually formalized by Euclid in the remarkable little treatise aptly titled *Katatomē Kanonos*, *Division of the Canon*, or *Sectio Canonis*.[7]

[6] As Russell explains in the *Principles of Mathematics*, pp. 407ff., in metrical geometry, the numbers adduced "can be given a one-to-one correspondence with the various relations of the class in question." The class in question in this instance is *distance*, which the Pythagoreans treated as a class of magnitude, or *megethos*. If the distances be magnitudes, any two must be equal or unequal. Since the distance in this case is a rational number, and rationals are one-one relations, additions and subtractions of such magnitudes are pursued not arithmetically, but by relational multiplication and square roots, respectively. Cf. Russell, *Principles*, pp. 254ff.

[7] The treatise is contained in *Jan*, pp. 148–66. It is translated by Barker, II, pp. 191–208. The text, translation, and commentary, together with the commentaries of Porphyry and Boethius, are provided by André Barbera,

The opening lines of the *Sectio Canonis* bring into decisive action those fundamental forces of the Greek genius for charting the road to truth – thought and perception, the way of reason and the way of the senses. They do this with the effortlessness of a master thinker and with the presentiments of a genius at thought. For only the power and purpose of a wisdom unrealizable by the average intellect could have inferred, as does Euclid, so much from so little, and express it withal in the majestic simplicity of the opening words of the *Sectio Canonis*.[8] It begins with stillness, the element of stricken silence, the better to introduce motion, the physical cause of musical pitch:[9]

> If there were stillness and nothing astir, there would be silence; with silence and nothing at all in motion, nothing would be heard. If, therefore, something is to be heard, there must first be percussion and motion. Consequently, since all musical notes derive from the advent of some sort of percussion, and there can be no percussion without there first being motion – some of which motions are more densely distributed, others more widely separated, the motions of greater density producing the higher-pitched notes, those more widely separated producing the lower-pitched notes – it follows necessarily that notes are higher in pitch when they are constituted of more densely massed and more numerous motions, but they are lower in pitch when they are constituted of more rarified and fewer motions.

The Euclidean Division of the Canon. See also, Barker, "Methods and Aims in the Euclidean *Sectio Canonis*," 1–16, for critical analysis.

[8] The grounds for assuming that the *Sectio Canonis* is the unified treatise by the single author, Euclid, were presented by Levin, "Unity in Euclid's 'Sectio Canonis'," *Hermes* 118 (1990), 430–43. This view is far from a unanimous one, however. Over the centuries, opinions as to authorship have varied widely. The arguments for and against Euclid as author have been collected and carefully evaluated by André Barbera, *The Euclidean Division of the Canon*, pp. 3–29, whose even-handed analyses do not favor one side of the debate over the other. In an earlier work, "Placing *Sectio Canonis* in historical and philosophical contexts," *JHS* civ (1984), 161, Barbera committed himself to the following view: "The style and language of the *Sectio* are like those of Euclid's *Elements*, and there can be hardly any objection to calling the musical treatise 'Euclidean'." For a history of the controversy about the date and authorship of the treatise, see Burkert, *Lore and Science in the Ancient Pythagoreanism*, p. 375, n. 22.

[9] This translation is from the text of Jan, pp. 148–49.

Euclid then goes on to explain that number and ratio are implicit in the increase or decrease of the vibratory motions responsible for the rise and fall of pitch. Number thus refers to the multiplicity of the vibratory motions, whereas ratio refers to the relation of such numbers to one another. That being the case, musical pitches can be thought of solely in terms of number and ratio:

> As a consequence, notes that are pitched too high can be lowered to the proper level by a reduction of motion, while those that are pitched too low can be raised to a proper level by an increase in motion. Since notes arrive at a proper level by an increase or decrease in motion, they should be spoken of as being constituted of parts. And since all things composed of parts are spoken of in terms of the ratio of numbers obtaining between them, notes should also be spoken of in terms of the ratio of numbers obtaining between them. In the case of numbers, some are described as in multiple proportion, others in superparticular proportion, and others in superpartient proportion. The necessary consequence is that notes should also be spoken of in reference to one another in terms of proportions such as these. In the case of these [numbers], however, those which are in multiple and superparticular relations to one another are designated by a single name. We also know of musical notes that some are concordant, while others are discordant; and that the concords result when two notes effect a single blend between them, while discords do not. This being the case, it is reasonable to assume that notes are concordant when they produce a single blend of the voice from their two pitches and that they are of the class of numbers which are designated under a single name between them, whether they be in multiple or superparticular relation.

Insofar as Euclid found motion to be the basis upon which to express the melodic facts – pitch and interval – in terms of quantitative values, he could effectively formulate a science of relationships between the sounds of melody while ignoring the prerogatives of those sounds themselves: those tensions and remissions that operate with living vigor between the sounds. Harmonics, as practiced by the Pythagoreans and Euclid, was therefore distinct from the study of melody, as practiced by Aristoxenus, because the object of Pythagorean harmonics was quantity and not those musical phenomena that cannot be represented by

mathematics. Nonetheless, for Euclid, certainly, mathematical names had a wonderful power of suggestion, such names, for example, as *pollaplasios* (multiple), *epimorios* (superparticular), and *epimeres* (superpartient). Indeed, they were invitations to him to deal with the concords and discords of melody in the same way one does with the mathematical names by which the concords and discords are designated. The result is that Euclid made the objects of harmonic knowledge – notes and intervals – as similar as possible to those of mathematics. But, as Aristoxenus saw all too clearly, there is a danger in this procedure: one can easily make mathematics arbitrary in its results instead of making the results of harmonic investigation mathematically evident.

Strictly speaking, the introduction to the *Sectio Canonis*, all of which is translated earlier from Jan, for which see note 9, is concerned solely with musical pitch, the product of vibratory motion; but its net effect comes from the attention it turns on the vibratory nature of that motion as it applies to the production of pitch. For, by turning his attention in this direction, Euclid has in effect given a physical account of how the sound of all musical instruments, the human voice included, is propagated into space, an account that is uncannily prescient in its comprehension. He accomplished this by relating vibratory motion, whether it be of plucked strings, vocal cords, air-columns, or drum surfaces, to the rate of percussion induced thereby on the surrounding air. In so doing, he actually set the provisions, and even provided the technical terms, for what would be discovered almost two thousand years later – namely, that the succession of condensations (*pyknoterai*) and rarefactions (*araioterai*) in the air, which reaches the ear and causes the sensation of sound, constitutes a sound wave.[10] This, the discovery of the nineteenth-century mathematician, Joseph Fourier, was of unprecedented importance in

[10] The periodic succession of these condensations and rarefactions was understood by Isaac Newton to be "pulses" of air. As he put it in his *Philosophiae Naturalis Principia Mathematica*, Book II, Section VIII, Proposition 43: "Every tremulous body in an elastic medium propagates the motion of pulses on every side straight forwards. Case 1: The parts of the tremulous body, alternately going and returning, do in going urge and drive before them those parts of the medium that lie nearest, and by that impulse *compress* and *condense* them; and in returning suffer those compressed parts to recede again, and *expand* themselves" [italics mine].

that it made possible the mathematical analysis of those properties of sound – loudness and quality – which had theretofore escaped all scientific representation.[11]

To be sure, what is described in the introduction to the *Sectio Canonis* is the "condensed" and "rarefied" motion of a string set into vibration, a motion that appears to the eye as condensed by being closely packed or rarefied by being widely spaced; but what is implied in this description is the corresponding pattern of motion in the air itself, that pattern which Fourier found a way to reproduce graphically. In short, Euclid's words "condensation" and "rarefaction" carry in their meaning the weight of all they imply. For in their meaning, the motion of a plucked string, for example, is linked to the *time* during which that motion of the string transpires. Indeed, without the factor time, there would be no basis for imputing a greater or lesser density to the motion under observation. Thus, to arrive at a theorem which would express the loudness of a sound in the amplitude of a sound wave, for example, Fourier related the displacement of the air molecules in motion to the time of their travel from the point of origin, just as one would relate the distance an object falls to the time it takes to fall. In surmising, moreover, that any musical pitch can be analyzed into discrete components, each one being a function of the lateral displacement of a string from its initial position and all in concert being time-dependent, Euclid implanted in this introduction the first of the many signposts it would take to mark the road to the most dazzling discovery of acoustical physics: the harmonic analysis by which Fourier determined the components of periodic vibrational motion and the harmonic synthesis by which he determined the resultant periodic vibrational motion from a given set of harmonic components. For with this process of harmonic analysis and synthesis the tone quality of every musical instrument and every human voice could be represented in mathematical terms.[12]

[11] What Fourier showed is that any periodic vibration, or time-dependent tone, however complex it might be, is in reality a combination of simple harmonics whose frequencies are in the ratio of 1:2 : 3:4, etc. Cf. Wood, *The Physics of Music*, p. 68; Roederer, *Introduction to the Physics and Psychophysics of Music*, pp. 103–6.

[12] Fourier's analysis stands in a remarkable sense as the counterpart of Aristoxenus' method for dealing with the elements of melody. For where Aristoxenus found

From the time of Euclid to that of Fourier, there were myriad truths to be discovered about sound and its complex properties of pitch, loudness, and quality. Euclid succeeded in penetrating into the outer rim of one such truth – that concerning pitch and its relation to vibratory motion. To codify the laws governing this truth, Euclid began by observing so distinct a conformity between the visible motion of a stretched string and the pitch it produces, that the latter, the effect, seemed precisely to copy the former, its cause. For, as he evidently saw, the faster a string under tension moves, the higher the pitch it emits when plucked; the slower its motion, the lower its pitch. Seeing also that with the simultaneous production of two different pitches, the faster and slower vibrations, to which these pitches conform respectively, must themselves be occurring over the same period of time, Euclid deduced that the faster vibrations, if they were to occupy the same time span as the slower ones, had to be more numerous, and hence more densely packed, than the slower vibrations producing the lower pitch. On this basis, he inferred that the vibratory motion productive of musical pitch consists of discrete parts, the addition or subtraction of which caused a corresponding rising or lowering of the pitch produced. He reasoned, therefore, that because all things composed of parts can be spoken of in terms of the numerical proportions obtaining between them, musical pitch, because it conforms so precisely in its variations to the enumerative vibratory motion causing it, ought to admit of a similar mathematical construal. Euclid was, of course, right; vibratory motion can be analyzed quantitatively. But the problem for Euclid was that he had no reliable way of assigning to it the correct numerical values.

Theoretically speaking, the vibrations of a long string might have been slow enough for Euclid to count by eye, but practically speaking,

a way to synthesize the discrete series of notes and intervals into a *continuum* of one-to-one terms, Fourier found a way of dissolving the *continuum* formed by a steady, time-dependent tone into the discrete frequencies of the harmonic series. He did this by resolving steady tones into a superposition of harmonics whose frequencies are integer multiples (as in note 11) of the fundamental tone. Aristoxenus' method is discussed in the chapter to follow. Parts of the present chapter are based upon Levin, "Unity in Euclid's 'Sectio Canonis'," pp. 430–43.

those of a short string would have been far too fast for his or anyone else's eye to compute. With no scientific method for counting the wavelike parts of a vibrating current, a method that would be discovered only in the distant future by Marin Mersenne (1588–1648),[13] and with no oscilloscope to render those parts visible on a fluorescent screen, Euclid was at the same disadvantage as an astronomer without a telescope. No appeal to direct experience was possible. Even if such an appeal were possible, it would have been unnecessary, since he had in his possession a hypothesis that was dictated by the facts as given him by nature, even though they had to be extended beyond their warrant. Its truth consisted in this: the vibrational frequency of a stretched string is inversely proportional to its length.[14] This meant that the motion responsible for musical pitch is subject to laws of a mathematical nature. In writing these laws, Euclid could not, of course, represent the particular pitches with which he was concerned by numbers derived from the actual events of causal vibration. There was no way for him to derive the number 440, for example, from the fact that 440 actual vibrations per second produce the note A, that note by which the mid-point or m*esē* of the ancient Greek scale-system is conventionally represented.[15] In short, Euclid was in that blessed state of ignorance that makes it so easy for a genius to be original.

[13] Mersenne was the first to explain the true relations obtaining between tension and the frequency of vibration of a stretched string, relations subsequently codified in 'Mersenne's Laws.' As Mersenne discovered, the frequency of vibration of a stretched string is proportional to the square root of the tension. In order, therefore, to raise the pitch of a stretched string to an Octave, the tension exerted on it must be four times greater than that of the lower-pitched string. See Jeans, *Science and Music*, pp. 64–65. Cf. Levin, "πλήγη and τάσις in the Harmonika of Klaudios Ptolemaios," *Hermes* 108 (1980), 205–6.

[14] These facts of inverse proportion are an integral part of the Pythagorean tradition. They are detailed by Nicomachus, the Pythagorean exponent par excellence, in the tenth chapter of his *Manual of Harmonics*. See Levin, *Manual*, pp. 143–44.

[15] The ancient Greeks' notion of pitch was not like that of ours: a comparison with an external standard. Instead, they framed abstract scale-models and based their transcriptions on the relations between the notes of those scales. The procedure is explained by Chailley, *La musique grecque antique*, pp. 76–77. The scale-model called *Systēma Teleion Ametabolon* is an example of such a scale-model, or template, that accommodates the various modes and genera in a

To obtain results that agreed with the facts of his experience and observation, Euclid gave precedence to his own powers of inference and imagination. He based his computations on string-lengths instead of on vibratory motion and, in the process, was able to describe in his mathematical theorems – those twenty theorems that succeed the Introduction to the *Sectio Canonis* – not the actuality of motion as it pertains to musical pitch, but rather its correlative symptoms. To do

single unified system. Cf. West, *Ancient Greek Music*, pp. 222–23. The tables of all the scale-systems together with their notational symbols are preserved in the treatise by the otherwise unknown theorist, Alypius, called *Introduction to Music*, contained in Jan, pp. 367–406. The method whereby musicologists translate the Greek scales and pitch formations into modern notation is based on an equivalence between our A_2 and the sign C, which corresponds to the *mesē*, or middle note. See Fig. 5: Greater Perfect System. For note 15, Chapter 4: which depicts the linked tetrachords of the Greater Perfect System (Hypaton and Meson) and Diezeugmenon and Hyperbolaion, these disjoined from the first two tetrachords by the whole tone: A–B. It is this disjunction that is a characteristic feature of the Greater Perfect System. As Fig. 5 shows, the Lesser Perfect System contains only conjoined tetrachords.

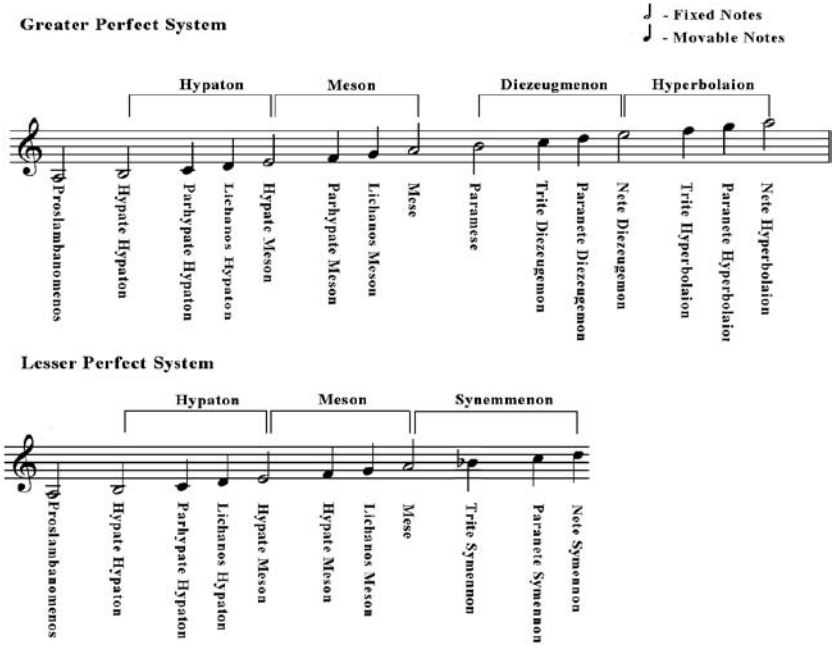


FIGURE 5. Greater Perfect System: Lesser Perfect System

this, he inverted the numerical proportions which would normally represent the greater and the lesser speeds of vibration. On his standard then, the larger number represents the greater length of string (of slower vibration) and the smaller number, the shorter length of string (of faster vibration). Because the full array of experiential facts involving motion was unavailable to him, Euclid went beyond the empirical by postulating truths of reason for truths of experience. And under the former he assembled those propositions of logic and mathematics which signify relations that are universally valid.

Taken in its entirety, the *Sectio Canonis* seems bent on giving a roundly affirmative answer to the question posed in Ps-Aristotle *Problems* 19.23:

> Why is it that *nētē* is the double of *hypatē*? First of all, is it because the string when struck at half its length gives an octave with the string that is struck at full length?

To frame his answer, Euclid in his Introduction took up one side of the question, that according to which *nētē* is thought of as the double of *hypatē,* because its string vibrates twice as fast as that of *hypatē*, its speed increasing in inverse proportion with its length. To round out his answer, Euclid's theorems account for the other side of the question, that according to which the string of *nētē*, producing the faster vibrations, is half as long as that of *hypatē*, its string length thus decreasing in inverse proportion with its speed of vibration. But since the theorems do not mention vibratory motion and the introduction does not mention string-length proportions, we are left to infer from the numerical ratios that are applicable to one side of the question – namely, string-length proportions – those properties that are imagined to be true of the other side of the question – namely, speed of vibration.[16]

[16] Because the vibration theory, as enunciated in the Introduction, must give the larger numbers to higher pitches and smaller ones to lower pitches, while the Propositions assign the numbers the other way round, scholars have seen reason to question the authenticity of the treatise as a whole. Barker, in his "Methods and Aims in the Euclidean *Sectio Canonis*," *JHS* (CI), 1981, 1–2, sees evidence of a missing link: "This anomaly may seem insufficient to justify suspicion. So it is: it is less a sign of separate authorship than of the existence of a lacuna.

To be sure, Euclid knew full well that true knowledge should be positive, that it should not be made up of what is unknown, that it should be grounded in the fullness of what we do know. The problem for him was, therefore, to find a knowledge that would surmount the limitations imposed on him by the reality of vibratory motion. This was the task he faced: how to establish facts and theorems about a motion whose speed could not be rationalized in terms of actual numbers, but could only be imagined in such terms. It required a choice: to think without appealing to experience, or to imagine with the assistance of common sense. Euclid chose to do both, by deciding to treat vibratory motion as mathematics, that is, to substitute for the actual, experienced world of sound, whose causal factors could not be measured accurately, a geometric world made real in numbers, the terms of which would describe the symptoms of vibratory motion as they are imagined to be.

Shifting his focus from frequencies of string vibration to string-lengths as the physical correlates of musical pitch, Euclid arrived at three types of numerical ratios – multiple, superparticular, and superpartient – that would represent the two types of relations between any two musical pitches – concordancy and discordancy. He thereupon postulated that concords, or consonances, correspond only to those types of ratios that are designated under a single name. And the only ratios that are designated under a single name are, he says, multiples and superparticulars. Because the names of the ratios that fall under the class of multiples and superparticulars express what is a greater unity between the numerical elements than do the names of the superpartient class, which leaves the elements discrete and unblended, it should follow that the more unified consonances – such as octaves, fifths, and fourths – must correspond to those ratios with more unified names. Accordingly, of the multiple classes of ratios, that which is expressive of the octave is called by the single name *diplasios* (2:1), that which is expressive of the octave and a fifth is called by the single name *triplasios* (3:1), and that which is expressive of the double octave is called by the single name, *tetraplasios* (4:1). Of the superparticular class of ratios, that

What is missing is a way of relating the primary ratios of movements to the reversed ratios of the lengths of strings." Cf. Tannery, "Inauthenticité de la 'Division du Canon' attributée à Euclide," 213–19.

which is expressive of the fifth is called *hemiolios* (3:2) and that which is expressive of the fourth is called *epitritos* (4:3).[17]

As Euclid readily acknowledged in his Introduction, what is known must be derived initially from the ear's own perception. In other words, the ear must know in advance of what it hears that which enables it to recognize as concordant or discordant those relations between the notes *that* it hears. It is this knowledge on the part of the ear that requires the canonician to do exactly what Euclid has described in his Introduction. This is to increase or decrease the rate of vibration of a string until the pitch produced arrives at the level which the ear accepts as proper. Clearly, then, the ear must know in advance of what it hears how an octave or a fifth or a fourth should sound. Otherwise, an adjustment of the sort detailed by Euclid would not be needed to satisfy the ear's expectations. Recognizing this necessity, Euclid gave the ear its proper priority in the canonic enterprise; but he did so just long enough for that knowledge, which only the ear can provide, to set into motion that irresistible engine of truth – mathematical reasoning. From that point on, Euclid did what every orthodox Pythagorean would have done under the circumstances: he dispensed with the testimony of the ear and introduced mathematical reasoning as an independent criterion that is authorized to work on its own initiative.

By taking this strictly Pythagorean approach, Euclid was led to build his *Sectio Canonis* on assumptions that Aristoxenus felt compelled to call into question. For the moment that Euclid bypassed the evidence of the ear and licensed reason to equate musical consonances with certain kinds of ratios, he began to entangle the melodic elements in a maze of stubborn facts. And although these facts may be geometric marvels in their own right and completely acceptable, if not delightful, on the grounds of pure mathematics, they can contradict the ear's testimony in surprising ways. Indeed, the ear's peculiar knowledge consists not merely in

[17] Barker (note 16), 2, explains the situation in these terms: "Whereas, in Greek, superpartient ratios such as 5:3 can only be designated by compound expressions like 'five to three', there is a one-word name for every ratio in the other two classes. ... Of the superparticulars, the ratio 3:2 bears the special name '*hemiolos*', that is, 'half-whole', while all the others bear names generated by adding the prefix '*epi-*' to an ordinal adjective." On the problems raised by Euclid's "one name" requirement for concordancy, see also Barbera, *The Euclidean Division of the Canon*, pp. 55–58. See Fig. 2.

its ability to discriminate between consonances and dissonances or to distinguish one pitch from another; its more remarkable capacity lies in its ability to interpret what it discriminates as melodious or not and thus to find what it interprets as completely acceptable or not – and even delightful or not – on the ground of the purely musical. In short, hearing, like seeing, is not simply a passive process by which the ear duplicates meaningless sensations that it has no power to interpret. On the contrary, the great power of the ear, as all musicians know, is to construe forms out of the raw data given it. And once supplied with these forms, musical reason is then prepared to discover the strictly musical intelligibility in the melodies that the ear is framed to understand. This blending together of what is in effect musical intuition (*synesis*) and empiricism made for an explosive combination in the mind of Aristoxenus; it also made for a radical departure from the Pythagorean method as exemplified in the *Sectio Canonis*.

Unity was presumably the common feature by which Euclid related the musical consonances to certain kinds of ratios. To conceive of the intervals of melody as being, on this or on some similar basis, the same as the ratios of numbers may perhaps answer a natural aptitude on the part of the human mind to see all things as fundamentally the same – whether they are seen to be the same as water, as with Thales, or air, as with Anaximenes, or fire, as with Heraclitus, or an infinite, eternal primal substance, as with Anaximander, or number, as with the Pythagoreans. In the case of the Pythagoreans, certainly, their failure to represent the facts of audition faithfully stems from their unguarded use of a principle of unity that is present solely in the mind. It is, therefore, Euclid's predilection for fitting the harmonic relations into a preconceived mathematical pattern on the basis of such a principle that makes his *Sectio Canonis* so typically Pythagorean. Paradoxically, though, the more Euclid proceeded as a pure mathematician in this effort, the farther he removed himself from the material he was treating. And the less enslaved he was to the material he was treating, the more he could, like a composer of music, be a free creator of his own world of ordered beauty, one that differs from that of music by belonging to a world of facts as opposed to a world of becoming.[18]

[18] This notion comes from Aristotle *Post. An.* 100a6–9, in which he explains the universal as the one that corresponds to the many, which provides the starting

Of all the relations between two notes, unisons, or *isotones,* were for the Pythagoreans the exemplars of unity, because the ratio by which they are represented on the canon, namely 1:1, is, itself, of all the relations between two terms, the epitome of unity. For there is no interval between the respective terms of either relation – the mathematical or the melodic. That being the case, the Pythagoreans had a basis for comparing the equality of distances on the canon, or ruler, with all the melodic intervals of identical pitch (*isotones*). And if that comparison held true, they could, on the same basis, compare the inequality of distances on the canon with melodic intervals of different pitch (*anisotones*). They saw, therefore, that in the case of unequal distances the ratios of the terms involved certain characteristic differences, and they assumed that where the relations between notes of different pitch were concerned, characteristic differences of a comparable sort should obtain. It is on this line of reasoning that the *Sectio Canonis* is framed.

Given this line of reasoning, of all the relations between unequal terms, double ratios had to have been considered by Euclid and the Pythagoreans the very best, on the obvious grounds that they come closest to the unity that is epitomized in the ratio 1:1. This is because it is only in the double ratio that the excess of one number over the other is equal to the original number. And because the double ratio is representative of that relation in which string-lengths productive of the octave stand to one another, it followed for the Pythagoreans and Euclid that the octave had to be the most consonant of all the intervals between notes of different pitch. In this instance, they had the full concurrence of the ear; for of all the melodic elements, the octave is that which sounds to the ear most like the unison. To follow this Pythagorean hypothesis to its logical conclusion, then, the closer to unity or oneness the relations between string lengths are to one another, the closer to a unison will be any *anisotone* that is produced; and, by contrast, the farther from unity the relations of string lengths are to one another, the farther from a unison will be any *anisotone* that is produced. One need only add to this line of thought that the farther from an *isotone* any melodic interval turns out to be, the more dissonant it will sound to the ear. Thus, if distances on

point of art and science: art, if it pertains to the world of becoming, science, if it pertains to the world of facts.

a canon are defined by being a class of one-to-one relations with certain mathematically assignable properties, then the melodic elements for which they stand must themselves belong to a similar class of one-to-one relations with certain melodically assignable properties.[19]

Armed with such an hypothesis, Euclid and the Pythagoreans could dispense with the ear's testimony altogether, for the numerical ratios derived from the division of the canon told them all they needed to know.[20] With the best of Pythagorean intentions, then, Euclid directed his efforts in the *Sectio Canonis* to the establishment of certain truths about the canonic ratios without taking into account the facts of perception to which they apply. Instead, it was solely from the mathematical properties of the straight line – the canonic symbol par excellence – that Euclid inferred the concordant and discordant properties of the melodic elements. Given this approach, it could not but be that what was true of the canonic ratios would be true of the melodic elements also. This meant that if a certain property such as divisibility was the outstanding

[19] Barbera (note 17), pp. 52–54, calls this line of reasoning the Fundamental Principle of Consonance, a principle that is the "central tenet of the Pythagorean musical creed."

[20] The truth to be derived solely from the numerical ratios is elegant in its simplicity: the smaller the numbers in the ratios, the greater the concordancy; and, conversely, the larger the numbers in the ratios, the greater the discordancy. To this day, no one has been able fully to explain why this is so. Thus Jeans, *Science of Music*, p. 154: "And though many attempts have been made to answer it, the question is not fully answered yet." The truth lies in the following relations:

Interval	Ratio	Largest Number
Unison	1 : 1	1
Octave	2 : 1	2
Fifth	3 : 2	3
Fourth	4 : 3	4
Major Third	5 : 4	5
Major Sixth	5 : 3	5
Minor Third	6 : 5	6
Minor Sixth	8 : 5	8
Major Second	9 : 8	9
Minor Second	19 :18	19

feature of a particular ratio such as 4:1, the same property could be posited of the melodic interval to which that ratio corresponds. The interval in this case is the double octave. It is this divisibility that is at the heart of Euclid's *Proposition* 1 of the *Sectio Canonis*. This is Euclid at work (the letters B, G, and D refer to points on a line):[21]

> If a multiple interval when doubled forms a certain interval, this too will be a multiple interval. Let there be the interval B G and let B be a multiple of G; and let B be to D as G is to B. I say, then, that D is a multiple of G. For since B is a multiple of G, G therefore measures B. But as G was to B as B was to D, the result is that G measures D also. Therefore, D is a multiple of G.

The proposition may be diagrammed as follows:

D	B	G
8	4	2

This means that 8:4 :: 4:2; the double octave is therefore divisible.

By contrast, if indivisibility is the outstanding feature of a particular ratio such as 3:2, the same property could be posited of the melodic interval to which that ratio corresponds, namely, the fifth. Using the geometric method in *Proposition* 3, Euclid proved that between any two distances on the straight line that are in the ratio of 3:2, there is no number that will fall at the proportional mean between the one distance and the other. For such a number would be less than the one distance and greater than the other. The same thing holds true of the melodic interval, the fifth: there is no note in Pythagorean tuning that will divide the interval into two identical halves. As Euclid proved, this is an impasse that arises from the fundamental laws of mathematics, which musicians are powerless to change. The problem of the fourth in the superparticular ratio of 4:3 is the same as that with the fifth. The fourth is indivisible for the same reason that the fifth is: no number will fall at

[21] Jan, 150. 1–11. Cf. Barker, II, p. 194, n. 9. Barbera, *The Euclidean Division of the Canon*, p. 119, explains that here as well as in the Propositions that follow, the most essential parts of propositions and theorems are exhibited: enunciation, proof, and conclusion. Cf. Heath, *Elements*, I. pp. 129–31.

the proportional mean between the two distances on the line (or canon) that are in the ratio of 4:3. That being the case, the melodic intervals, fourth and fifth, like the mathematical ratios to which they correspond, cannot be divided into two equal parts. If, to follow Euclid's argument, the property of a ratio be simplicity or the "oneness" that is expressed in the more unified names by which such ratios are designated, as, for example, *diplasios*, the double ratio, then the more unified the name of the ratio, the more concordant the corresponding melodic interval can be expected to sound to the ear. And, by the same token, the less unified the name of the ratio, the less concordant the corresponding melodic interval must necessarily be. The names of the ratios had therefore a profound power of suggestion for Euclid, because he had what seemed to him a firm basis for extending to all melodic relations those very properties that are expressed in the names of the ratios themselves.[22]

To the human mind, which delights in uniformity and order, nothing could be more gratifying than these irrefragable mathematical facts. For it was on the grounds of that unspeakable unity to which all Pythagorean speculation tends that the names of the ratios could be applied by Euclid to all logical relations of order, even to those of a strictly melodic sort.[23] Thus, it was that Euclid and the Pythagoreans came to grief on the empirical fact that what the ear knows for a certainty to be concordant, the geometry of the straight line confutes utterly on the basis of its own logical necessity. By contrast, what ought to have been incontrovertibly concordant on the basis of that same necessity of mathematical logic, turned out on occasion to be distinctly discordant to the ear. If the Pythagoreans were aware, as they must have been, of

[22] Thus Lippmann, *Musical Thought in Ancient Greece*, p. 154: "Consonant tones . . . are sounds that unite and mix together, or mutually blend; consonance is the creation of a common character or a common principle. But in the Pythagorean view, the auditory manifestation is in essence numerical; accordingly, Euclid postulates that the consonant sounds are in ratios that are either multiple or superparticular (2:1 or 3:2, for example), since only such ratios can be designated (in Greek) by a single word; like two consonant tones, their constituent numbers also unite in a common character!"

[23] As Lippmann (note 22) emphasizes with his exclamation point, the correspondences that Euclid alleges between the technical names and the sounds have no aptness in languages other than ancient Greek.

such discrepancies between the ear's apprehensions and mathematical logic, they suffered no real uneasiness on finding that the abstract connections they made between numerical ratios and melodic intervals had such incongruous consequences. Quite the contrary; they proceeded, as did Euclid, by ignoring those discrepancies that were all too obvious to musicians, or else they simply charged the ear with being aberrant. As they saw it, their theory had to be right, however much the ear might hold it to be wrong, because their theory was one which had been discovered by reason itself (*logos*). And reason could not be the origin of imperfect knowledge. Euclid saw nothing objectionable, therefore, in asserting that concordant melodic intervals are designated by simple names, whether they be in multiple or superparticular relations.

From the very beginning, the Pythagoreans had pledged themselves to give mathematically true demonstrations of everything – every proportion in the heavens and on earth that is magnified in the logic of lines and angles, the long lines of every human and planetary law, whatever the direction and however far afield they might extend. The Pythagoreans had no use for mere probabilities; indeed, they made it a cardinal point that number was all and that mathematical reasoning was, as its agent, simple, permanent, uniform, and self-existent. Where number is right, all sciences of number are bound to be right. With number the Pythagoreans held a copyright on the world, and this conviction commended them to men of thought, to such men as Euclid. The numbers in Euclid's *Sectio Canonis* are, of course, mathematically right. The problem is that Euclid's generalizations, based as they are on the geometry of the straight line, have the unavoidable consequence of disagreeing with the testimony of the ear. For, on the ear's reckoning, not *all* the multiples are ratios of concords; and neither do *all* the concords belong to the class of multiple and superparticular ratios only, nor do *all* superpartient relations on the canon yield discords, as would be expected from Euclid's statement.

To begin with, then, two distances on the canon in the multiple ratio of 5:1 gives not an expected concord but an interval that was not only accounted by the ancients to be discordant but one that did not even have a place in their scale systems: the double octave and a major third. Yet another exception to the rule of concordancy crops up in the case of the superparticular ratio, 5:4, this producing not a concord, as prescribed by

Euclid, but an interval which the ancients construed to be discordant: the major third. In addition, the ratio, 9:8, being of the class of superparticulars, does not yield an expected concord, but the discordant whole-tone interval. Moreover, two distances on the canon in the superpartient relation of 8:3 gives not an expected discord, but that interval which the ear registers as an indisputable concord: the eleventh, or an octave and a fourth, that interval by which the scale system called Lesser Perfect is circumscribed.[24]

In the final analysis, the *Sectio Canonis* appeals essentially to the eye. Its various comparisons between distances on the canon can be put down on paper for everyone to measure visually. And when it enlists the aid of mathematics, the symbols by which these distances on the canon are represented can themselves be verbally expressed. Thus, for example, in *Proposition* 6, to prove that all such distances in a double relation are composed of the two greatest superparticulars, Euclid puts the numbers standing for the distances in question in the following relations:[25]

$$a : b : c$$
$$12 : 8 : 6$$

[24] The Lesser Perfect System, or *Systēma Teleion Elasson*, comprises three conjoined tetrachords. Its contradiction of the Pythagorean and Euclidean theory of concords was noticed early on by Ptolemy *Harmonics* I. 6 (Düring, 13–15): "Such being the hypothesis of the Pythagoreans concerning the consonances, the octave and a fourth which is quite clearly a consonance, puts out of countenance the ratio fitted to it by them." As Ptolemy has it, the ratio 8:3 is itself insulted by the discrepancy. The discrepancy so noted by Ptolemy has occasioned much discussion among scholars. Barker in his "Methods and Aims in the Euclidean *Sectio Canonis*," 9ff, argues that the criterion by which any interval is judged to be concordant or discordant is acoustic, not mathematical. He goes on to conclude that "From this point of view the disqualification of the octave plus a fourth is plainly illegitimate." Euclid's failure to mention the octave and a fourth at all indicates how far the Pythagoreans were prepared to go in placing reason (*logos*) over perception (*aisthēsis*) as the final arbiter of intervallic quality. Opposing views on the question of the octave and a fourth are presented by Barbera, "Placing *Sectio Canonis*," 161, who observes that "The consonant character of the octave plus a fourth . . . is in no way certain, and the issue was hotly debated in musical treatises throughout antiquity and the Middle Ages." See Fig. 5: Lesser Perfect System.

[25] Jan, 154. 15–155. 22. This proposition consists of two parts, the first of which is geometrically conceived as given here.

Given this arrangement, *a* stands to *b* in the hemiolic ratio of 3:2 and *b* stands to *c* in the epitritic ratio of 4:3. Therefore,[26]

$$2a = 3b$$
$$3b = 4c$$
$$2a = 4c$$
$$a = 2c$$

To interpret such a complex, one uses words such as hemiolic and epitritic to express what are the two largest components of the double ratio. The question is, however, what in fact has been interpreted when one uses such technical language? Not the formal elements of melody, certainly, because these, on Aristoxenus' conception especially, cannot stand still long enough to be embraced by such interpretations. What is being interpreted here instead are the formal elements of arithmetic and geometry.

The geometry of the *Sectio Canonis* is strictly metrical. The class of relations that it analyzes is called *diastēma*, or interval, the outstanding property of which is the single dimension it occupies, namely, the straight line. If the notes of melody are conceived of as points on a straight line forming a continuous series, then the distances between these points must be a continuous series also. What is required for the measurement of these distances is, therefore, *number*. That being the case, the elements of melody can be translated into points and lines on one dimension and interpreted in the discursive language of mathematics. The assumption that any two commensurable magnitudes on a straight line can find their equivalent in a corresponding interval between two musical pitches is that on which the geometric method of the *Sectio Canonis* is based. Limiting himself, therefore, to those relations prescribed by the geometry of the straight line, Euclid succeeded in calculating the mathematical ratios to which the various distances on the canon correspond. This done, he proceeded to explore the inherent properties of the ratios themselves and thereby revealed certain discrepancies between the melodic elements and the geometry of the straight line. The most telling of these discrepancies is that the octave is less than six whole-tones, a mathematical fact of enormous implications in music.[27]

[26] This is a representation of the second part of Proposition 6, in which the proof is rendered arithmetically. Cf. Barker, II, p. 197, n. 17.

[27] Euclid's proof is given in the ninth and last mathematical *Proposition* of the *Sectio Canonis*, which states: "Six epogdoic [whole-tone] intervals are greater

Euclid kept an intentionally tight rein on his subject, allowing nothing to impede his progress from pure mathematics to canonics, or applied mathematics. To this end, he divided his twenty *Propositions* into problems and theorems: the problems of *Propositions* 1 to 9, being strictly mathematical, concern the generation, division, subtraction, and addition of intervals, or distances on the canon; the theorems of *Propositions* 10 to 20, being strictly harmonic, exhibit the melodic attributes of these same distances on the stretched string of the canon. From *Propositions* 10 to 20, Euclid thus applied his mathematical conclusions to the musical facts – that is, to the distances between the musical notes, whose names he now introduced for the first time.[28] In *Proposition* 10, for example, he applied to the melodic elements those facts that he had proved mathematically in *Proposition* 1: "If a multiple interval is doubled and forms an interval, it, too, will be a

than one double interval [octave]." This is found in Jan, 157. 5–14. The application of this proof to harmonic analysis is given in Proposition 14 (Jan, 160. 20–161. 3), which states: "The octave is less than six whole-tones." Since a whole-tone ratio is 9:8, the six whole-tones of the octave form a progression such that to the number assigned to the first one there is added an eighth of that number, and so on successively to the completion of the octave. For example, if the first assigned number is 64, then $(64+\frac{64}{8}=8)=72$.

The end result of this process will be a number that exceeds the limits of the double ratio. As Barker, II, p. 199, note 22 explains it: "Each number must be such that one ninth of it is a whole number." Thus, one ninth of 72 is the whole number 8. The method of proof is given also in Euclid's *Elements*, Book VII, *Proposition* 2. These proofs are adamant. Yet Aristoxenus, in the face of them, remained unshaken in his belief that six whole-tones are exactly equal to an octave. This gives some indication of the value that he placed on his ear's perception.

[28] The Greek nomenclature is considered, even by classicists, to be forbidding. But, on examination, it turns out to be no more complex than, and just as systematic as, the modern terms: tonic, supertonic, mediant, subdominant, dominant, submediant, subtonic. The Greek names are derived from the position of the strings on the tilted lyre and, with the exception of *lichanos*, or forefinger, are adjectives modifying *chordē.* Relative to the performer, in ascending pitch they are: (*hypatē* the string of the highest position), which emits the lowest pitch, *parhypatē* (next to the highest), *lichanos* (the string plucked by the forefinger), *mesē* (middle), *tritē* (third from the top), *paranētē* (next to the top), *nētē* (the topmost string, which emits the lowest pitch). See West, p. 64; pp. 219–20.

multiple interval." *Proposition* 10 is designed, therefore, to show how the melodic elements – those forming the double octave – share the property of the mathematical formulations in *Proposition* 1 – those of the multiple ratio. Accordingly, there is between the note *proslambanomenos* (A_1)[29] and *nētē Hyperbolaion* (A^1) a proportional mean at *mesē* (A); the entire distance between these limits cannot be represented by a superparticular proportion, since there is no mean number which falls proportionally between the limits of the superparticular proportion. The entire distance between *proslambanomenos)* (A_1) and *nētē Hyperbolaion* (A^1) must consist, therefore, of two multiple intervals taken together. Of the three distances in question – which may be represented as *AB*, *BC*, *AC* – one must be the greatest. This is *AC*. Then in virtue of the definition, *B* (*mesē*) will be fixed between *A* (*proslambanomenos*) and *C* (*nētē Hyperbolaion*). Therefore, *AB* and *BC* must be multiple intervals. And this is to bear out the mathematical axiom that things which are double of the same thing are equal to one another.

Proceeding in this manner, Euclid went on through *Propositions* 11 to 13 to prove that octaves are composed of fourths and fifths; that fourths and fifths, being of the superparticular class of ratios, cannot be divided in half; that if intervals which are not multiples be doubled, their total will be neither a multiple nor a superparticular and, therefore, not a consonance; that a triple interval is formed from an octave and a fifth (3:1); that if a fourth be subtracted from a fifth, the left-over interval is an *epogdoic* or sesquioctave ratio of 9:8, the ratio of the whole-tone. From this point on, that is, from *Propositions* 14 through 20, the *Sectio Canonis* can be read as an unrelenting and meticulously argued polemic against Aristoxenus' principles of harmonics. Everything that Aristoxenus had stated axiomatically about melodic intervals is systematically confuted by Euclid: that the octave consists of six whole-tones; that the fifth consists of three whole-tones and a semitone; that the fourth consists of two whole-tones and a semitone; that whole-tones are not only divisible into two equal semitones, but also into quarter-tones by which the Enharmonic genus is defined as such, as well as into other micro-intervals that characterize the various *chroai* or nuances. In short,

[29] On *proslambanomenos*, the "note added" to the five tetrachords of the Immutable System, bringing its total range to two octaves. See Fig. 1: Immutable or Changeless System.

Euclid's mathematical dismemberment of everything that Aristoxenus held to be axiomatic about melodic intervals appears to be so pointed and well targeted that it is hard to believe he did not have Aristoxenus' work before him when he composed the *Sectio Canonis*.[30]

Euclid was, as it happens, only about a generation younger than Aristoxenus. He is said to have been younger than the first pupils of Plato (d. 347 B.C.) but older than Archimedes (287–212 B.C.), which would place his *floruit* at about 300 B.C. He was apparently not long in winning fame as a geometer, for his first appearance in the literature of antiquity has him conversing on familiar terms with one of the most resplendent and powerful figures of the age – Ptolemy I Soter, who reigned over Egypt from 306 to 283 B.C. Ptolemy had just asked him if there were not some shorter route to a knowledge of geometry than through the *Elements.* "Not even for a king," Euclid dared to reply in defense of geometry's crowned truths. The time must have been 300 B.C. or thereabouts and Euclid, established by now in Alexandria, where he founded his own school of mathematics, was perhaps not much more than thirty years old. His birthplace is unknown, and no reliable genealogy has come down with his name. But his chronicle of mathematics' advancement allows us to infer something about its author and how different from Aristoxenus he must have been. For one thing, he seems to have had no quarrel with the work of his predecessors. On the contrary, he was noticeably respectful of all that had entered into the mathematical tradition, even to the point of including for the sake of completeness much that he knew to be of little current application. Where he himself invented, it was not to controvert but, rather, to complement past endeavors in the field. His aim was to offer as complete an account of mathematical science as his talents permitted. His talents rewarded posterity with his *Elements*, a work of which it was said in October 1848:[31]

[30] On the polemic nature of Euclid's last few *Propositions*, see Barker, II, p. 204, n. 57. Whereas Euclid seems to have had Aristoxenus' method in mind as he proceeded to frame his polemic, Aristoxenus, in his turn, seems to have had Euclid's method in mind as he proceeded to formulate the third book of his *Harmonic Elements*. For in writing the rules of melodic topography in this book, Aristoxenus adopted a Euclidian style for composing his strictly musical Propositions and Theorems. Paradoxically, Aristoxenus chose the very style of Euclid's *Elements* to oppose his strictly mathematical method.

[31] Heath, *Euclid's Elements*, Volume 1, Preface.

> There never has been, and till we see it we never shall believe that there can be, a system of geometry worthy of the name, which has any material departures (we do not speak of *corrections* or *extensions* or *developments*) from the plan laid down by Euclid.

Euclid was original but, unlike Aristoxenus, he never claimed to be so. Yet, like Aristoxenus, he too could be impatient when he felt that the assumptions of pure knowledge were being challenged. But that natural sense of irony, so lacking in Aristoxenus, could encourage him to say for the benefit of all students to come, what he once had occasion to say to a student of his own. This student had asked him a question that would be repeated in one form or another through the ages: "What shall I gain by learning all this?" "Give him three obols," Euclid told his servant, "since his purpose in learning is to make a profit."[32] The student's reaction may be surmised. Unlike Aristoxenus, who took such unabashed pride in his own contributions to knowledge, Euclid seems to have been a paragon of modesty. He seems also to have had by instinct the fairness and generosity of spirit to advance all who would serve knowledge, neither anticipating by design what their methods and discoveries prefigured, nor withholding credit when their contributions could enrich mathematics, even if minimally. He was a man absorbed, not with himself, but with his work. In fact, his individuality merges so closely with his work as to vanish in it entirely. As a result, knowledge of Euclid, the person, and the external circumstances of his life have had to suffer accordingly. Only the thinker with the objects of his thought appears, the unshakable certainty embedded in the axioms and basic propositions of his mathematics proclaiming what he was and what he meant.

Unattended then by the persona of its author, Euclid's *Elements* is concerned solely to document a knowledge – one whose truths it assembled from the work of many schools of mathematicians that dominated Greek intellectual life from the time of Pythagoras on. Mobilized for the first time in a single unified work, these truths represent nature's objective realities in such terms as points, lines, angles, circles, triangles, cones, cubes, and more – terms that do not represent the actual physical objects of nature but, rather, the concepts abstracted from such objects. The

[32] The source of the anecdote is the 5th century A.D. compiler, Stobaeus, *Eclogae* II. 228. 30 (Wachsmuth).

stretched string is such an object, and from it may be abstracted the concept of a straight line. When Euclid defined a mathematically straight line, therefore, as a line which lies evenly with the points on itself, the extremities of which are points,[33] the abstraction from a stretched string lying evenly between its fixed ends is obvious enough. That the properties of the stretched string are reflected in the mathematical abstraction to which it gave rise required only that a string thus stretched be pressed against a finger-board – as on the ruler of the canon – at any point whatever and plucked accordingly. Because any alteration, however slight, at the point of finger-pressure produces a corresponding difference in pitch, a single string can be made to yield a multiplicity of pitches. And so it came about that the harmonic canon, or monochord, a lute-type instrument, said to have been invented by Pythagoras himself, was favored by specialists like Euclid and the Pythagorean harmonicians.[34]

Euclid's mathematically driven researches on the canon produced in the *Sectio Canonis* a clearly articulated skeleton of the Immutable System in the Diatonic genus (*Proposition* 20). He achieved this by locating the fixed notes, so-called, which form the iron-bound consonantal scaffolding on which the five linked tetrachords of the Immutable or Changeless System are built. These are the notes that remain fixed at one and the same pitch, whether the genus be Enharmonic, Chromatic, or Diatonic. The names of the five tetrachords are defined by these fixed notes to reflect their position in the Immutable System: *Hypaton* (lowest), *Meson*

[33] Euclid, *Elements*, I. Definitions 3 and 4.

[34] Nicomachus *Manual*, 4 (Jan, 243. 14–16) says of the monochords that they were "commonly called *pandouroi* but which the Pythagoreans called canons." Because the monochord and canon were, like modern guitars, provided with a finger-board upon which marks (like frets) could be placed, indicating the measurements of string length required for the production of desired pitches, they were ideally suited for acoustic experiment. Aristides Quintilianus, a theorist of late antiquity, says in his *De musica* 3. 2 (Winnington-Ingram 97. 3–4) that Pythagoras' last words, just before he died, were to exhort his disciples to use the monochord in their researches. Of the lute-type instruments mentioned by Nicomachus, the *pandouros* (*pandoura*) was the most ancient, having been used by the Egyptians as early as the New Kingdom (c. 1570 B.C.). The monochord, or canon, is described by Ptolemy *Harm*. I. 8 (Düring, 18. 1ff.) in complete detail together with a diagram of its construction. For additional references, see Levin, *Manual*, pp. 69–71.

(middle), *Synēmmenon* (conjoined or linked), *Diezeugmenon* (disjoined or unlinked), *hyperbolaion* (highest). The complete array of these tetrachords, together with their interior pitches – the movable notes, so-called, because they change in pitch with the changing melodic genera – was computed by Euclid in *Proposition* 20 using only those ratios that he had already assigned to the consonances: Double Octave (4:1), the defining limits of the Immutable System; Octave (2:1), formed by the two tetrachords, *Hypaton* and *Meson*, linked together on the note *mesē*, with another note added at the base (*proslambanomenos*); Fifth (3:2), formed by the tetrachord *Meson* and the whole-tone disjoining it from tetrachord *Diezeugmenon*; Fourth (4:3), forming the boundary limits of each tetrachord.[35]

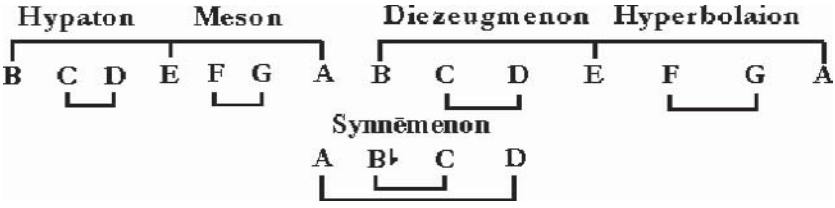
Using the ratio of the whole-tone interval that is left over on the subtraction of the Fourth from the Fifth – 9:8 – Euclid went on to locate the two whole-tones in each tetrachord of the Immutable System: the one that intervenes between the moveable notes *parhypatē* (next to the lowest note) and *lichanos* (finger-note) of the lowest tetrachord (*Hypaton*); that intervening between *lichanos* and *mesē*, the lowest note of the middle tetrachord (*Meson*); that intervening between *tritē* (the third note from the highest) and *paranētē* (next to the highest) in the conjoined tetrachord (*Synnēmmenon*); that intervening between *tritē* and *paranētē* in the disjoined tetrachord *Diezeugmenon*; that intervening between *paranētē* (next to the highest note) and *nētē* (the highest note) in both the disjoined and the highest (*Hyperbolaion*) tetrachords. In this

[35] The guiding note is *mesē*, the middle note (A), and the paradigmatic scale formed in relation to *mesē* has the same structure as the modern key of A natural minor (i.e., minus the G♯). This paradigmatic scale, or template, was the ancient Hypodorian. Cf. Chailley, *La musique grecque antique*, p. 79. The addition of the tetrachord *Synnēmenon*, whose linkage to *mesē* provided a critical note (B♭, C D), made for a range of an eleventh, an octave and a fourth. See note 24. According to Ptolemy *Harm*. II. 6 (Düring, 55. 19–22), the modulation (*metabolē*) effected by this linking tetrachord presents the ear with an unexpected melodic deviation of the "very finest" sort, if it be properly managed. In modern terms, such a deviation is comparable to a modulation from C Major to F Major and was considered by the ancients as particularly melodic (*emmelēs*).

way, Euclid marked out on the canon all those divisions that define the Immutable System in the Diatonic genus.[36]

Euclid's purpose in setting out these metrical divisions on the canon was twofold: to reinstate all the facts that he had developed in *Propositions* 17–19; to prove that the location of the interior, or moveable, notes of any tetrachord can be determined with mathematical certainty in one melodic genus only – the Diatonic. This disclosure of *Proposition* 20 was in all respects a carefully measured refutation of Aristoxenus' teachings on the divisions of the tetrachord.[37] For, according to Aristoxenus, any division of the tetrachord was possible as long as it was melodically intelligible. And for this to be so, it was necessary, and even mandatory at times, to ignore the canonic rules for locating the moveable notes of the tetrachords. Directly inspired by mathematics then, Euclid assembled the facts of *Propositions* 17–19, which lead inexorably – and almost insolently – to the sharply etched disclosures of *Proposition* 20. To accomplish this full-scale attack on Aristoxenus, Euclid unabashedly used the very method that Aristoxenus himself had invented for locating the moveable notes, a method which Aristoxenus called "Intervals ascertained by the principle of Concord."[38] Nor was that all. The genus on which Euclid focused for this purpose was the one

[36] The whole-tones in question are C–D, F–G, B♭–C, and, an octave higher, C–D and F–G. They appear in the tetrachords as follows:

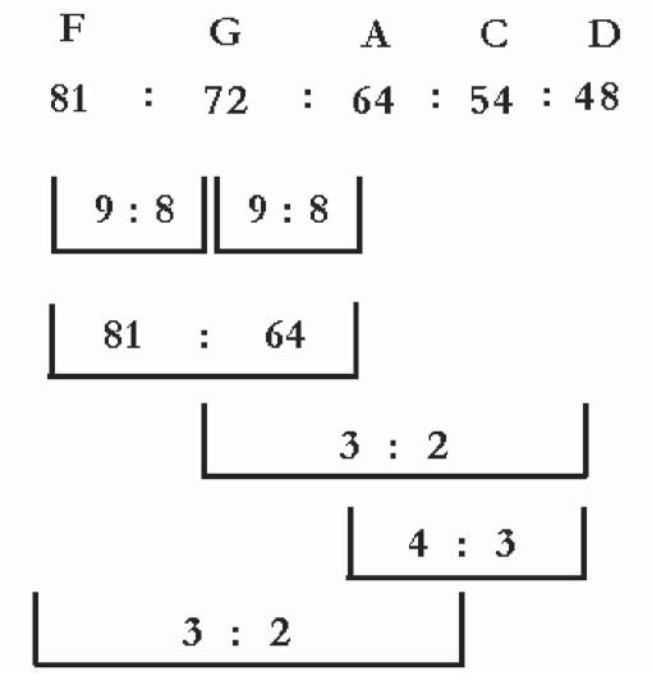


[37] *Harm. El.* I . 24 (Da Rios, 31. 1). Euclid's point here was apparently to prove that ascertaining intervals by the principle of concords cannot determine the enharmonic quarter-tone with any exactitude. Aristoxenus was, as it happens, fully aware of this problem. For he is quoted by Ps.-Plutarch *De mus*. Ch. 38, 1145B (Ziegler–Pohlenz, p. 32. 4–7) as saying: "Then, too, there is the impossibility of determining the magnitude [of the quarter-tone] by concords, as can be done with the semi-tone, the whole-tone, and the rest of the intervals."

[38] *Harm. El.* I. 23 (Da Rios, 29, 14–16): "That there is a type of melodic composition requiring a ditone *lichanos* [a *lichanos* distant from *mesē* by a Major Third] and that far from being most contemptible, but indeed probably the most beautiful

which Aristoxenus had proclaimed, against all canonic dictates, to be the most noble of all the melodic genera – the Enharmonic.

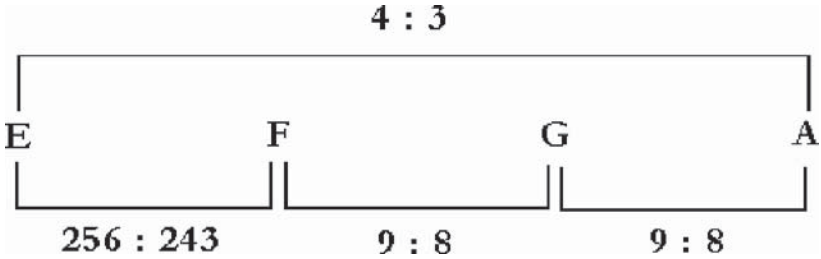
What conferred an especial nobility on the Enharmonic genus was, in Aristoxenus' view, the prominence that it gave to the ditone, or major third, an interval that he felt to be particularly beautiful in the context of the Enharmonic tetrachordal division.[39] To bring this about, that is, to bring the ditone into moving prominence, it was necessary that *parhypatē* be a quarter-tone (*diesis*) distant from *hypatē*. Euclid begins his attack, therefore, in *Proposition* 17 by showing that while the ditone is formed of two whole-tones and is mathematically expressible in the ratio 81:64, there is no way to divide the whole-tone itself into equal halves by the process of tuning by consonances described in *Proposition* 18. That being the case, there can be no mathematically secure way of representing the *diesis*, or quarter-tone, of the Enharmonic genus. According to Euclid then, the furthest one can go in dividing the canon accurately is as follows:



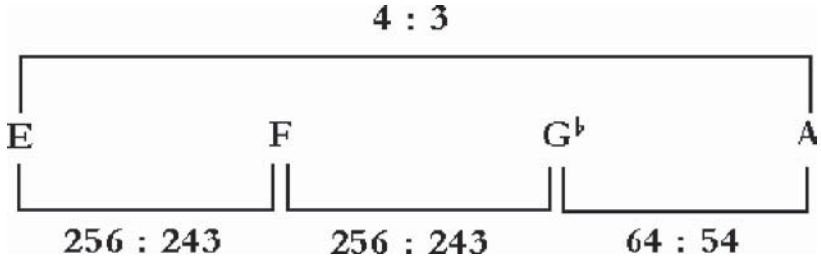
style, is not at all evident to most people who concern themselves with music today; yet, it might become evident to them if they were given examples of it."

[39] These are the ratios arrived at by Plato in *Timaeus* 36A1–B5. On Plato's elaboration of the full diatonic scale, see Levin, *The Harmonics of Nicomachus*, pp. 89–91.

An added (and unlovely) consequence of these computations is obtained on the subtraction of two whole-tones (81:64) from the Fourth (4:3), this being the semi-tone which the Pythagoreans called *leimma*, the "leftover" 256:243, approximated by modern acousticians to 19:18. The Diatonic tetrachord had theretofore to be computed as:[40]

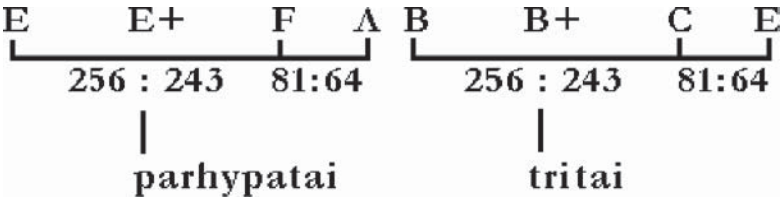


The Chromatic tetrachord was omitted by Euclid from his calculations presumably because its two semitones were subject to the same mathematical afflictions as those befalling the Diatonic *leimma*, and could speak for themselves in these terms:



[40] From the time of Philolaus and Plato on, these unruly ratios provoked an almost endless array of rationalizations and correspondences for numerologists to ponder. For example, Philolaus extracted from these relations the number 27, a number of great cosmic significance in the *Timaeus* and the *Republic*. For 27^2 is the number of days and nights of the year. According to Boethius *De inst. mus.* 278. 11ff.), Philolaus derived the number 27 from his efforts to divide the whole-tone. Computing the whole-tone as 243 : 216 (= 9: 8), and subtracting 216 from 243 (= 27), Philolaus proceeded to halve 27 and found 13 and 14, which he called *diesis* and *apotome*, respectively. Subtracting 13 from 14, he found the *comma* (= 1) and halving the *comma*, he found the *schisma* (= ½). He could have gone on to infinity without finding intervals of equal size, even as Aristoxenus maintained. Cf. Frank, *Plato und die sogenannten Pythagoreer*, pp. 263–76.

Finally, in the Enharmonic tetrachord, as detailed by Aristoxenus, the division of the semitone by the *parhypatai* and the *tritai* into two equal quarter-tones was shown by Euclid to be mathematically anomalous:[41]



Aristoxenus had two things to say about all such computations as these. One has already been mentioned but bears repeating here. Speaking of some of his predecessors, who can only be the Pythagorean mathematicians, he says: [42]

> For some of these introduced extraneous reasoning and, rejecting the senses as inaccurate, fabricated rational principles, asserting that height and depth of pitch consist in certain numerical ratios and relative rates of vibration – a theory utterly extraneous to the subject and quite at variance with the phenomena.

His second observation seems to have been aimed directly at Euclid himself:[43]

> It is usual in geometric constructions to use such a phrase as "Let this be a straight line"; but one must not be content with such language of assumption in the case of intervals. The geometrician makes no use of his faculty of sense-perception.

As for all else, Aristoxenus simply ignored all computations such as those of Euclid, not because they were untrustworthy, but because they were irrelevant. Failing therefore to agree with Euclid and the Pythagoreans as to how mathematics could be made useful to the study of melody, he proceeded with the view that it should at least be made harmless.

[41] *Harm. El.* II. 33 (Da Rios, 42. 13–17).
[42] *Harm. El.* II. 32 (Da Rios, 41. 19–42.3).
[43] *Harm. El.* II. 32 (Da Rios, 41. 19–42.3).

5 The Topology of Melody

A solution was still to seek. Jane Austen

MUSIC SEEMS ALWAYS TO HAVE PROVIDED PHILOSOPHERS, PHYSICISTS, and cosmologists with an ideal image of the universe in action. To Pythagoras, the whole cosmos was a musical domain, the moving planets acting as the physical surrogates of musical notes, the spaces between these notes defined by harmonic boundaries indelibly etched in the fabric of the heavens. The musical space in this domain was in no sense understood as metaphorical; on the contrary, it was perceived to be as real as the movements of the planets themselves. In a giant leap from earth to heaven, Plato made of these planetary notes and the spaces between them a mathematically realized diatonic scale covering four octaves and a major sixth. This, he said, was the work of the Demiurge; and this, his masterpiece, was the World-Soul defined.[1]

With the Demiurge's archetypal scale now fixed to a mathematical certainty, the study of the harmonic truths embedded in this scale had to lead to a direct understanding of the physical structure of the entire universe. Ptolemy applied these truths to astronomy and astrology, thereby wedding his own geocentric planetary system to the

[1] See Chapter 1, note 29. As Taylor says in his *Commentary on the Timaeus of Plato*, p. 140: "The compass of Plato's progression is much greater than any which was employed in contemporary music." The Immutable System of Music (for which, see Fig. 1) did not exceed two octaves. But Plato was not constructing a scale for humans; rather, his was for the "unheard melody" of the universe, a melody always in evolution, with no real beginning and no certain ending. On Plato's method, see Brumbaugh, *Plato's Mathematical Imagination*, pp. 227–29.

numerical ratios determining the musical consonances. As Ptolemy saw it, if the musical consonances are numerical in origin, then the astrological symmetry, because it involves the same numerical ratios as those of the consonances, must itself be a perfect analogue of the musical domain.[2]

The proof of Ptolemy's hypothesis lay in the natural divisions between the constellations of the zodiac that mark the yearly path of the sun through the heavens. To Ptolemy, these constellations stood as signs marking off twelve equal sectors of the ecliptic, or apparent circle described by the sun, a circle whose plane intersects that of the equator and forms with it an angle of 23° 30′.[3] He was impressed by the fact that just as the zodiac is divided by nature into twelve equal sectors, so too is the two-octave Greater Perfect System of music constituted of twelve whole-tones which, if not exactly equal, are approximately so.[4] To show how the zodiac and the double-octave are comparable, Ptolemy instructs us to bend the double-octave of musical theory into a circle by conjoining *nētē Hyperbolaion* (A^1, the highest note of the system) to *proslambanomenos* (A_1, the lowest note of the system). For when these two notes are united, the one, *proslambanomenos*, and the other, *nētē Hyperbolaion*, now locked together, are seen to lie diametrically opposite to *mesē* (A, the middle note of the Greater Perfect System).[5] In this way, Ptolemy showed that the octave, the most perfect consonance, has properties equivalent to those of the circle, the most

[2] Thus Barker, II, p. 274: "He [*sc.* Ptolemy] traces the ways in which the mathematical relations underlying the structures of audible music also constitute the 'forms' that are the essence and cause of perfection in other domains, in the human soul and in the movements and configurations of the stars."

[3] On the discovery by Anaximander of the ecliptic's obliquity, see Heath, *Greek Astronomy*, pp. xxxvii–xxxviiii; cf. Dicks, *Early Greek Astronomy*, pp. 157ff.

[4] Ptolemy *Harm*. III. 9 (Düring, 103. 13–104. 2): "the two-octave perfect system, being approximately equal to twelve whole-tones (δώδεκα τόνων ἔγγιστσ), the whole-tone interval was adapted to the twelfth part of the circle." See Fig. 6. Aries and Libra are usually placed on the diameter; I have given precedence here to the notes A^1 and A_1, thus realigning the constellations. For a more conventional representation, see McClain, *The Pythagorean Plato*, p. 151, Fig. 48.

[5] Ptolemy *Harm*. III. 8 (Düring, 101. 20) observes that *mesē*, the middle note of the perfect system, is the source (*archē*) of the circle's equality by being at its very center. This is diagrammed by Barker, II, p. 382.

perfect geometric figure. Or, as Ptolemy says, the circle, by virtue of its component parts, bears a marked resemblance to the octave. Ptolemy says accordingly:[6]

> If one bends the double octave around into a circle in keeping with its function, and attaches the *Hyperbolaion* {*nētē Hyperbolaion*} to *proslambanomenos*, making the two notes into one, such an attachment will clearly stand diametrically opposite to *mesē*, and will be relative to it {*mesē*} in the homophone of an octave. The rationale for the comparison as described consists in the fact that there occurs a near likeness between the diameter on the circle and the attributes displayed in the octave.

With the double-octave now bent into a circle, the position of *mesē* relative to the conjoined notes, *proslambanomenos* and *nētē Hyperbolaion*, is seen to be on the line with the diameter of this conceptual circle. This means that a line drawn from *mesē* through the diameter of this circle to the opposing point on its circumference, namely, that of the conjoined notes, can be represented by the double ratio – or, as Ptolemy says:[7] "For it is in the diameter that the double ratio of the whole circle to the semicircle is contained."

Following Ptolemy's line of reasoning then, if the whole circle of the zodiac be divided into twelve equal segments of thirty degrees each, with every point on the circumference of the circle representing a sign of the zodiac, each point (or sign) will stand to its opposite one on the circumference in the same ratio as that of an octave.[8] Thus, the opposition between any two constellations of the heavens is as that between the two boundary notes of any octave. Space, as it appears to the senses, and as it is assumed in astronomy, has, therefore, a real counterpart, namely, the constitutive elements of the double-octave: twelve

[6] Ptolemy *Harm*. III. 8 (Düring, 101. 12–18).

[7] *Harm*. III. 8 (Düring, 101. 18–19).

[8] *Harm*. III. 8 (Düring, 101, 24–26): "Hence the configurations of stars that are diametrically opposite one another in the zodiac are the most invigorating [or 'active', *energetikotatoi*] of all of them, as are those among the notes that make an octave with one another" (trans. Barker). As Barker, II, p. 381, n. 61 points out, this passage shows "that Ptolemy is prepared to treat the movements and configurations of the heavens both from the point of view of scientific astronomy and astrologically."

approximately equal whole-tones. On this conception, the Greater Perfect System of music appears to mirror the universe, there being in evidence a pre-established harmony of opposites between all the constellations of the zodiac: Aries ♈ and Libra ♎; Pisces ♓ and Virgo ♍; Aquarius ♒ and Leo ♌; Capricorn ♑ and Cancer ♋; Sagittarius ♐ and Gemini ♊; Scorpio ♏ and Taurus ♉.[9]

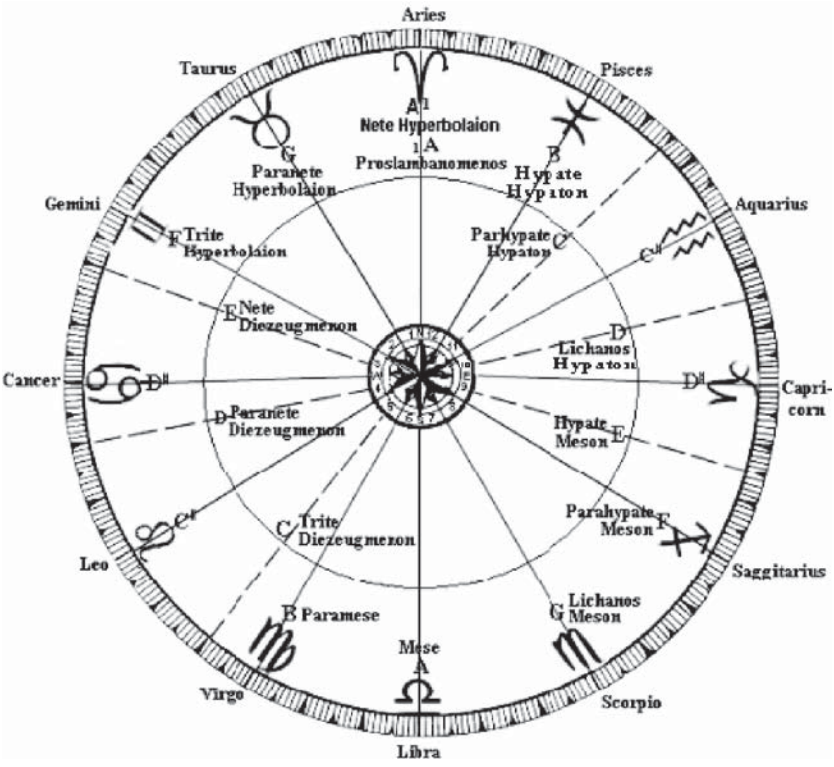


FIGURE 6. The Greater Perfect System Projected on the Zodiac

As is shown in Fig. 6, not even Ptolemy could reconcile this perfect universal arrangement based on twelve equidistant sectors with the

[9] These configurations are most conveniently set out by Jocelyn Godwin, *The Harmony of the Spheres*, p. 30, both in linear form as well as on the zodiac. In this way, Godwin shows in graphic detail that the effect of the constellations is strongest when they are to one another as are the notes of an octave.

melodic framework of the Greater Perfect System of music. For although the Greater Perfect System does add up to twelve approximately equal whole-tones, its distribution of these elements is not that of twelve whole-tones in a sequential order, but a sequence of whole-tones and semi-tones whose order is dictated by the laws of tetrachordal consecution. Therefore, the moment Ptolemy superimposed the Greater Perfect System with its own distinctive arrangement of intervals on the circle of twelve equi-distant degrees, he was bound to confront a series of discrepancies.[10] That being the case, Ptolemy resorted to a strictly geometrical division of the circle. For, inasmuch as twelve is the smallest number which can be a common denominator for the multiple ratios 1:2 (octave), 1:3 (octave and a fifth), and 1:4 (double octave), Ptolemy was able to locate by means of these ratios the octave, the fifth and the fourth on the zodiac. He found the octave, the fifth, and the fourth each represented on the zodiac three times; the twelfth or octave and a fifth twice; the eleventh, or octave and a fourth, once; the double-octave once; and the whole-tone once.[11]

By mapping the double-octave with its twelve approximately equal whole-tones on the ecliptic, Ptolemy produced an image of one great

[10] As Ptolemy has stipulated (note 4), the resolution of these discrepancies consisted in treating the Greater Perfect System as the equivalent of twelve nearly equal whole-tones. He could thereby "adapt" the fifteen separate pitches of the two-octave scale to the twelve zodiac positions on the ecliptic. This required sharping two pitches, C and D, to C♯ and D♯, and flatting one pitch, E, to E♭. With twelve approximately equal whole-tones now in place, *lichanos Hypaton* (D) and *hypatē Meson* (E) are made to share virtually the same pitch (D♯ = E♭); the same change appears in the higher octave where *paranētē Diezeugmenon* (D) and *nētē Diezeugmenon* (E) are sharing the almost identical pitches (D♯ = E♭). This meant that the nearly identical pairs in each octave are allotted to the same constellation, namely, Cancer and Capricorn, which are diametrically opposite to one another. The series of whole-tones appears as follows with astrological symbols:

A	B	C#	D#	E♭	F	G	A	B	C#	D#	E♭	F	G	A
♈	♉	♊	♋		♌	♍	♈	♎	♐	♑		♒	♓	♈

This comports with Aristoxenus' analysis of the octave constituents.

[11] These relations are charted by Godwin (note 9), p. 414, note. Cf. Solomon, *Ptolemy*, p. 156.

recurring scale – a scale as cyclical in nature as the rotation of the sphere of fixed stars around the axis of the earth. For half of this scale – an octave – like half of the ecliptic, is at any given moment above the horizon; while the other half – an octave, also – is below the horizon. Harmonized in this way, the structural boundaries of the universe are defined in a single scale of stupendous unity, the cross-sections of which appear as recurring parts of the unified whole.[12] Thus, just as geometry could succeed in reducing the complex activity of the sun, for example, to a daily motion and a yearly motion, which appear to be altogether uniform, so too could Ptolemy's geometrically induced harmonics interpret all scales, whatever their apparent individuality, as segments of one and the same harmonically unified system. In this harmonious cosmos, where every end is a beginning, where there is always another *proslambanomenos* rising on the ecliptic, no primordial contradiction can dispel the balance of perfection, and no primordial pain can dwell in the heart of the primal unity.[13] For here the laws of *harmonia* are granite; they reconcile all seeming discordancies as expressions of one immutable law.

[12] In Chapters 10, 11, and 12 of his *Harmonics* III (Düring, 104. 18–107. 18), Ptolemy correlates three types of stellar movements with three melodic phenomena: continuity, genera, modulation: (1) continuity = the longitudinal (*kata mēkos*), which is the diurnal orbit of the fixed stars and planets around the earth, wherein the Sun moves from East to West; (2) genera = the vertical (*kata bathos*), literally, motion "in depth," or planetary epicycles wherein each planet appears to approach the earth and then to recede from it; (3) modulation = the lateral movements (*kata platos*), literally, motion "in breadth" or the planetary declinations as they move through the zodiac from Cancer in the North to Capricorn in the South, thus moving away from the powerful tonal center that is the celestial equator. Cf. Godwin (note 9), p. 414, note 18; Barker, II, p. 384, note 71. Bruce Stephenson, *The Music of the Heavens*, p. 36: "The sounds described in all this seem to be essentially a low tone as the planet rises in the east, ascending to a high tone when the planet culminates, and then descending again until the planet sets in the west." See also, Liba Chaia Taub, *Ptolemy's Universe*, p. 128.

[13] The reference here is to F. Nietzsche, *The Birth of Tragedy* in *The Philosophy of Nietzsche*, p. 979: "Language can never adequately render the cosmic symbolism of music, because music stands in symbolic relation to the primordial contradiction and primordial pain the heart of the Primal Unity, and therefore symbolizes a sphere which is beyond and before all phenomena."

Ptolemy's solution to cosmic truth by way of music produced a geometrical masterpiece – a complex and elegant edifice of cycles and epicycles that explained the irregularity of planetary motion to the satisfaction of astronomers for fourteen centuries, until Nicholas Copernicus dismantled it in 1543 with the publication of his *De revolutionibus orbium coelestium*. For Copernicus, just as for Ptolemy, the universe appeared as a system of concentric spheres in which the sphere of fixed stars contained all the others, but with this difference: the sun, and not the earth, as with Ptolemy's system, occupied the center of the universe.[14] Additional facts drawn from observation and confirmed by experiment, as by Giordano Bruno (1548–1600), Galileo Galilei (1564–1642), and Tycho Brahe (1546–1601), would ultimately bring down Ptolemy's radiant mirror of the macrocosm.[15] It was only the harmonic astronomy of Johannes Kepler (1571–1630) that prevented it from being shattered under the superincumbent weight of scientific skepticism.

Kepler's interest in Ptolemy's many analogies between music and the heavens was aroused when he was still a youth in his early twenties. This was not because of any innate tendency toward mysticism or heliolatry on

[14] On Ptolemy's geocentric model of the universe, the Sun is construed to be as mobile as all the planets: but on the heliocentric model, the Sun is construed to be as immobile as the fixed stars. Cf. J. V. Field, *Kepler's Geometrical Cosmology*, p. 17. On Copernicus' spherical universe, see Kuhn, *The Copernican Revolution*, pp. 145–50.

[15] Bruno advanced the hypothesis – most remarkable in his day – that the stars are really Suns and that their extension in the universe is infinite. See Field (note 14), p. 77. Almost a hundred years after Copernicus displaced the Earth by placing the Sun in the center of the universe, it was still possible for Galileo to be condemned to death for confirming with his telescopic observations the heliocentric hypothesis of Aristarchus (c. 310–230 B.C.) as well as that of Copernicus. Cf. Merleau-Ponty and Morando, *The Rebirth of Cosmology*, pp. 63–64. Tycho Brahe, the bon-vivant of Astronomy, employed Kepler as his assistant at the observatory near Prague. When he died a year later (1601), Tycho left his vast astronomical observations to Kepler, who became his successor as imperial mathematician. How Kepler used these observations to construct his *Astronomia nova* is a story fascinatingly told by Bruce Stephenson, *Kepler's Physical Astronomy*.

his part. Quite the contrary: he was responding to the sheer rationality that he recognized in Ptolemy's harmonic theories.[16] For, as Ptolemy had proposed, the distributive motions of the heavens were as orderly as those of music, and Kepler saw reason enough to agree with him in this instance. As Kepler saw it, there was indeed a rational basis in the geometrical relationships between the heavenly bodies that Ptolemy had discovered. These relationships were made evident in Ptolemy's alignment of the musical intervals on the circle of the zodiac, where, in addition to the diametrical octave-positions that he outlined, other consonances showed up which themselves corresponded geometrically to certain aspects of the zodiac.[17] Ptolemy showed, for example, that the musical consonance of a fifth in the proportion of 3:2 arises from that portion of the circle – namely 120° – that is, two-thirds of the way around the zodiac from any given point. Thus, two-thirds from that point is in a trine aspect to that given point. So, too, the consonance of a fourth in the proportion of 4:3 corresponds to 90°, or the quartile aspect. Ptolemy explains it in this way:[18]

> Those parts of the zodiac that are similarly situated are the first ones in affinity with one another. These are the ones that are in diametric opposition, comprehending two right angles and six of the twelve parts [of the zodiac] and 180 degrees; those which stand in a trine position, comprehending one and a third right angles, four parts of the twelve and 120 degrees; those which are said to be in quartile aspect, comprehending one right angle and

[16] Stephenson (note 12) has put to rest for all time the notion that Kepler's *Mysterium cosmographicum* of 1596 and the *Five Books of the Harmony of the World* (*Harmonices mundi libri v*) of 1619 reveal a mystical bent on his part. Thus Stephenson, p. 249: "Kepler was not a mystic. He was, undeniably, an astrologer; but in that age astrology was not yet entirely irrational. His own theories about why astrology worked (and he did believe that it worked) rested on the same theoretical foundations in geometry as his theories on the harmonies of the world."

[17] Geometry was for Ptolemy the basis of the universe's whole design. Cf. Mark Riley, "Theoretical and Practical Astrology: Ptolemy and His Colleagues," *TAPA* 117 (1987), 246, n. 27.

[18] *Tetrabiblos*, 13.

> three of the twelve parts and 90 degrees; and finally, those that make up the sextile position, comprehending two-thirds of one right angle and two of the twelve parts and 60 degrees.

Ptolemy thus recognized four geometrical relationships between the constellations: opposition (octave), trine (fifth), quartile (fourth), and sextile (sixth). Kepler himself is said to have discovered several other aspects of the circle based on other aliquot parts of 360 degrees.[19]

Inspired by Ptolemy's geometry, Kepler was eventually able to link the perfect polygons – triangles, squares, pentagons, and so on, depending on the degree of commensurability of their sides with the diameter of the circle – to all the musical consonances. For example, the ratio of the arc on the circle created by the two sides of a pentagon to the rest of the circle, being 3:2, becomes a visual embodiment of the perfect fifth. In similarly geometrical terms, the ratio between the equilateral triangle and the circle, being 3:2, also makes for the fifth. The ratio between the square and the circle, being 4:3, is productive of a perfect fourth. Working along geometrical lines such as these, Kepler proceeded to find correspondences between all the consonantal ratios of music and the circle. In this way, he elucidated what was implicit in Ptolemy's account of the heavenly harmony: the musical correspondences, as is proved by their ratios, do not always involve the entire circle of the zodiac; the aspects between the constellations, based upon the common denominator 12 for all the musical consonances, always involve the entire circle of the zodiac.[20] The consonances, being the most fundamental elements in the formation of musical scale-systems, thus epitomized for Kepler the startling perfection of the cosmos in all its classical geometrical complexity.

[19] Robbins, *Tetrabiblos*, p. 72, n. 2. It should be noted that Ptolemy's sextile of sixty degrees equals two whole-tones, or a major third.

[20] Field (note 14), p. 133 quotes Kepler, *Harmonikes Mundi*, Book IV, Chapter 5, to this effect: "The consonances do not depend immediately on the circle and its arcs on account of their being circular, but on account of the length of the parts, that is, their proportion one to another, which would be the same if the circle were straightened out into a line." Field observes: "Kepler adds that the Consonances do not always involve the whole circle, but sometimes only ratios of parts of it, whereas Aspects do always concern the whole circle."

Assuming then that there is a consonantal relation between the movements of the planets, Kepler said:[21]

> Accordingly, perfect consonances are found: between the converging movements of Saturn and Jupiter, the octave; between the converging movements of Jupiter and Mars, the octave and minor third approximately; between the converging movements of Mars and the Earth, the fifth; between their perihelial, the minor sixth; between the extreme converging movements of Venus and Mercury, the major sixth; between the diverging or even between the perihelial, the double octave.

But it was apparently Ptolemy's comparison of the planets' daily lengthwise east-to-west motion to actual melodies that inspired Kepler to reproduce in the fifth book of his *Harmonikes mundi* the harmonious constructs that he believed to pervade the universe.

With the harmonic sectioning of the heavens as his guide, Kepler persevered in studying the physical problems involved in the variations of the planetary motions, until he was led, at last, to his revolutionary discoveries: the planets move in ellipses, not in circular orbits; their speeds – and hence the "songs" they produce – vary with their distance from the sun. Accordingly, the closer a planet is to the sun – its perihelial tuning – the faster it travels and the higher the pitch of its "voice"; the more distant a planet is from the sun – its aphelial tuning – the slower is its speed and the deeper is the pitch of its "song." Thus, Mercury, being closest to the sun, and the fastest-moving planet, sings a high-pitched ascending and descending melody; the deep bass voices of Saturn and Jupiter produce a major and minor mode, respectively; Venus, being more circular in its orbit and more limited in its range, sings on one pitch only, while Mars produces a portion of an F-Major scale, and the Moon

[21] *Harmonies of the World* (trans. Wallis), p. 1033. Thus Stephenson (note 12), p. 153: "To summarize all this, Kepler has found, by examining the 'observed' harmonies between the converging motions of adjacent planets, reasonably precise harmonies between the converging motions of Saturn and Jupiter, Jupiter and Mars, Mars and Earth, and Venus and Mercury; between the diverging motions of Venus and Mercury; between the aphelial motions of Earth and Venus; and between the perihelial motions of Mars and Earth, Earth and Venus, and Venus and Mercury."

a segment of the G-Major scale. The saddest song of all the heavenly bodies is that of the Earth: a semi-tone, an interval verberant with tension.[22] Bringing all his calculations within the range of one octave, Kepler poised himself to say this:[23]

> Accordingly, you won't wonder any more that an excellent order of sounds or pitches in a musical system or scale has been set up by men, since you see that they are doing nothing else in this business except to play the apes of God the Creator and to act out, as it were, a certain drama of the ordination of the celestial movements.

The conception as to the harmonic configuration of the universe that began with Pythagoras' discovery of the mathematical basis of musical intervals became the archetype underlying Kepler's physical astronomy. For in his astronomy, Kepler verified mathematically the geometric postulate of an analogy between the infinitely large – the cosmos – and the infinitely small – musical intervals. By uniting these two infinities mathematically, Kepler progressed through one doggedly painful analysis after another to his ultimate solution of planetary motion.[24]

[22] Kepler's planetary songs are given by Elliott Carter Jr. In *Harmonies of the World*, p. 1039, in Kepler's notation with his moveable clefs, and in modern notation as well: Saturn: G A B A G; Jupiter: G A B♭ A G; Mars (approximately) F G A B♭ C B♭ A G F; Earth: G A♯ G; Venus: E E E; Mercury: C D E F G A B C D E C G E C; Moon: G A B C B A G. This is discussed by Stephenson (note 12), pp. 166–68, who explains the basis for Kepler's different tunings, p. 168: "Kepler has simply chosen a note that expresses the aphelial motion of each planet in one of the primary tunings and based its up-and-down melody on that note. The melodies of the different planets are not intended to be comparable in pitch."

[23] *Harmonies of the World* (trans. Wallis), p. 1038.

[24] Kepler speaks of the problem in locating the intermediate positions within the intervals traversed by the planets in almost the same terms used by Aristoxenus in connection with the location of the moveable note, *lichanos* (Ch. 4, note 39). Thus, Kepler in *Harmonies of the World* (trans. Wallis), p. 1039: "They [*sc.* notes] do not form articulately the intermediate positions, because they struggle from one extreme to the opposite not by leaps and intervals but by a *continuum of tunings* and actually traverse all the means (*which are potentially infinite*) – which cannot be expressed by me in any other way than by a continuous series of intermediate notes" [italics supplied].

Thus, what began with Pythagoras' conception of *harmonia*, the union of opposites, was carried to those scientific heights of commanding objectivity with which Kepler laid the groundwork of modern astronomy. Kepler's theory – a "sacred madness," he called it – was framed within the context of ancient astronomical thought; it is expressive of a universe that is as rational and as beautiful as the interplay of forces in a well-formed melody. Its ruling principle is *harmonia*, wherein the affinity between music and mathematics is fully realized. In *harmonia* then, all things human and animate are linked together, and through its offices universal life is made to be nothing less than an endless musical performance. From the most distant planet to the nearest bird, every participant in its choreography is designed and proportioned according to mathematical law to work like an articulate musical instrument. As Kepler so movingly describes it, this is a universe that is governed by a Supreme Being whose mind is as that of a musician. Given the grandeur of his conception, it could not but be that music is of all the arts the most useful; for of all the arts it held for Kepler the key to universal truth, the key with which he unlocked the laws of planetary movement.

Kepler's conception of a harmonically active universe finds intellectual support from a most unexpected quarter: a little-known scholium of Isaac Newton for his *Proposition* VIII in Book III of the *Principia*. The proposition has to do with gravity and reads as follows:[25]

> In two spheres gravitating each towards the other, if the matter in places on all sides round about and equidistant from the centres is similar, the weight of either sphere towards the other will be inversely as the square of the distance between their centres.

Newton's scholium for this proposition would remain little known had Jamie James not introduced it in full into the literature on Pythagorean doctrine.[26] It is, as James says, "most startling," because it "relates directly to the great theme of the music of the spheres."[27] What Newton states in this scholium is that the inverse-square relationships between the weights of the

[25] *Mathematical Principles of Natural Philosophy* (trans. Motte; rev. Cajori), p. 282.
[26] James, *The Music of the Spheres*, pp. 164–65.
[27] James (note 26), p. 163.

planets and their distances from the sun is adumbrated in the inverse-square relationships between the tensions and the pitch of vibrating strings. Not only did Newton ascribe the discovery of this inverse relationship in vibrating strings to Pythagoras, he also asserted that Pythagoras:[28]

> applied to the heavens [the proportions that he had discovered] and consequently by comparing those weights with the weights of the Planets and the lengths of the strings with the distances of the Planets, he understood by means of the harmony of the heavens that the weights of the Planets towards the Sun were reciprocally as the squares of their distances from the Sun.

In the views expressed by the scientists of classical antiquity, by Kepler in his *Harmonikes mundi* and, unexpectedly, by Newton in his scholium to *Proposition* VIII, music and the laws of *harmonia* were acknowledged as offering valuable insights into the mysteries of the universe. As a distinct science, *harmonia mundi* was concerned to illuminate the formal aspects of the universe, thereby offering the key to rationality in all of nature. But subsequent advances in physics have shifted the focus so far from the role of music as a guide to scientific inquiry that the whole notion of a harmonic universe had eventually to end up as an antiquarian curiosity, a pseudo-science unworthy of serious intellectual pursuit. Jamie James has said it best:[29]

> Science has drifted so far from its original aims that even to bother with the question of its relationship to music might appear to be an exercise in irrelevancy, like chronicling the connection between military history and confectionery. Yet every scholar of the history of science or of music can attest to the intimate connection between the two. In the classical view it was not really a connection but an identity.

This classically warranted identity between music and the cosmos has recently reemerged with an urgent promise to throw a blinding

[28] James (note 26), p. 165. James rightfully observes of this statement (pp. 165–66): "Pythagoras did nothing of the sort. . . . The Master erroneously taught that a simple arithmetical relationship existed between the weights of stretched strings and their tones, just as it did between the lengths of plucked strings of different lengths and their tones." Cf. Chapter IV, note 13.

[29] James (note 26), p. 10.

light – more properly, an all-pervasive vibration – into those very edges of knowledge that have heretofore escaped understanding. Most remarkable, this new theory, called Superstring, which gives priority to a sounding universe over the visible or material one, was predicted with uncanny prescience by Jocelyn Godwin, an avowed adherent of the ancient tradition of a harmonious universe. In his words:[30]

> The first postulate of speculative music immediately sifts the believers from the profane. It is that *sound* (or tone, or music) is *ontologically prior to material existence*. One way of giving assent to this is through recognizing that underlying the apparent solidity of matter there is nothing but a network of vibrations. . . . Speculative music often goes further and asserts that the whole cosmos is audible in its superior modes of existence, just as heaven and its inhabitants are visible to certain mystics, even when there are no light vibrations striking the eye.

Godwin wrote these words in 1989, some years before the particle properties in Superstring theory were beginning to be heralded as the solution to an unimaginably difficult problem: how to reconcile two mutually incompatible theories with one another. The one – Einstein's theory of general relativity – deals with the nature of the universe on its most macroscopic level: stars, constellations of stars, galaxies, and clusters of galaxies. The other – quantum mechanics – treats the nature of the universe on its most microscopic level: molecules, atoms, electrons, protons, neutrons, quarks, and, finally, vibrating strings.[31] The problem for physicists has been this: when the calculations of the one theory – the macroscopic – are merged with those of the other theory – the microscopic – the answer turns out to be infinity. For physicists, such an answer offers no solution at all.[32] Superstring theory,

[30] Joscelyn Godwin, ed. *Cosmic Music*, Introduction, p. 13.

[31] Throughout this discussion, I have relied on Brian Greene, *The Elegant Universe*, a book which has helped to moderate my ignorance of particle physics and cosmology. Greene's lucid explanations of arcane matters have brought me a modicum of understanding where hitherto there had been none at all.

[32] Thus Greene (note 31, p. 129): "Calculations that merge the equations of general relativity and those of quantum mechanics typically yield one and the same ridiculous answer: infinity. Like a sharp tap on the wrist from an old-time

it is hoped, will eventually unify these two incompatible theories under a single incontrovertible principle: *harmonia*.

What Jocelyn Godwin had perceived independently of particle physicists to be a network of vibrations "underlying the apparent solidity of matter," has come to be postulated by the proponents of Superstring theory as a cosmic symphony of strings that are vibrating under different resonant patterns. In a book chapter entitled "Nothing but Music: the Essentials of Superstring Theory," Brian Greene explains how the properties of elementary particles observed in nature have taken on the musical characteristics of the type that Godwin had predicted intuitively. Thus Greene:[33]

> With the discovery of superstring theory, musical metaphors take on a startling reality, for the theory suggests that the microscopic landscape is suffused with tiny strings whose vibrational patterns orchestrate the evolution of the cosmos. The winds of change, according to superstring theory, gust through an aeolian universe.

According to Superstring theory then, the elementary ingredients of the cosmos – its fundamental building-blocks – are infinitely small oscillating filaments hidden deep within the heart of universal matter. These filaments are believed to vibrate so harmoniously with one another and in so elegant a concinnity as to provide a framework for uniting the infinitely large with the infinitely small. The arena in which this intense symphonic activity takes place is thought to be a spatial region that is as smooth as a *continuum* and as sensitively responsive to all cosmic events in the universe as a living membrane is to life itself. This cosmic space is anything but passive; on the contrary, it is as dynamically reactive to all occurrences within its precincts as is the *topos* of melody to the myriad movements of the singing voice.[34]

school-teacher, an infinite answer is nature's way of telling us that we are doing something that is quite wrong."

33 Greene (note 31), p. 135.

34 As Greene (note 31), p. 72 explains, immersed as we are within the three-dimensional fabric of space, what we in fact *feel* is gravity, and "space is the medium by which the gravitational force is communicated." On the "cosmic

Just as cosmic space is thought by cosmic physicists to respond to objects such as planets moving in the vicinity of the sun, so do musicians think of the *topos* of melody as reacting forcefully to the movements of the singing voice. And whereas cosmic space is profoundly affected by the gravitational force exerted by one body, such as the sun, on another, such as a planet, the *topos* of melody is felt to be similarly excited by the tension that erupts when one melodic note moves into the vicinity of another. On this analogy, cosmic space and the *topos* of melody are linked at the deepest level by the three categories of experience: motion, time, and the forces of gravity. What is essential to music, therefore, are properties that are truly cosmic. If physicists and cosmologists are given to finding music in the universe, then musicians must be equally justified in discovering the universe in music.

In music, the passage of time depends on the speed with which a melody moves through melodic space. The sharing of melodic motion – tempo – with time and space thus underlies all musical utterance. As is commonly understood, speed is a measure of how far an object travels between two points in a given length of time: and distance is a measure of how much space is traversed between two points in a given period of time. Applied to music, however, all such notions are contingent on a sovereign kind of causality that renders music independent of everything in the visual world. This allows music to set its own pace, make its own time, and order its own space. Roger Scruton explains the situation in these terms:[35]

> The phenomenal space and phenomenal time of music are matched by phenomenal causality that orders the musical work. . . . The notes in music follow one another like bodily movements – with a causality that makes immediate sense to us, even though the *how* of it lies deep in the nature of things and hidden from view.

symphony" produced by the infinitely small vibrating strings – each being thought of as an elementary particle – see Greene (note 31), pp. 146–51.

[35] Scruton, *The Aesthetics of Music*, p. 76. Aristoxenus, as mentioned in Chapter 2, note 96, thought the *how* of it to lie hidden deep within the soul.

The same point is made even more vividly by Wayne D. Bowman, who says:[36]

> The time music "takes" is a contingent affair, whereas the time music "has" or "is" is an essential or fundamental aspect of musical experience. The time in music is musically critical, while the time music is in is incidental.

In discussing time and motion in music, it is important, therefore, to specify who or what is doing the measuring. If, for example, a singer is seen striding across a stage while singing a slow-moving adagio, the observer to the scene is made to experience two kinds of time simultaneously. Although the voice of the singer may be said to be moving with the singer – like a slow-moving passenger on a speeding train – the observer – namely, the listener to the song – will have a perspective of time and distance that differs essentially from that of the witness to the walk alone. The walk of the singer may abstract from objective or universal time ten full minutes as measured by a clock; but the melody being sung makes of time a fusion of the present with its own past and future. Bowman characterizes this peculiar relativity of time and motion in music as a dilation of the present:[37]

> When music dilates the present, one is carried along by the inflections of a "moving" passage. When music broadens present in the direction of the past (as when vividly recalling and reliving a musical experience) it squeezes "real" presence and future to one side, making past present again.

Time – the paradigmatic *continuum* – and music – the temporal art par excellence – can be measured only while they are moving or passing us by. Nonetheless, there really is a past time and a future time, also; but it is only the present time that really "exists" long enough to be measured.[38] If music has the power to annul all such temporal

[36] Bowman, *Philosophical Perspectives in Music*, p. 272. Bowman has quoted in this passage Thomas Clifton, *Music as Heard: A Study in Applied Phenomenology*, p. 62.

[37] *Ibid*.

[38] Thus St. Augustine, *Confessions*, XI. 16 (trans. Watts): "As for the past times, which now are not; or the future, which yet are not, who is able to measure them? Unless perchance some one man be so bold to affirm to me, that that

contradictions, it is because music allows us to think of the past and the future as vividly present before us. The "past" in music is identified with memory, the "future" with expectation, with the consequence that memory and expectation are in effect both present knowledge.[39] This present knowledge that is so actively supplied in music is exactly what St. Augustine was describing when he said:[40]

> If the future and the past do exist, I want to know where they are. I may not yet be capable of such knowledge, but at least I know that wherever they are, they are not there as future or past, but as present. For if, wherever they are, they are future, they do not yet exist; if past, they no longer exist. So wherever they are and whatever they are, it is only by being present that they *are*.

According to Aristoxenus, they are in music. In fact, as he sees it, the only way to deal with music is to follow all three times – past, present, and future – with the ear and with the mind. In stating this case, Aristoxenus seems to have been thinking with the very mind of St. Augustine, whose own relativistic theory of time has always been a matter of amazement to philosophers. Here is Aristoxenus:[41]

may be measured, which is not. Therefore, while time is passing, it may be observed and measured: but when it is once past, it cannot, because it is not."

39 This present knowledge is in reality a consciousness of necessity, or, as Kant would have it, a "causality of reason." It is this that is imposed by the laws of melodic order upon the notes of melody. Thus Scruton (note 35), p. 76, says: "A tone is heard as the *response* to its predecessor, as tending towards its successor, as continuing an action which makes sense as a whole."

40 *Confessions*, XI. 18 (trans. Pine-Coffin).

41 *Harm. El.* II. 38–39 (Da Rios, 48. 11–18). There is reason to believe (and to lament the fact) that lacunae of unknown length intruded into and succeeded this important statement. Cf. Macran, p. 269; Da Rios, *criticus apparatus*. What has been lost must have had to do not only with the time that music imposes on its *topos*, but also with the motion of melody, whose processes cannot be captured by mathematics. For in speaking of melody as "a process of coming into being" (ἐν γενέσει), Aristoxenus had clearly in mind something that lies well beyond the grasp of mathematics. Thus Barker, II, p. 155, n. 37: "The emphasis on 'coming to be' may conceal a thrust against Pythagorean theory, whose mathematical representations deal only with relations between notes located at specific pitches."

> It is clear that the comprehension of melodies consists in following with both the ear and the intellect things that are transpiring in their every distinction. For melody, even as all the other parts of music, consists in a process of coming into existence. The intuitive understanding of music comes from these two things: perception and memory. For it is necessary to perceive what is taking place and to remember what has taken place. There is no other way of following what is occurring in music.

Just as the musical intellect is connected with time, so musical intuition is connected with the notion of musical space. In fact, it is really quite impossible to speak of musical time without dealing with the question of musical space. It is on this latter question – musical space – that theorists and aestheticians are often at odds not only with one another, but with musicians also. Thus, Bowman:[42]

> The extraordinary difficulty of describing music's temporal character without spatial terms shows that musical time and space are experientially inseparable. … However, music's spaces are very unlike the "objective" spaces we ascribe to visual, physical, or geometrical forms. Music's spaces are phenomenal spaces, spaces that move without going anywhere, that change while staying the same.

The problem with musical space is this: Does music move in space or does space become an actuality through the motion of music? Is there in fact a "place" in which music moves? Quoting Geza Revesz, Zuckerkandl writes:[43]

> The space that becomes alive through sound entirely lacks the essential spatial characteristics of optical space, such as three-dimensionality, spatial order, multiplicity of directions, form, and above all occupancy by objects; it has no direct relation to the world of bodies, is related to neither of the two sensory spaces which are given [visual space and tactile space], either in its structure or in its phenomenal elaboration; it knows no geometric relations, and possesses no spatial finiteness.

[42] Bowman (note 36), p. 273.
[43] Zuckerkandl, *Sound and Symbol*, pp. 280–81.

To Zuckerkandl, Revesz' argument against the reality of melodic space is applicable only to noise; for noise and all such disordered acoustical phenomena are localized, as it were, or fixed in place. Where there is noise, there is no melodic space to be ordered; in this case, there can be no music at all. It is in music only that melodic space becomes "alive." To make this point, Zuckerkandl seems to have been thinking with the very mind of Aristoxenus. Here is Zuckerkandl:[44]

> Even where there is nothing to be seen, nothing to be touched, nothing to be measured, where bodies do not move from place to place, there is still space. And it is not empty space; it is space filled to the brim, space "become alive," the space that tones disclose to us. Far from being unable to testify in matters of space, music makes us understand that we do not learn all that is to be said about space from eye and hand, from geometry, geography, astronomy, physics. The full concept of space must include the experience of the ear, the testimony of music.

Others see melodic space in metaphorical terms drawn from the world of vision. Thus Roger Scruton:[45]

> There is no *real* space of sounds; but there is a phenomenal space of tones. It is modeled on the phenomenal space of everyday perception – the space in which we orientate ourselves. It has "up" and "down," height and depth; its single dimension is understood not only geometrically but also in terms of effort and motion, attraction and repulsion, heaviness and lightness. It is permeated by a phenomenal gravity, to the law of which all tones are subject, and against which they must strive if they are to move at all. . . . Yet, try as we might, we cannot advance from this phenomenal space to an objective spatial order. The topological character of space, as a system of places and surfaces, is not reproduced in the acousmatic realm.

44 Zuckerkandl (note 43), p. 292. Cf. Chapter 4, the motion in space of the voice.
45 Scruton (note 35), p. 75. Cf. Lippman, "Spatial Perception and Physical Location as Factors in Music," 24–34.

Like Aristoxenus, Rothstein seems to have accepted melodic space as a given, that is, as a fact of experience that qualifies as a first principle. He states the case in these words:[46]

> A composition is an exploration of musical "space," then, which creates its own topology. It establishes which events are continuous, which discrete; it creates connections between small events and large … . The composer determines what paths may be taken through a musical space and where they will lead.

To others, such as Nicholas Cook, all such concepts as space, motion and melodic lines, or *continua*, can only be understood as metaphors from the visual world:[47]

> Musical lines have no material existence; they only exist in terms of the metaphor of space, a metaphor which Scruton considers to be so deeply entrenched in the experience of music as to constitute one of its defining properties.

Cook thereupon quotes Roger Scruton to this effect:[48]

> It seems then that in our most basic apprehension of music there lies a complex system of metaphor, which is the true description of no material fact. And the metaphor cannot be eliminated from the description of music, because it is integral to the intentional object of musical experience. Take this metaphor away and you take away the experience of music.

The difference in the theories expressed in the above statements and that of Aristoxenus derives from this fact: what Scruton, Bowman, Cook, et al. treat as metaphors, Aristoxenus accepts as reality. In other words, the spatial metaphors of Scruton, Cook, and Bowman express concepts or mental reflections of objects in which one kind of thing – melodic space – is understood in terms of another – visual space. To Aristoxenus,

[46] Rothstein, *Emblems of Mind*, p. 126.
[47] Nicholas Cook, *Music, Imagination, and Culture*, p. 24.
[48] *Ibid*.

however, melodic space is real in itself by virtue of being natural to the domain of music. As such, it requires no construal from any source outside of music in order to be understood.

What musical space does need, as Aristoxenus so keenly realized, is a logical analysis that is compatible in every respect with the testimony of the ear, an analysis that would "save the phenomena," as astronomers understood the case. To "save" the phenomena of music, such an analysis would have to conform to the rule of greatest simplicity: musical space, being the *topos* of melody, *is* what it seems to be. It has been described in the following ecstatic terms by Proust:[49]

> . . . the field open to musicians is not a miserable stave of seven notes, but an immeasurable keyboard (still, almost all of it, unknown), on which, here and there only, separated by the gross darkness of its unexplored tracts, some few among the millions of keys, keys of tenderness, of passion, of courage, of serenity, which compose it, each one differing from the rest as one universe differs from another, have been discovered by certain great artists who have done us the service, when they awaken in us the emotion corresponding to the theme which they have found, of showing us what richness, what variety lies hidden, unknown to us, in that great black impenetrable night, discouraging exploration, of our soul. . . .

Little wonder, then, that Pablo Casals could say so authoritatively of musical space:[50] "The most difficult aspect of music is not the notes, but the spaces between them."

To the musically cognizant ear, musical space is not identical with space in the material world; it stands over against that space as something unmistakably music's own. It is the *topos* in which every kind of melodic change occurs. All such changes – or motions, as they are

[49] Marcel Proust, *Remembrance of Things Past* (trans. C. K. Scott-Moncrieff). I, p. 268.

[50] David Blum, *Casals and the Art of Interpretation*, p. 19, quotes Casals: "'Each note is like a link in a chain – important in itself and also as a connection between what has been and what will be.' When he [*sc.* Casals] played, these links became living art. Every phrase was borne upon a movement of energy which flowed from one note through the next, going towards a point or coming from another, ever in flux, ever formulating a contour."

understood by Aristoxenus – result from the forces of certain tonal functions (*dynameis*); but the forces produced by these tonal functions are not identifiable by anything that geometry can represent.[51] Like musical space itself, tonal functions stand apart from physical functions in the material world as things which belong to music alone. They are functions that set limits to each and every melodic change that occurs in musical space. But musical space itself does not arise, because composers construct it out of points of pitch as though *they* were the only substantial elements of music; rather, points of pitch can be posited only through a synthesis in which the *form* of melodic consecution originates. According to these teachings of Aristoxenus then, it is in the concept of musical functions and their invariability that the potential for this strictly musical synthesis subsists. That being the case, no melodic configuration and no succession of musical notes can contradict what is embodied in the general procedure of spatialization or in the synthesis of melodic consecution and remain music. Multiform and coherent, this synthesis of form and function is secreted in the material quality of sound. It is impervious to definition;[52] but

[51] On the contrary, they are identified by the musical intellect, according to Aristoxenus *Harm. El.* 33 (Da Rios, 42. 11–13): "For we judge the sizes of the intervals by ear, but we contemplate their functions with our intellect (*dianoia*)." Commenting on this important statement, Lippman, *Musical Thought in Ancient Greece*, p. 149, says: "It is possible, then, for the infinitude of pitches comprised in the locus of any moveable tone to be recognized by the intellect as discharging a single function." While Barker, "Music and Perception: A Study in Aristoxenus," 13, finds the passage disappointingly vague, he allows this much to be said: "The role of *dianoia* is to identify the sequences not merely as sequences of intervals, which would be musically meaningless, but as forming or implying structures within which the notes stand in functional relationships to each other."

[52] This strictly melodic phenomenon, the experience of which has been likened to a "dynamic knowledge" by Zuckerkandl (note 43), resists definition, but invites discussion by theorists and aestheticians. Thus Zuckerkandl (p. 313): "What makes the tone an element of musical order is not its pitch but its audible relation to other tones; differences in direction and tension, not differences in pitch, are the constituents of the musical order of tones. And this characteristic of tone sensation, its dynamism, is, unlike pitch, closely connected with the spatial component of tone sensation." *Dynamis* is a state of space that

the musically cognizant ear knows how to follow every nuance in the domain of music, which has been referred to as "le domain si malléable, illimité, de la musique."[53]

The domain of music, or the "topos" of melody, as Aristoxenus understood it, is malleable in that it is acutely responsive to all the motions of the singing voice; it is illimitable in that it is theoretically capable of infinite extension and infinitesimal diminution;[54] it is also a homogeneous *continuum* in that it imparts no difference to the melodies that move in its precincts, for what can occur at one point or pitch range in the melodic *topos* can also occur at every other point or pitch range.[55] This attribute of the melodic *topos* makes all and any transpositions possible.[56] These notions of spatial homogeneity, malleability, and infinitude are prominently featured in Aristoxenus' theory and, as he insists, they are all derived from one source only: the special knowledge of the

is implicated in the tensions and resolutions that arise in melodic progressions and, as Scruton (note 35), p. 266 observes, "is of inexhaustible interest, and has inspired some of the most important ventures in music theory." Cf. Chapter 3, note 46.

53 Léon-Paul Fargue, *Maurice Ravel*, pp. xxv–xxvi.

54 Thus Arnold Schoenberg, *Theory of Harmony*, p. 129: "The analogy with infinity could hardly be made more vivid than through a fluctuating, so to speak, unending harmony . . ."

55 The changes imparted to scales and the melodies formed on those scales arise from differences between pitch-keys or *tonoi*, one of the most disputed issues in ancient theory, one that produced a considerable mass of scholarly writing. See, for example, Barker, II, pp. 17–27. The question of the relation between the *tonoi* and the modal scales is complicated by the fact that the Greeks' notion of pitch was not, as is ours, that of a comparison with an external standard.

56 In *Harm. El.* I. 7 (Da Rios, 11. 19–12. 8), Aristoxenus seems clearly to be speaking of keys of transposition: "Since each of the scales (*systēmata*), when placed in a certain region (*topos*) of the voice, is sung, the scale when taken by itself, admits of no difference, while a melody, composed in that scale, takes on no accidental difference, but the greatest difference. Therefore, it is necessary for one who would deal with the subject before us to speak of the *topos* of the voice in general and in detail so far as is appropriate, that is, so far as the nature of the scales themselves signifies." On Aristoxenus' view of the *tonoi*, see Mathiesen, *Apollo's Lyre*, p. 318; for musicians' similar view, see Zuckerkandl, *Sound and Symbol*, pp. 310–11.

musically intuitive ear. The problem that challenged Aristoxenus was to find a musically logical formalism capable of dealing with the three major problems raised by these notions: (1) how to reconcile the concept of a homogeneous *continuum* with the fact that it is constituted of indivisible notes when, according to Aristotle, what is continuous cannot be constituted of indivisibles;[57] (2) how to deal with the fact that the melodic *topos*, however perfect and homogeneous it may appear to the ear, is contradicted by the mathematical laws of Pythagorean harmonics;[58] (3) how to determine the true size of any musical interval and how to arrive at the true location for any melodic note in a tonal *continuum* that is infinitely divisible.[59]

As Aristoxenus saw it, if any solution to these problems was still to seek, three well-defined areas had to be avoided: (1) geometry (which is the stronghold of Pythagorean harmonics);[60] (2) the properties of musical

[57] See Chapter 3, note 34.

[58] See Chapter 4, note 41.

[59] As noted in Chapter 3, note 59, the infinite divisibility of the tonal continuum stems from the characteristics of the square-root of two, the number obtained on the division of the whole-tone. The square root of two is irrational (*alogos*), for its constituents can be written with unending, nonrepeating decimal parts. As such, it is a natural inhabitant of the world of the continuum and, as a root of the polynomial equation $x^2 - 2 = 0$, can submit to being counted. Thus, John D. Barrow, *Pi in the Sky*, p. 211: "The continuum is made up of all the rational numbers, which are countably infinite, plus the irrational – those quantities like the square root of 2 – which cannot be expressed as the ratio of two natural numbers." As mentioned earlier, in Chapter 4, the Pythagorean harmonicians dealt either with whole numbers, like 1, 2, 3, and so on, or with the harmonic fractions, like 1/2 (octave), 2/3 (fifth), and 3/4 (fourth), all being numbers which are yielded by dividing any whole number by another whole or rational number. And since they could not express the square root of 2 as the ratio of any two whole numbers, they found the octave itself impossible to measure out precisely. Cf. Bélis, *Aristoxène*, pp. 65–66.

[60] In arguing against the geometrical method of the Pythagoreans, Aristoxenus makes the point that musicians are, in no sense of the word, craftsmen. Unlike carpenters, lathe-turners, or other kinds of craftsmen, they are not trained to discriminate the straight line, the circle, or any other figures. Rather, the objects of their perception (which must be acute) are magnitudes of intervals and their functions. Cf. note 51.

instruments;[61] (3) musical notation.[62] The search had to be conducted instead within the natural boundaries of what Aristoxenus called "attunement" (*to hērmosmenon*). He thought of this attunement as something utterly different from the Pythagorean concept of *harmonia* – the "fitting together" of opposites that is encapsulated in the geometrically-secured formula, 6:8 : 9:12. For Artistoxenus, attunement was a topological framework from whose unifying principles the phenomena of music could be rationally deduced and in whose statutes the testimony of the ear would be honored. Attunement had to do solely with those musically logical sequences of intervals that would offer composers the freedom to launch melodies into a musical space of wonderful flexibility and adaptability. There, melody would be able to live independently of the laws of mathematics.[63]

[61] As Aristoxenus expressed himself in *Harm. El.* II. 39 (Da Rios, 49. 3–7), he saw little value for the student of melody in the study of musical instruments. For in his view, musical instruments and their physical properties, as studied in isolation from music, would lead one as far from the truth about attunement as does geometry. His argument seems so eerily directed against everything that Schlesinger maintained in *The Greek Aulos* (as though time does not exist), that there is little wonder why she had vigorously to refute Aristoxenus. Cf. Barker, II, p. 154, note 33, who explains: "Armed with her complex and original theories about *auloi* and their scales, Schlesinger was often prepared to explain away claims made by Aristoxenus about them, arguing that he was largely ignorant of the instrument." In truth, Aristoxenus was a leading authority on *auloi*. Cf. Bélis, *Aristoxène*, p. 105, who explains Aristoxenus' position on instruments by saying "the auloi are 'inanimate instruments' which require being attuned by the ear of the musician." Cf. p. 26.

[62] In criticizing those who considered notation to be the goal of harmonic science, Aristoxenus had this to say in *Harm. El.* II. 39 (Da Rios, 49. 6–9): "So far from being the limit of harmonic science, notation is not even a part of it, unless writing down metres is also a part of the science of metre" (trans. Barker). On the notation of Phrygian melodies, see Chapter 2, note 91. Bélis, *ibid.*, makes this important point on the question of notation: "The notation of which Aristoxenus speaks is a notation of intervals and not of notes; the sign used thus defines a succession of intervals, whose size alone is important … notation is done to oblige the uninitiated, the ignorant." Moreover, as Aristoxenus states explicitly in *Harm. El.* II. 40 (Da Rios, 50. 4ff.), notation fails to distinguish the differences in functions (*dynameis*) of the various tetrachords. Cf. Barker, II, p. 156, n. 42.

[63] Musical space, or the *topos* of melody, admitted of three conditions according to Aristoxenus, none of which can be defined by mathematical

The moment Aristoxenus probed deeply into the nature of this topological framework – either experimentally or theoretically – he saw that no mathematical formula could describe or predict the natural course of those intervallic sequences that are countenanced by the mind's ear. The Pythagoreans had proved to a mathematical certainty that many of the intervals conceived of as melodic by composers could not be represented by ratios of whole numbers – intervals such as semitones, quarter-tones, and other micro-intervals that characterize the melodic genera and nuances (*chroai*). This, as the formulas of Euclid and Archytas demonstrated, is because musical space is irrational mathematically speaking and no amount of geometric approximations can do away with its innate incommensurability. The geometers working with divisions of a straight line were therefore of no help at all since, as Aristoxenus argued, there could be no agreement between their abstract calculations and the real world of melody.[64]

Being an authority on the construction of musical instruments, Aristoxenus knew that if the properties of strings and winds are integrated with anything, they are not with attunement, but most emphatically with the laws of geometry. For the design makeup of strings and winds alike is a matter of geometrical measurements and proportional processes. These procedures entail the estimation of major ratios like those of body-containing rectangles, for example, or the proper alignment

formulae: (1) the place where the intervallic motion of the singing voice occurs; (2) the place to which the moveable notes of the tetrachord are confined; (3) the sonorous space, or *tessitura*, extending between high and low pitch, whose divisions by the voice or instruments are apprehended solely by the ear. For as Aristoxenus understood it, there is nothing in music that is not a fact of our perception. Cf. Bélis, *Aristoxène*, pp. 134–35.

64 The real world of melody is, as Bowman (note 35), p. 138, explained it, "always and unavoidably a world of the ear." He describes Aristoxenus' orientation in these words: "Attend to the sounds, he [*sc.* Aristoxenus] urged: music's significance must be explained in terms of these sounds, their relationships, their functions within a musical system – not extra-musical affairs like mathematical proportions. Music consists, he in effect argued, not in isolated acoustical 'data', but in tendencies, connections, and functions within a musical system. A truly musical theory cannot be built from acoustical information about discrete tones or intervals, but must address the ways these function within musical practices."

of finger-holes. The question of attunement does not enter into the case at all. In his typically no-nonsense style, he says:[65]

> In general terms, the greatest and most egregious of errors is that which refers the nature of attunement to musical instruments. For it is not because of the properties of musical instruments that attunement has the sort of character that it has.

He goes on to argue that attunement is something that exists in the musical mind, even if he could offer no proof of its existence other than its presentation in an actually existing melody. Of this he was certain: attunement, as he understood the term, is not a property of musical instruments:[66]

> For as there is no attunement in the strings save that which the skill of the hand confers upon them, so there is none in the finger-holes save that which has been introduced by the same agency. That no instrument puts itself into attunement, but that it is sense-perception which is the principal authority over this operation is obvious and requires no discussion.

Aristoxenus' remarks on musical notation are precious: they are the earliest reference in Western literature to the art of casting music into written form. He speaks of musical notation as already a long-standing institution in his own day, but at the same time he repudiates it as a source of musical knowledge. As he sees it, musical notation can make the sizes of intervals discernible to the eye, but it can never succeed in explaining the musical properties of such intervals as they are presented to the ear in melody. That being the case, the mere perception

[65] *Harm. El.* II. 41 (Da Rios, 52. 5–9).

[66] *Harm. El.* II. 43 (Da Rios, 53. 13–18). Commenting on Aristoxenus' refutation of all musical theories that refer melodic attunement to the constitution of musical instruments, Bélis, *Aristoxène*, pp. 61–62, says: "The key idea of the text is that one cannot take for a goal, for an aim and a criterion, a simple medium, a common instrument: for, Aristoxenus says, the aulos and the lyre are the object of a judgment (τὸ κρινόμενον) and cannot be an authority and a goal (κύριόν τε καὶ πέρας); in fact, the strings of the lyre and the perforations of the aulos are not tuned by themselves; the tuning of an instrument requires a manipulation by a musician, whose ear hears and judges of the sounds produced."

by the eye, or even by the ear, of the magnitudes of intervals does not constitute musical knowledge. In his words:[67]

> Regarding what is accounted the final goal of the study called Harmonic, some people say that it is in the writing down of melodies, asserting that it [notation] is the end-all of our understanding of every kind of melody. . . . But the fact is that notation is not the end-all of harmonic knowledge, nor is it even a part of it. . . . For the person who knows how to write down a Phrygian melody does not necessarily know best what a Phrygian melody is. It is clear then that notation should not be the end-all of the study in question. That what has been said is true should be clear to those who study the subject: notation is necessary only for making the sizes of intervals discernible.

In Aristoxenus' view then, musical notation does not in and of itself constitute musical knowledge; it is, rather, a necessary, but not a sufficient skill for the composition of music. In the centuries succeeding him, many composers would make the same point about the knowledge of notation and the understanding of music, perhaps no one more powerfully than Beethoven:[68]

> I carry my thoughts about with me for a long time, often for a very long time, before writing them down. I can rely on my memory for this and can be sure that, once I have grasped a theme, I shall not forget it even years later . . .

[67] *Harm. El.* II. 39 (Da Rios, 49. 1–18). As Bélis, *Aristoxène*, p. 105, points out: "Rarely does Aristoxenus show himself to be so virulent as he does towards these professors of writing: he taxes them with ignorance and describes their ideas as absurd . . . . He explains in substance that one can know very well how to notate a melody that one hears, without knowing however in what it consists (understand: without knowing the rules of the art); the notation of meters is no more a part of metrics than is the notation of intervals a part of the science of harmonics." The most important fact to be learned here is that the notation of which Aristoxenus speaks is that "of intervals and not of notes; the sign used thus defines a succession of intervals, of which only the magnitude matters."

[68] From a written conversation with Louis Schlösser (1822 or 1823), for which see Morgenstern, *Composers on Music*, p. 87. For Beethoven, then, notation was simply a necessary chore, a written record for the purpose of publication. His genius lay not therefore in his capacity to write down what he heard in his mind's ear, but, rather, in his ability to develop, in the absence of notation, monumental forms from the germinal bits and pieces that had taken root in his mind.

> since I am aware of what I want to do, the underlying idea never deserts me. It rises, it grows, I see and hear the image in front of me from every angle, as if it had been cast [like sculpture], and only the labor of writing it down remains, a labor which need not take long, but varies according to the time at my disposal, since I very often work on several things at the same time. Yet I can always be sure that I shall not confuse one with another.

And Robert Schumann makes the case in these purely Aristoxenian terms:[69]

> He is a good musician who understands the music without the score, and the score without the music. The ear should not need the eye, the eye should not need the outward ear.

The Pythagorean harmonic method was derived from the most fundamental and original mathematical model of continuity – the simplest geometric figure – the straight line. Born of this mathematical model of continuity, Pythagorean harmonics had to yield results that are mathematically true.[70] But, as Aristoxenus discovered, these results were not always musically true. Nonetheless, he could ill afford to ignore them so long as they stood opposed to his goal: a standardized system of attunement that would not only be practical, but also reliable and convenient for all musical purposes. The task he faced was forbidding. He had first to ameliorate the hostility between the geometry of the straight line and the perceived symmetrical continuity of the melodic *topos*. Euclid and the Pythagoreans had proved that the most perfect interval, the octave, is in fact asymmetrical in that it cannot be

[69] Morgenstern (note 68), pp. 148–49. The passage in full reads as follows: "As Eusebius observed a young student of music diligently following a rehearsal of Beethoven's *Eighth Symphony*, score in hand, he remarked: "*There* is a good musician!' – 'By no means,' said Florestan. 'He is a good musician who understands the music without the score and the score without the music. The ear should not need the eye, the eye should not need the (outer) ear . . .'."

[70] Music, by its very nature, or *physis*, does not conform to the precision of mathematical concepts such as obtain for the straight line. Euclid had demonstrated as much himself when he proved mathematically that octaves are always something less than six whole-tones and whole-tones themselves are not divisible into two equal semitones. Cf. Chapter 4, note 27.

reconciled mathematically with its internal components. For the ratio of the octave (2:1) – the macrocosmos, as it were – when merged with the ratios of its innermost interval – the microcosmos – yields infinity in the form of .66666666....[71] In practical terms, this meant that the process of fixing the locus .of the interior or moveable notes of any tetrachord could never be completed because, at any stage of the process, it could always be continued.[72]

Aristoxenus came upon this problem the moment he attempted to locate by conventional mathematical means such interior notes as *lichanos* in the two lower-pitched tetrachords – *Hypaton* and *Meson* – and *paranētē* in the three higher-pitched tetrachords – *Synnemenon*, *Diezeugmenon*, and *Hyperbolaion*. For example, in mapping the interval from E (*hypatē Meson*) to A (*mesē*), he found by a process of dividing magnitudes that there was no position at which the magnitudes thus yielded could not be potentially redivided. This was because at every stage in his process of division, there was always a remaining interval which was itself capable of being divided. Therefore, if, as Aristotle had observed, "every magnitude is divisible into magnitudes," how could a *lichanos* be assigned an exact location between *hypatē Meson* (E) and *mesē* (A) in any one of these three genera, let alone in any one of the three differentiae or nuances (*chroai*), if the possible sites for a *lichanos*

[71] The problem of infinity arises in the relationship between octaves and fifths. Creating fifths on the monochord twelve times, as Pythagoras is reputed to have done, will yield a series of fifths that is about one-ninth of a whole-tone sharper than a note produced by an octave series. This discrepancy is expressed mathematically by the ratio 531,441:524,288, an interval known as the Pythagorean *comma.* To judge from what Aristoxenus tells us on this subject, he seemed to feel that such an interval as a comma and such an unmanageable interval as the fifth were nature's – that is, musical nature's – way of informing us that we are doing something wrong. Cf. note 59. See Helmholtz, *On the Sensations of Tone*, p. 548; Isacoff, *Temperament*, pp. 102–5.

[72] What is implicated in this problem is the geometrical criterion for the incommensurability of two line segments. It is stated by Euclid *Elements*, Book X, Proposition 2: "If, when the lesser of two unequal magnitudes is continually subtracted in turn from the greater, that which is left never measures the one before it, the magnitudes will be incommensurable."

were potentially infinite? In other words, the limits of the tetrachord, being a perfect fourth, can be calculated on the Pythagorean standard as 4:3; but by the same standard, the precise locations of the various notes intervening between the limits of the tetrachord cannot be fixed by measuring continuous magnitudes.[73]

Using the conventional method of dividing magnitudes, Aristoxenus discovered a linear array of an actually infinite collection of discrete notes such that between any two notes there could be yet a third. This made it altogether impossible to locate *lichanos* in the *pyknon*, the "dense" ensemble of the two smallest intervals of the tetrachord.[74] For the *pyknon* called for the most delicate variations in the pitch of *lichanos*, each of which lent a recognizable and distinct tonal color to the

[73] As Bélis, *Aristoxène*, pp. 43ff. reminds us more than once, much of Aristoxenus' teachings are lost. But from what remains, this much emerges: Aristoxenus was determined to deal with the infinite, the *apeiron* – not with the immeasurably large, but with the immeasurably small. The task he undertook had to do with number and not, as was the case with the Pythagoreans, with geometry. Thus, Aristoxenus showed intuitively that at the start of the process in seeking the movable notes there is always a remainder, and that this remainder continues until it becomes so small that the operation of pin-pointing a note such as *lichanos* becomes so limited as to compel one to stop altogether, or to settle for an approximation. Similarly, no matter how many sides a polygon may have, it will never fit into a perfect circle; it can only approximate a perfect circle.

[74] This characteristic density of intervals occurs in the Enharmonic genus, in which the sum of the two quarter-tone intervals is less than the remainder of the tetrachord, as, e.g.:

E E + F A

pyknon

A second occurrence of the *pyknon* is heard in the Chromatic genus, in which the sum of the two smaller intervals is a whole-tone, this – the *pyknon* – being less than the remainder of the tetrachord by a semi- tone, as, for example:

E F G♭ A

pyknon

Cf. Michaelides, *s. v.*

tetrachord.[75] In seeking a place for *lichanos*, Aristoxenus, as he tells us, came upon infinity:[76]

> In the first place, if we seek a specific name for each increase and decrease of the notes forming the *pyknon*, it is obvious that we will need an infinite number of names; since the locus of *lichanos* is divisible into infinite parts.

A series of such infinite *lichanoi* could not but give an impression of vagueness wherein there must be a general obliteration of distinctions. There must be a "fog," as it were, of *lichanoi* such that no one *lichanos* can be distinguished from another, let alone from its immediate neighbor. This enigma of the *lichanoi*, as posed here by Aristoxenus, recalls the paradoxes of Zeno the Eleatic (c. 490 B.C.), whose notions of continuity, unity, and infinity have vexed the mind of man since their inception. For the paradoxes of Zeno, being based on the geometry of the line, to the exclusion of the factor, time, demonstrate outstandingly that there is no "next point" or "next instant" that can possibly be

[75] In the examples given in note 74, the note F is *lichanos* in the Enharmonic genus; the note G♭ is *lichanos* in the Chromatic genus. Neither of these notes can be located using the standard Pythagorean method. The *chroai* (literally, "colors") present even greater difficulties, since they answer to the most subtle shiftings of pitch imaginable. As Barker, II, p. 142, note 93 explains: "the placing of the two moveable notes at any definite position within the ranges proper to a genus constitutes a shade (*chroa*) of that genus." For a complete analysis of the genera and shades, see Fig. 7.

[76] *Harm. El.* II. 48 (Da Rios, 59, 9–60.3). As Aristoxenus explained earlier in *Harm. El.* I. 26 (Da Rios, 34. 3–7): "We must regard the *lichanoi* as infinite in number; for wherever you station the voice in the *topos* of the *lichanos* as designated, the result will be a *lichanos*, the *topos* of the *lichanos* not being an empty space, that is, not a space that is not capable of admitting a *lichanos*." To this statement, Aristoxenus adds what must be considered the greatest understatement of his treatise (Da Rios, 34. 7–8): "So that the matter we are arguing is one of no small importance." On the contrary, the matter being argued by Aristoxenus is of the greatest importance: namely, that while the individual movable notes such as *lichanos* are themselves finite, they are, as a class, infinite. Aristoxenus' recognition of this fact places him among the great thinkers of the millenia. Cf. note 78.

specified.[77] So, too, Aristoxenus revealed the potentially infinite *lichanoi* through the same procedure: the geometry of the line, to the exclusion of the ear and the voice. And this led him to a similar result: there is no *lichanos* immediately succeeding a preceding note, *parhypatē*, that can be accurately specified in theory or in practice. The problem raised here by Aristoxenus had been anticipated a century or so earlier by the philosopher, Anaxagoras of Clazomenae (c. 500–c. 428 B.C.), who observed:[78] "In the case of the small, there is no smallest thing, but always something smaller."

All these things being true of the straight line, a line that has the characteristics of a physical *continuum*, Aristoxenus came to realize that by relying solely on the geometry of the line, he could never resolve the problem of potentially infinite *lichanoi*. This meant that there could be no way to establish a system of attunement whose elements would be related to one another by chains of logical implications such that each member of the system would imply every other member. To be sure, mathematics had succeeded in securing the boundaries of the consonances: octave, fifth, and fourth. But when it came to fixing the locations of the notes intervening between these linear boundaries, the mathematical method not only produced discontinuities in the numerical proportions defining the consonances, but also irrational numbers arising from the division of the consonances. Indeed, as

[77] Thus, Richard Sorabji, *Time, Creation and the Continuum*, p. 322: "Aristotle takes Zeno to have supposed that an infinity of sub-distances would require an infinite time. To that he gives the right answer, that we must distinguish between infinite divisibility and infinite length. The distance is infinitely divisible, not infinitely long, and therefore the time available is adequate, because it is infinite in the appropriate way, that is, infinitely divisible."

[78] *Vors.*, 59B3 (D-K, 33. 14–18). See Rothstein, *Emblems of the Mind*, p. 64, who observes that Anaxagoras anticipated Georg Cantor in this statement. As Rothstein explains, Cantor, in studying the problem of infinity, dramatically changed the way in which we think of lists of things. What Cantor has made us realize is that "space or time itself is so dense with points that any area surrounding a given point contains an infinite number of other points, which, in turn, connect 'smoothly' with one another." Aristoxenus, in his turn, makes us realize that if any *lichanos*, or point on the line of pitch, approaches another point of pitch in the infinite *topos* admitting such a point, it must also be included in the continuous collection of pitches known as *lichanoi*.

Euclid had proved, intervals of the superparticular variety – fourths, fifths, and whole-tones – are not divisible rationally at any point whatever, despite all that the ear records to the contrary.[79] In sum, then, where Aristoxenus intuited a seamless melodic continuity, he met with theoretical imprecision, ambiguity, and incompleteness in the form of infinite pitch possibilities; and where he perceived by ear and memory a "wondrous order" in the logical succession of melodic notes and intervals, he came upon mathematical discontinuities in the form of incommensurable magnitudes.[80]

Long before Aristoxenus' time, musicians dealt with problems such as these by the simple expedient of ignoring them. With only their innate musical instincts to guide them, they tuned their winds and strings by ear and often produced music of such surpassing beauty as to drive incommensurable numbers like $\sqrt{2}$ out of the mind's ear. One such performer prompted the composer Telestes (c. 420–c. 345 B.C.) to speak of his Lydian airs as the work of the "King of the auloi." The music reduced Telestes to this rapturous outpouring:[81]

79 Macran, *Aristoxenus*, p. 248, says accordingly: "But the ear ignoring the mathematical differences attends to the common features in the impressions which these divisions make upon it, and constitutes accordingly three genera, the Enharmonic, Chromatic, and Diatonic . . ."

80 The term "incommensurable" reflects the Greeks' geometric view of number. This is made explicit by Plato in the *Theaetetus*, the dialogue which he dedicated to the memory of the young hero who fell in battle in 369 B.C. As Plato tells it (147D3–6), Theaetetus credits the mathematician Theodorus of Cyrene (born c. 460 B.C.) with the full explanation of incommensurability: "Here, Theodorus was describing something about the sides of squares [square roots, or *dynameis*], showing that the sides of squares of three or five feet are not commensurable in length with those of one foot, and in this way he kept taking up one after another until he reached seventeen feet. At this point he stopped." As Thaeatetus surmised (147D7–9), Theodorus stopped because he came upon square roots "that appeared to be infinite in number, so that one had to try to collect into a single term one by which all these infinite roots could be spoken of." Incommensurable in length thus means that no common measure exists between these entities. See B. L. van der Waerden, *Science Awakening* I, pp. 141–42. Cf. A. Wasserstein, "Theaetetus and the History of the Theory of Numbers," *CQ*, n.s. 8 (1958), 165–67.

81 Athenaeus, *Deipnosophists* xiv. 617b.

> Or that Phrygian, king of the sacred, fair-breathed auloi, who was the first to attune a shimmering Lydian air that rivalled the Dorian muse, interweaving on his reeds the lovely-winged strain in the melodious voice of his life's breath.

Others of Aristoxenus' predecessors were equally outstanding, not only as performing artists but, also, like Aristoxenus himself, as leaders of their own pedagogical or theoretical schools. Of these, the best known is the celebrated theorist, Damon of Athens (mid-fifth century B.C.), who was fortunate in having had Plato to represent his views on music's powers to change the human soul for better or for worse.[82] Also well known is the composer, dithyrambic poet, and virtuosic aulete, Lasus of Hermione (sixth century B.C.). Among his many accomplishments in the field of acoustics and musical theory were his innovations in the technique of aulos playing, some of which were considerable enough to have brought about a veritable revolution in the performance of music.[83] Equally interesting is the virtuoso harpist, Epigonus of Ambracia (sixth century B.C.), whom Aristoxenus characterizes as a master-teacher of his own school, his many accomplishments on the harp having been prodigious enough to win him the epithet: Virtuoso (*mousikotatos*).[84]

[82] On Damon and his ethical theory of music, see Chapter 1, note 71. His enormous influence on the musical ideas of Plato has been the focus of intense study, as, for example, by Lasserre, *Plutarque*, pp. 80–84; Richter, *Zur Wissenschaftslehre von der Musik*, pp. 22–26; Moutsopoulos, *La musique dans l'oeuvre de Platon*, pp. 67–80. This influence, as treated by Aristotle, is considered by Bélis, *Aristoxène*, pp. 56–60.

[83] Lasus' innovations on the aulos are described by Ps.-Plutarch *De mus*. Ch. 29. 1141C (Ziegler-Pohlenz, 23. 13–17): "Lasus of Hermione, by adapting the rhythms to the dithyrambic movement, and by using more notes obtained by dividing them fractionally (διερριμμένοις), brought the music that existed before his time into an altered state." My translation, "by dividing them fractionally," is influenced by Lasserre, *Plutarque*, p. 37, who renders the passage, "par l'adjonction de sons obtenus par fractionnement." This translation suggests that Lasus was seeking a way to accommodate within the limits of a tetrachord more notes than mathematical theory would allow. See, however, Barker, I, p. 235, who translates: "(and so making use of more notes, *widely scattered* about)." The italics are mine.

[84] Epigonus was of the same era as Lasus (sixth century B.C.). According to Athenaeus *Deipnosophists* xiv. 637–38, he, too, was seeking to enlarge the

These and other theorists mentioned by Aristoxenus all seemed to have had one thought in common with him: how to reconcile the inherent problems of musical space with the evidence of the ear and the limitations of their instruments. Lasus and the followers of Epigonus, for example, apparently tried to ameliorate these difficulties by assigning "breadth" to individual notes so as to have them fill up the spaces denied them by mathematics but allotted to them by the demands of melody.[85] This practice is not unlike that of singers who, when singing with instrumentalists, deliberately "torque" their intonation as a way of intensifying the ambient colors of the pitches being sung. Aristoxenus disdained all such tactics, on the grounds that they violated the concept of notes as dimensionless points, whose single occupancy of every melodic space is a function of their fixed or moveable position in the attunement. The theorists who provoked Aristoxenus' sharpest criticisms, however, were the *Harmonikoi* (so-called by him), who based their theories solely on the evidence of their ears and the attributes of their musical instruments.[86] Eratocles

melodic capacities of the harp, whether by adding strings, or by increasing the size of the sound-box, the result of his efforts having been the introduction of new and beautifully colored (*euchroa*) variations into his playing.

[85] Aristoxenus *Harm. El.* I. 3 (Da Rios 7. 19–21) emphatically warns against this practice if the distinguishing features of musical notes are to be ascertained: "Anyone who does not want to be forced into the position of Lasus and certain of the followers of Epigonus, who thought that a note has breadth [*platos*] must say something rather more precise about it: and once this has been defined, many of the subsequent issues will become clearer." (trans. Barker). Barker, II, p. 128, n. 12, explains Aristoxenus' position by stating his "idea that the moveable notes have ranges of variation which abut, but do not overlap, so that though each is dimensionless, it is the sole occupant of a determinate region of pitch."

[86] One of the most important papers on the subject of Aristoxenus' position with regard to his predecessors, the *harmonikoi*, is that by Andrew Barker, "`OI KALOUMENOI `ARMONIKOI": "The Predecessors of Aristoxenus," *PCPS* 24 (1978, 1–2). As Barker points out (p. 5), the criticisms which Aristoxenus leveled against the *harmonikoi* are in general to the effect that they presented no principles, or *aitiai*, to support their conclusions, many of which were based on the structures of their instruments. Most important, as Barker argues, Aristoxenus took it upon himself to criticize "*all* the fumbling, relatively unscientific attempts to establish the basic outlines of

(fifth century B.C.), a leading exponent of this strictly empirical school, was roundly denounced by Aristoxenus, not only for being totally unsystematic in his approach, but, even worse, for violating the phenomena of music.[87]

Theorists like Eratocles are the very sort whom Socrates singles out in the *Philebus* as proof that music is a matter of guesswork and, hence, is not worthy of being considered a superior art. As Socrates explains to Protarchus, musical skills are imprecise at best, and at their worst, are a matter of industrious drills by rote:[88]

> First of all, music is full of it [guesswork], tuning the consonance not by measurement but by the lucky aim of a practiced hand; and all of this [guesswork] is in the art of the aulos, too. In the art of the cithara, a hunt goes on to guess at the exact measure of each string as it sounds its note, so that there is little certainty, but a great admixture of unreliability.

that science of which Aristoxenus sees himself as the Newton or Darwin." Aristoxenus was in fact quite right in his assessment of his own contributions to the field.

87 Eratocles is not known from any other source than that of Aristoxenus. Barker, "Music and Perception: A Study in Aristoxenus," *JHS* 98 (1978), 11, makes a number of important points in connection with Aristoxenus' criticism of Eratocles' methods, one of which is that the phenomena that Eratocles was violating are "the facts of experience ascertainable by ear, which the Pythagorean system cannot readily accommodate." Another is that the points raised by Aristoxenus take us "beyond musicology and into philosophy. Aristoxenus is not simply investigating agreed phenomena in standard ways: he is expressing, and vigorously arguing for, a particular conception of what music is, and in what the science of the study of music properly consists" (p. 10). Eratocles' interest in the cyclical ordering of the octave in the Enharmonic genus amounted in Aristoxenus' estimation to little more than mechanical manipulations. Cf. André Barbera, "Octave Species," *Journal of Musicology* 3 (1984), 229–41. Aristoxenus' own aim was far more profound and expansive, as Da Rios, *Aristosseno*, p. 55 states so well: "he constructed a homogeneous and regularly-connected system of tonalities ("construì un sistema di tonalita omogenee e regolarmente") that would provide a grand and uniform possibility for modulation." See Fig. 4.

88 Plato *Philebus* 56A3–7. See, however, the view expressed in Chapter 1, note 28.

The musicians who were performing these tasks liked this sort of guesswork as little as did Socrates. In fact, they had long been struggling for a way to base their attunements on something in music to which the ear bears testimony, something so fundamental that without it, all possibilities of attunement vanish.[89] Using their ears then, to the exclusion of any instruments of measure, they tried to isolate what could not be identified mathematically: the smallest unit-interval into which the melodic *topos* could be divided. This mathematically elusive interval is the *diesis*, or enharmonic quarter-tone. These strivings after musical accuracy provoked Socrates to say in the *Republic* that they were "wasting their time in measuring the audible consonances and notes against one another." Glaucon, Socrates' interlocutor, concurred, with the following revealing words:[90]

> Yes, by the gods, it is ridiculous how they speak of quarter-tone groups and such like, bending their ears alongside their instruments as though they were hunting after their neighbors' conversation. Some of them claim that they hear still another pitch in the middle and that this is the smallest interval which ought to be measured as the unit-interval. Others argue that the notes sound alike. Both prefer their ears to their minds.

Aristoxenus agreed completely with this position, stating more than once that no theory could ever be successful if it is uniquely determined by the empirical evidence. As he put it, a theory "must rest instead on an appeal to the two faculties of hearing and intellect

[89] For references to this crucial principle, see Barker, II, p. 129, n. 24.

[90] Plato *Rep*. 531A4–9. The quarter-tone groups of which Glaucon speaks are set forth by C. F. Abdy Williams, *The Story of Notation*, p. 34, Fig. 2. As he points out (p. 33), it is just this sort of "scheme of pycna" or *katapycnosis* at which Aristoxenus scoffed (see note 92). Barker, "Music and Perception," p. 15, thus notes that, according to Aristoxenus, what is indeed needed for musicians is a *metron*, a standard of measurement to which musicians can refer the melodic intervals *as they are heard*. But, as Aristoxenus realized, the enharmonic *diesis*, or quarter-tone, could not under any circumstances fulfill the requirement of a principle, or starting point (*archē*), for definition. This, as Aristoxenus carefully explained in *Harm. El*. II. 55 (Da Rios, 68. 10ff.) is because the ear relies to begin with on those intervals which it can most easily discern: concords. See Barker, II, p. 168, n. 110.

(*dianoia*)." Thus, for example, where Eratocles and his followers were attempting to systematize the progressions of tetrachords, Aristoxenus saw only an impromptu affair unguided by any logical principles:[91]

> They neither assert any logical reason for their method, nor do they investigate how the other intervals are combined with one another or whether there is a rational principle of synthesis that delineates every interval from every other one.

And when the Harmonists (*harmonikoi*) made diagrams of quarter-tone sequences in an effort to plot the various scales in use onto some sort of symmetrical framework, Aristoxenus was too clear-sighted not to perceive the musical inadequacy of these *katapyknoseis*, or "close-packed" formations. As he put it:[92]

> We must seek continuity not as the Harmonists do, who attempt to render it in their "close-packed" diagrams, showing that among the notes that are successive with one another, it happens that they are separated from one another by the smallest interval. For it is impossible for the voice to sing twenty-eight quarter-tones in succession.

However much Aristoxenus had to say against these procedures, they may in fact have provided him with the wellspring for his own innovative and revolutionary harmonic system. For the Harmonists permitted Aristoxenus to see that the topology of melody is made up of groups,

[91] *Harm. El.* I. 5 (Da Rios, 9. 18–22). As Barker, II, p. 129, n. 24, observes: "Aristoxenus is firmly committed to the view that these matters are orderly, subject to fixed principles." The most important of these fixed principles is that enunciated by Aristoxenus in *Harm. El.* I. 29 (Da Rios, 37. 8–13). See note 89.

[92] *Harm. El.* I. 28 (Da Rios, 36. 1–7). Commenting on the makers of enharmonic diagrams, Bélis, *Aristoxène*, pp. 95–96 has this to say: "Aristoxenus does not willingly cite the name of his adversaries, and when he does, it is always to denigrate their theories; he seems, however, to consider Eratocles with a little more indulgence, because Eratocles, alone among them all 'tried to enumerate the different forms of a single system in a single genus: the enharmonic octave.'"

not unsingable or haphazardly composed groups, but well-organized structures, which together form a closed world of relations.

In presenting his harmonic theory, Aristoxenus seems to have fallen victim to his own genius, for he reminds the reader at almost every turn how novel, how original, and how penetrating are his insights. All of this provoked even Henry Macran, that most generous scholar, to speak of his "endless repetitions, his pompous reiterations of 'Alone I did it'," [93] And Andrew Barker was similarly moved to observe of Aristoxenus that he apparently saw "himself as the Newton or Darwin of harmonic science." [94] However much Aristoxenus' arrogant posturing may offend scholars and critics, one thing is certain: his doctrine of harmonics is all that he thought of it and everything that he said of it. In particular, his belief that a theory of harmonics must rest ultimately on the doctrine of limits, and his demonstration of this belief, set him so far afield from his predecessors and contemporaries that he marks himself as a man *per se*. Indeed, he had to have had the genius to which he fell prey.

That his theory has not reached us in complete form is evident from his numerous references to its presentation in prior and in subsequent works.[95] But what does lie before us tells sufficiently of the new ground he was breaking. In the end, he would prevent music from degenerating into pure empiricism and, at the same time, he would preserve music from being absorbed into the field of acoustical physics. For Aristoxenus believed that music merits a science of its own, the key to which he offers in the following statement.[96]

[93] Macran, *Aristoxenus*, p. 87.

[94] Cf. note 86.

[95] This is especially true of the second book, the loss of its ending, which must have contained much analysis, being most cruelly felt. Cf. Bélis, *Aristoxène*, p. 43 on the end of Book II: "Here, the break between the plan and the explanation leaps before the eyes; similarly, on considering that the pages which concerned keys, modulation and melodic composition, have disappeared, it is hard to reconcile our texts of plan 2 [analysis]; moreover, it is tempting to resolve the problem by making of this book 2 another version of book 1."

[96] *Harm. El.* I. 19 (Da Rios, 24. 7–11). It is this principle that guarantees that a melody be melodious; for despite the immense variety of forms which melody can assume, there is an immutable law that governs the successions of sounds

> For the present, let it be said in general that while attunement admits of many different possibilities in its synthesis of intervals, there is nevertheless something of such sort which we shall assert to be one and the same in every attunement, something that embodies so important a function that when it is taken away, the attunement disappears also.

This "something" that must underlie every truly melodious attunement is presented by Aristoxenus as a logical principle of similitude. By implementing this principle, he was able to work towards two goals: the elimination of the Pythagorean principle of proportion from the substructure of harmonic science; the combination of the ear's evidence with that of the deductive powers of the mind. His aim being to purge harmonic science of any fallacious or slipshod reasoning, he began by believing in the relativity of infinitesimals, as his revelation concerning the infinite series of *lichanoi* demonstrates. For he realized that these infinite series had no sound basis in mathematical fact. He also believed in the homogeneity of the melodic *topos*, but this led him into vagueness and Eleatic-type conundrums, the result being that the moveable pitches such as *lichanos* and *paranētē* could not be demonstrable logically. For if, as he observed, the number of *lichanoi*, for example, could not be obtained by counting, then they could not be identified each by a separate name.

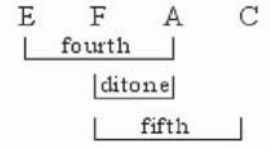
Aristoxenus had somehow to find a way of producing a well-ordered series of consecutive notes between any two of which no other note could be inserted. That is, he was trying to make certain that every note which could not be specified by rational coordinates could be specified in some other way, namely, as the limit of a progression or series of notes whose own particular coordinates are rational. As he intuited, every note in this progression would be an upper limiting point on the one hand, and a lower limiting point on the other.[97] These notions of

in all truly melodious utterances. This is the law of consonances, the enunciation of which is lost from Book II.

[97] An example of this delimiting process is given by Aristoxenus in *Harm. El.* 56–57 (Da Rios, 70. 14–71. 4): "When these progressions have been set up, we must refer to the ear's recollection of the outermost limits of the notes that have been defined; if they seem discordant to the ear, it will be obvious that the fourth is not two and a half tones; but if they sound

limit and continuity are, on Aristoxenus' formulation, not mutually exclusive. For the continuity of which he speaks is not that of a fog whose minuscule particles of moisture have become imperceptible; it is, rather, the kind of continuity that can, by proper mathematical means, be reduced to the continuity of uniform progressions. To accomplish this task, Aristoxenus had to devise a completely new and specialized convention that would facilitate the treatment of continuous magnitudes – magnitudes whose indivisible elements – notes – are intuited to be discrete. The convention adopted for this purpose by Aristoxenus is a highly sophisticated mathematics of inequalities that is based on a concept of continuously varying magnitudes which can approach certain values in the limit. It is rooted in the difference between magnitudes (*megethē*) and multitudes (*plēthē*), and is considered to be the invention of Eudoxus of Cnidus (c. 408–355 B.C.), the greatest mathematician in the era of the Academy.[98]

to the ear as the consonance of a fifth, it will be obvious that the fourth is two and a half tones. For the lowest of the notes under consideration was attuned to form a consonance of a fourth with the upper note that is the note delimiting the lower ditone, the result being that the highest of the notes under consideration forms with the lowest of them the consonance of a fifth." Thus, for example:



As Barker, II, p. 169, n. 114, points out, for all this to be possible, all of the elements – namely, semi-tones – must be equal.

98 In speaking of the remarkably sophisticated mathematics of Eudoxus, Owen, *The Universe of the Mind*, p. 40, observes most pointedly: "Although the historical accounts of his [*sc.* Eudoxus] contributions to mathematics are not as extensive as one might wish, they suffice to suggest that the theory of proportion in Euclid V are his. The mathematics of inequalities, introduced here, is highly sophisticated. Of the importance of this contribution no more need be said than that its definitions of equal ratios are the same as those in the theory of Dedekind, more than two thousand

If the most remarkable thing about Eudoxus' theory is its applicability to incommensurable as well as to commensurable quantities, then the genius of Aristoxenus derives from his application of this remarkable theory to the science of harmonics. He was in fact the first to do so. By adapting this theory to the incommensurable intervals that appear in all melodious attunements, he accomplished something whose importance cannot be overstated: he freed the science of harmonics from the bonds of the Pythagorean theory of proportions, the numerical theory that is applicable only to commensurables. The method that Aristoxenus adapted to harmonic science is one that involves *plēthē*, multiplicities that are countable, as opposed to *megethē*, magnitudes that are measurable.[99] As practiced by Eudoxus, Aristotle, and Archimedes, this method has been called a "careful manipulation of finite magnitudes." In reality, it is a logically rigorous way of setting limits to what would otherwise be an infinite and, hence, unspecifiable series of terms. A theory of proportion that is wholly independent of commensurability, it is treated by Euclid in the Seventh Book of his *Elements* with respect to multitudes. In all probability, it was Aristotle who familiarized Aristoxenus with its particulars.[100]

years later, and that its structure is identical to Weierstrass' definition of equal numbers."

[99] As Aristoxenus defined it in *Harm. El.* I. 15 (Da Rios, 20. 16–17), "a note (*phthongos*) is the incidence of the voice on a single point of pitch (*tasis*)." He also determined, as seen in note 97, that a note is either the locus of a potential division of an interval (magnitude), or an end-point of an interval after the division of an interval into smaller intervals. Thus, the single point of division becomes two points – the respective limits of two parts. Continuing along this line of reasoning, Aristoxenus began to think of intervals as defined multitudes rather than as measurable magnitudes. From that point on, he could treat intervals as collections of units, thus avoiding the problem of incommensurables.

[100] Cf. Sir Th. L. Heath, *Euclid's Elements*, Vol. II, p. 280, on Definition 2: "A number is a multitude composed of units." Thus Heath: "Aristotle has a number of definitions which come to the same thing: 'limited multitude'... 'multitude (or combination) of units' or 'multitude of indivisibles'.... The definition that is most apposite to Aristoxenus' concerns is "multitude measured by one" (*Metaphysics* 1057a3), the measure being unity (*Metaphysics* 1088a5).

Thus armed, and with his ear as a guide, Aristoxenus ignored the incommensurability of such intervals as the fourth, the fifth, and the wholetone; instead, he arithmetized all such magnitudes as these by making each interval between the notes he heard open to equivalent consideration. The result of this operation was that there was no longer a discontinuity between what is numerable (*plēthē*) and what is measurable (*megethē*), but only a symmetrical array of notes. To arrive at this point, Aristoxenus treated the potentially infinite collections of notes as a totality whose limits are set by the natural laws of melody. Attending, therefore, to the common features of all well-attuned melodies, Aristoxenus discovered the constant limits between notes of well-attuned (*emmelēs*) melodies to be those of the consonances: the fourth and the fifth. As he says:[101]

> Let it be assumed that when notes are arranged in a melodic series in any genus, each note will form either a consonance of a fourth with the fourth note distant from it, or a consonance of a fifth with the fifth note distant from it, or both; any note of which this is not the case is unmelodic by forming a dissonance relative to the other notes.

By counting the notes intervening between the limits of the fourth and fifth, Aristoxenus was in effect establishing a one-to-one correspondence between each note and each number in the sets: 1, 2, 3, 4 and 1, 2, 3, 4, 5, respectively. When he could no longer continue this set of correspondences, that is, when the process exceeded the capabilities of the human voice, he stopped the process, which could otherwise have been extended to infinity. In this way, Aristoxenus produced sets of notes whose amount was as large as the amount of numbers in the set of the first four and the first five natural numbers. Such countable sets are said by mathematicians to be denumerable.

Having done this much, Aristoxenus refined his concept of length and continuity even further by establishing correspondences between all the interior, or moveable notes of the tetrachord and the points at which they divide the melodic *topos*. He then computed each of these points as a rational number. In this way, he effaced the distinction between rational and

Aristoxenus thus posited a unit of measure such that "not one single unit of those among myriads differed from any other." Cf. Plato *Philebus* 56E2–3.

[101] *Harm. El.* I. 29 (Da Rios, 37. 8–13).

irrational, such as the infamous $\sqrt{2}$. This in turn enabled him to avoid all such mathematical problems as the knotty semitone or *leimma* computed by Plato and the Pythagoreans to be 256 : 243. In sum, Aristoxenus treated every note on the line of pitch equivalently and, by doing so, he was able to specify the location of any note in any genus or nuance of attunement. He began by assuming that what his ear told him was true: the octave consists of 6 equal whole-tones or their equivalent, that the fourth consists of two equal whole-tones and a semitone or their equivalent, and that the fifth consists of three equal whole-tones and a semitone or their equivalent. This meant that the full complement of semitones in the fourth is 5, that in the fifth it is 6, and that in the octave it is 12. He thereupon adopted as his quantum model the number 12 and in the end succeeded in reducing the geometrical idea of a magnitude (*megethos*) to the arithmetic idea of a collection (*plēthos*) of discrete units.[102]

Without the assistance of mathematics, and with his ear alone to guide him, Aristoxenus had discovered that musical space is incommensurable with the acoustical laws of attunement. He did this by tuning his lyre or cithara using perfect fourths and fifths, thus demonstrating to himself that the six whole-tones that seem so equal to the musical ear do not really fit into the space of an octave. Something, however minimal, always remains left over. The Pythagoreans and Euclid had proved this truth mathematically, and tried to resolve it by forcing musical space to conform to the laws of mathematics. Aristoxenus' master-stroke was to distribute the acoustical flaw in the heart of the octave throughout

[102] What Aristoxenus intuited was that the two classes of notes and their reciprocal intervals can be put into a one-to-one correspondence whereby the positive whole numbers are in a one-to-one correspondence with their reciprocals:

$$\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \ldots\ldots 12 \\ 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \ldots\ldots \frac{1}{12} \end{array}$$

Hence, these two classes have the same number of elements in them. The number that represents the quantity of elements in these two classes is 12. Twelve thus means something: it calls to mind the group of elements for which this number denotes the quantity. Aristoxenus arrived at the number 12 by a rational approximation whereby he had the square root of two define altogether new numbers; he then added these new numbers to the set of rational numbers of harmonic theory. Such a procedure is described by Alistair Macintosh Wilson, *The Infinite in the Finite*, pp. 379–80.

all its parts so evenly that the ear would accept this approximation to the truth for what it is: a "sweet division." Aristoxenus was doing for harmonics nothing less than what mathematicians had been doing for geometry since the days of Anaxagoras, Eudoxus, Theodorus, Theaetetus, and Archimedes, to mention only a few: express the values of transcendental numbers like π and irrational numbers like $\sqrt{2}$ as accurately as possible. In approximating the value of π, for example, Archimedes proved that the ratio of the circumference of a circle to its diameter is less than $3\frac{1}{7}$ but greater than $3\frac{10}{71}$, a minimal difference. And to approximate the value of $\sqrt{2}$, mathematicians had arrived at $\frac{7}{5}$. Consequently, when this approximation, being that of the diagonal of the square, is continued, the diagonal of the square turns out to lie between 1.414213 and 1.414214, a fraction so minimal as to be imperceptible.[103] To arrive at this degree of imperceptibility is what motivated Aristoxenus to compute the whole-tone as embracing twelve equal parts. The number 12 therefore means something: it calls to mind the group of notes for which this number denotes the quantity. Thus, the number 12, on Aristoxenus' standard of measurement, does not denote an interval per se in melody; rather, it denotes the quantity of a whole-tone. In speaking of such a thing as a twelfth of a tone, he explains:[104] "Such elements are not melodic: for we mean that an element is unmelodic which does not *per se* have a position in a scale-system."

The beauty of Aristoxenus' solution to the problems of musical space lies in its simplicity. It is in fact so simple a solution that it has gone unappreciated by almost everyone but the master of acoustical science, Hermann Helmholtz. Helmholtz did more than appreciate Aristoxenus' efforts; he cited him for having laid the foundations for Equal Temperament. He mentions the two critical steps taken by Aristoxenus that led to this discovery: (1) the tuning by fourths and fifths that revealed the excess of twelve fifths over seven octaves, this being the small interval called *comma* by the Pythagoreans; (2) distributing this acoustical flaw over the twelve fifths, making for a division of the octave into twelve equal semitones. With that accomplished,

[103] Cf. Heath (note 100), p. 119: "This gives the means of carrying the approximation to any degree of accuracy that may be desired."

[104] *Harm. El.* I. 25 (Da Rios, 33. 4–5).

Aristoxenus could state that the fourth consists of two whole-tones and a semitone and, as Helmholtz pointedly observed of that statement, "[It] is exactly true only in equal temperament."[105]

By dividing the octave into twelve equal semitones, Aristoxenus distributed the acoustical flaw among the various genera and nuances of melody so that no interval in any pitch range or *topos* would sound too far out of tune. This division offered musicians free modulation from one scale-system to another, from one genus to another, and from one key (*tonos*) to another. In selecting 12 as the number denoting a whole-tone, Aristoxenus was anticipating how acousticians would temper the octave centuries later: that is, finding the twelfth root of 2, or 1.05947631. For, as they calculated, on increasing the frequency of any note whatever by the factor 1.0594631, the pitch of that note will be raised a semi-tone. Had Aristoxenus gone on to multiply 1 by 2 twelve times, he would have arrived at $12\sqrt{2} = 1{:}1.0594631$, which modern acousticians approximate to 84:89. Carrying the number 12 even further, modern acousticians divide the interval between each pair of notes into 100 equal parts, these intervals now having the common ratio $1200\sqrt{2} = 1{:}10005778$. These parts are called *cents*.[106] This Archimedean principle of dividing entities into smaller and smaller parts, thereby achieving a closer and closer approximation to the truth, is what governs Aristoxenus' harmonic mapping of the octave.

Denoting the whole-tone by the number 12, the quantity of the semi-tone will be 6, that of the fourth which limits the tetrachord will be 30, and that of the octave will be 72. With these quanta in place, Artistoxenus was able to compute the exact proportions of the tetrachordal divisions in the three genera and the *chroai* or nuances using whole numbers:[107]

[105] Herman Helmholtz, *On the Sensations of Tone*, p. 548.

[106] Commenting on this procedure, Helmholtz (note 105), p. 431, says: "The object of temperament (literally, 'tuning'), is to render possible the expression of an indefinite number of intervals by means of a limited number of tones without distressing the ear too much by the imperfections of the consonances."

[107] See Macran, *Aristoxenus*, p. 249, for a concise table of the genera and nuances. Cf. Bélis, *Aristoxène*, pp. 157–58. See Fig. 7, in which the notes are placed on a graph.

Genera:				
	Diatonic:	½ tone	1 tone	1 tone
		6	12	12
	Chromatic:	½ tone	½ tone	1and ½ tone
		6	6	18
	Enharmonic:	¼ tone	¼ tone	ditone
		3	3	24
Chroai:				
	Soft Chromatic:	⅓tone	⅓tone	1 and ⅚ tone
		4	4	22
	Hemiolic Chromatic:	¾ tone	¾ tone	1 and ¾ tone
		4 ½	4 ½	21
	Soft Diatonic:	½ tone	¾ tone	1 and ¼ tone
		6	9	15

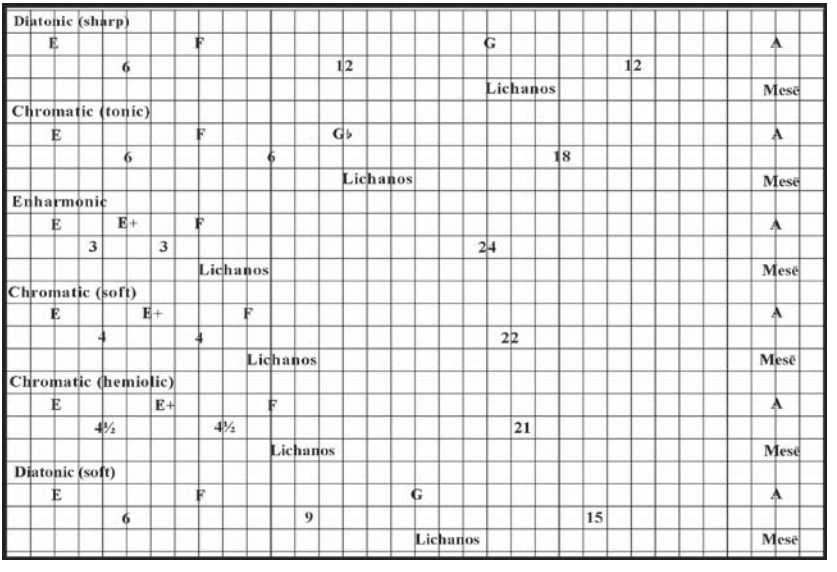


FIGURE 7. Six Meson Tetrachords Distributed Over Thirty Equal Parts

Aristoxenus' terms bear an interesting relation to what Nicomachus later identified as consecutive, or regularly occurring, numbers.[108] Nicomacus' most important observations relate to the properties of such (Aristoxenian) groups of three consecutive numbers taken from the natural series, as, for example, 1, 2, 3 = 6 (Nicomachus' second triangle), which equals Aristoxenus' semitones: 1, 3, 5 = 9 (Nicomachus' second square), which equals Aristoxenus' Soft Diatonic $\frac{3}{4}$ tone, and 3, 4, 5 = 12 (Nicomachus' second pentagon), which equals Aristoxenus' whole-tone. This operation with the number 12 also yields Aristoxenus' Soft Diatonic 1 and $\frac{1}{4}$ tone (= 15, or 4, 5, 6) and his Enharmonic ditone (= 24, or 7, 8, 9). In other words, this amounts to a special kind of decimal terminology initiated by Aristoxenus.

[108] Nicomachus *Intro to Arithmetic*, II. 21 (Hoche, 120. 5. 20–121. 6. 24). See D'Ooge, *Nicomachus of Gerasa. Introduction to Arithmetic*, p. 131. Cf. Heath, *History*, I, pp. 114–15.

6 Aristoxenus of Tarentum and Ptolemaïs of Cyrene

Rara avis in terris
nigroque simillima cycno — *Juvenal Satire* 6. 165

IT HAS BEEN MAINTAINED BY SCHOLARS CRITICAL OF HIS THEORY that Artistoxenus replaced the deductive and speculative method of the Pythagoreans with the empirical and experimental method of practicing musicians. But, as I argued in Chapter 5, this interpretation of Aristoxenus' accomplishments does not stand close scrutiny. For Aristoxenus' writings show clearly that there can be no empirical method for musicians without there first being speculative concepts and intuited forms of order.[1] What also becomes apparent in Aristoxenus' theory is that there is no speculative thinking whose musical concepts do not reveal, on close examination, the empirical material from which they stem. Aristoxenus obtained this material by means of his ear (*aisthēsis*), by experiments with tunings, and by close observation of the conclusions

[1] As argued elsewhere (cf. Levin, "*Synesis* in Aristoxenian Theory," *TAPA* [1972], 211ff.). Aristoxenus' epochal contribution to the theory of music was based upon the notion of *synesis*, a musical intuition, or competence, comprising one's implicit musical knowledge. This a priori notion of musical *synesis* stood for Aristoxenus "not in the thing that is adjudicated, but in the thing that does the adjudicating." (*Harm. El.* 41; Da Rios, 52. 3–4). As Aristotle would have put it, *synesis* was the efficient cause of music. Aristotle's dictum, as expressed by him in *Nicomachean Ethics* 1140A12–14, was not lost then on Aristoxenus: "All art is concerned with creation, and to practice an art is to contemplate how to create something that admits of existence or non-existence, and the efficient cause of which is in the maker, but not in the thing made."

he collected. He sought afterward the means to make these conclusions demonstrable.

Aristoxenus was bent on finding a correct standard for musicians. His goal was not simply to assert a higher ideal against Pythagorean theory; it was, rather, to vindicate the nature of music against the excessive demands of mathematics in Pythagorean harmonic theory. His own theory is not, therefore, as has been claimed, a slip-shod desertion of a traditional harmonic ordinance; it is, instead, a serious and systematic effort to impose an altogether new one. At the core of the matter is the presence of irrationality in mathematics: whereas the Pythagoreans tried endlessly to solve the problem of irrationality, Aristoxenus began by admitting its insolubility as a fact of musical life. What he had grasped with his naked ear (*aisthēsis*) and what he had parsed with his rational mind (*dianoia*) led him to his master-stroke: the inclusion in his system of numbers those irrationals that had so plagued the Pythagoreans. He did this by inventing a new number: twelve. By dividing the whole-tone into twelve equal parts, he split up the musical universe in such a way as to leave no distinction between rational numbers and irrational numbers. For by defining each division as a separate number, he discovered that he could avoid all the knotty mathematical problems involved in the standard method of dividing the whole-tone.[2]

Having made each point between the limits of the whole-tone open to equivalent consideration, Aristoxenus in effect homogenized the whole system of numbers and created what can truly be called a *continuum*: a smooth array whose center can be stipulated by one means only – the demands of melodic consecution. Thus, whereas the Pythagoreans

[2] As Louis Laloy and Annie Bélis have both demonstrated to a certainty, the influence of Aristotle is felt everywhere in the writings of his disciple, Aristoxenus. This is especially the case in Aristoxenus' inclusion of irrationals among the rational numbers, this leading to a line of numbers between which there are no gaps but only a seamless magnitude. The fundamental feature of this method is the postulation of magnitudes that are as great as need be for practical musical purposes and as divisible into whatever sizes one might wish for strictly melodic reasons. Here, Aristotle's doctrine, as expressed in *Nichomachean Ethics* 1094bllff., of seeking only that measure of accuracy which the subject-matter allows as acceptable comes very much into play.

had focused only on the discontinuity of musical space, Aristoxenus trained his thought on the perceived continuity of musical space and in his own innovative way succeeded in reconciling it with the mathematical facts of discontinuity. In so doing, he laid bare the source of music's ineffable energy: the creation of continuity out of discontinuity, or what Edward Rothstein so aptly calls "a sort of inversion of the calculus."[3] The compulsion of pure musical thought is that on which Aristoxenus' theory of music is centered; it not only defined the polemic tone of his writing; it also put him into direct opposition with all the mathematical formulations of the Pythagoreans and with all the empirical ideas of those professors of music, the *Harmonikoi*. He did not have a mathematician's passion for exactitude as an end in itself, but a musician's desire to penetrate to the ultimate melodic ground for every musical event. He regarded his quantum model, therefore, as an objective discovery, one which allowed him to reduce the geometrical idea of a *megethos* (a magnitude or interval) to the arithmetical idea of a *plēthos* (collection) of discrete units or notes.[4] The number twelve denoted for him the quantity of a whole-tone. On his conception then, given the quantum

[3] Rothstein, *Emblems of Mind*, p. 102. As Rothstein observes (p. 70), "The notion of continuity . . . was so hard to formulate mathematically because intuitively it seemed so transparent and obvious to the senses."

[4] As will be discussed in the chapter to follow, the one theory, that of the Pythagoreans, describes a discontinuous geometrically quantified reality; the other, that of Aristoxenus, a smoothly undulating continuum of melodious motion. The first is vividly manifest in the harmonic series, a phenomenon of acoustical nature; the second is recognized by the ear as equal temperament. When the geometry of the line, which encompasses the harmonic series, is integrated with a melodic *topos* that has been equally tempered, a most mysterious element is produced: melodic gravity. The greatest mystery about this element is that it is not itself smooth and continuous, but comes to us in discrete packets of tension and resolution, which Aristoxenus called *epitasis* and *anesis*, respectively. Cf. Chapter 5, note 50, on Casals' identification of these gravitational phenomena in melody. These are the mysterious phenomena that create a triangulation of forces whose combined effect produces magnetic fields of energy and invests melody with a third dimension. They are described rhapsodically by Bowman, *Philosophical Perspectives on Music*, p. 274, "as surfaces with varying degrees of relief, opacity, and translucence, and, indeed, even into three dimensional masses that may be penetrated, carved, and sculpted by silences."

twelve, every possible melodic interval is a finite multiple of the minimum quantum – twelve.[5]

The outcome of Aristoxenus' manipulation of finite numbers is a perfectly symmetrical melodic *topos*, the basis for a perfect harmony of parts. But the balance between this – Aristoxenus' invention of melodic symmetry – and the material world – the mathematical specifics of an imperfect geometry – is a delicate one. Aristoxenus' critics treated his method, therefore, as a flawed creation, a precipitate fall from the grace of mathematical truth. No one, it seems, appreciated the sophistication of Aristoxenus' method or the thoroughly modern concept on which it is framed: that one can make the difference between the original form – the whole-tone, for example – and the spaces filling up that form *as small as one pleases*. To be sure, Ptolemy saw far enough into Aristoxenus' method only to dismiss it as simply a way of "doing something with number and reason."[6] He pointedly objected, therefore, to the Aristoxenian method, because it focused on intervals and not on the notes themselves. As he said:[7]

> They [*sc*. The Aristoxenians] do not, in this way even define the differences, because they do not relate them to the things to which they belong [i.e., the notes]; for there will turn out to be infinitely many of them in each ratio if the things that make them are not defined first . . . (trans. Barker).

In a most useful note, Solomon, *Ptolemy* states the grounds for the criticism against Aristoxenus from the time of Ptolemy down through that

[5] On the cosmic implications of the number 12 in musical thought, see Chapter 5, Fig. 6. On the basis of this quantum, Aristoxenus could compute the *diesis*, or Enharmonic quarter-tone, with the whole number, 3. This meant that intervals such as the *eklusis* (= 3 *dieseis*) and the *ekbolē* (= 5 *dieseis*) could be computed by the simple addition of quarter-tones. Aristoxenus' own computation of these intervals is lost. They are defined, however, by Bacchius *Introduction to the Art of Music* I. 37 (Jan, 300. 17–20) and I. 42 (Jan, 302. 3–6). Bacchius thus explains in I. 37 (Jan, 300. 18–20) why they occur only in the Enharmonic genus. For discussion, see Solomon, "*EKBOLE* and *EKLUSIS* in the musical treatise of Bacchius," 113–14. On the accessibility of the number 12, cf. Georges Arnoux, *Musique Platonicienne*, p. 38.

[6] Ptolemy *Harm*. I. 9 (Düring, 20. 8–9).

[7] Ptolemy *Harm*. I. 9 (Düring, 20. 23–24).

of Mountford, Winnington-Ingram, and Schlesinger to the present day. As he makes clear, no one of Aristoxenus' critics saw anything rigorous in his method:[8]

> Ptolemy finds both inexcusable and incomprehensible the Aristoxenian method of intervals. They regard them spatially (essentially dividing the scale by an imaginary unit of measurement – the 1/12 tone) and not as ratios, i.e. differences, of string lengths.

What Ptolemy found both inexcusable and incomprehensible was Aristoxenus' disregard of all irrational ratios. But, as no one seemed to appreciate, Aristoxenus, by appealing to his own musical intuition, saw the problem of irrationals as a simple matter of calculating limits. By defining a certain limit – as of the *lichanos*, for example – Aristoxenus was in effect specifying a point beyond which all the member notes of the series would be within that small distance of the limit. Once he defined this notion of the limit, it was possible for him to view irrationals in a completely new way. Thus, the irrational whole-tone was treated by him not as something outside of the musically rational universe; it had simply to be calculated differently. This involved arranging whole numbers as points on a line, and, consequently, as treating the irrationals as just so many other points on a line. The whole-tone therefore contained twelve such points and the tetrachord thirty such points. Aristoxenus' method of limits was in all reality, therefore, a systematic and rigorous attempt to come as close as possible to the irrational – to the square root of two – even though he was never to reach it. For the bitter truth about irrationals such as the square root of two and *pi* is that they always lie beyond the reach of boundaries. Thus, Aristoxenus did as Eudoxus before him, and as Archimedes after him: he settled for approximations.[9] His approximations in the divisions of the tetrachord turned out to satisfy the ear, thus "saving the phenomena."

[8] Solomon, *Ptolemy*, p. 29, n. 149.

[9] The method used by Eudoxus and Archimedes is characterized by Dijksterhus, *Archimedes*, p. 130, as an "indirect passage to the limit" instead of the more usual "exhaustion method." Of the more common expression, Dijksterhuis says: "for a mode of reasoning which has arisen from the conception of the

In his *Introduction to Harmonics*, Cleonides treated Aristoxenus' tetrachordal divisions as self-evident knowledge. He did not understand the intellectual process by which Aristoxenus had arrived at these divisions; he simply accepted them for the immediate perception of notes and their relations whose existence was guaranteed by his ear. Cleonides had no difficulty, therefore, in computing the "Soft Diatonic" tetrachord, for example, as consisting of 6 + 9 + 15 units; nor did he find the "Soft Chromatic" any less difficult to represent in the Aristoxenian addition of 4 + 4 + 22 units.[10] Like Cleonides, Aristides Quintilianus was also convinced that the Aristoxenian divisions depicted true melodic forms, his only alteration of the master's computations being his own doubling of the quanta. He thus represented the tetrachord as consisting of sixty units of measure (instead of Aristoxenus' thirty) and thereby filled up the intervals with segments even smaller than those computed by Aristoxenus. Aristides' doubling of the quanta yielded these results:[11]

Diatonic:	12 + 24 + 24
Chromatic:	12 + 12 + 36
Enharmonic:	6 + 6 + 48
Soft Chromatic:	8 + 8 + 44
Hemiolic Chromatic:	9 + 9 + 42
Soft Diatonic:	12 + 18 + 30

Aristides' doubling of the units of measure had the obvious advantage of making the three-quarter tone interval of the Hemiolic Chromatic easier to compute with the whole number, 9, than with Aristoxenus'

inexhaustibility of the infinite, this is about the worst name that could have been devised."

[10] Cleonides, *Introduction to Harmonics*, Chapters 6 and 7 (Jan, 189.9–193.2). These Aristoxenian numbers are thus genetic assemblages such that each is a form (*eidos*) comprising unique eidetic units, or monads. Cf. note 29.

[11] Aristides Quintilianus *De Musica* I. 9 (Winnington-Ingram, 17.21–18.4). These divisions are tabulated by Mathiesen, *Aristides*, p. 85, n. 101. As explained by Barker, II, p. 419, n. 111: "Aristides doubles the figures, as does Ptolemy sometimes [for example, *Harm*. I. 11; Düring, 29. 12ff.] so as to make them all whole numbers."

fraction $4\frac{1}{2}$.[12] But his representation of the facts, like that of Cleonides, betrays no idea of how Aristoxenus came to replace classical mathematical reasoning with the quantum concept by which he succeeded in probing the finer structures of melody.

Aristoxenus had found in his system of correspondences a link between arithmetic and geometric sequences whereby he replaced the multiplication of string-length ratios with the addition of whole numbers; and he replaced the inverse operation, that of the division of string-length ratios, with the subtraction of whole numbers. This enabled him to define certain organized structures of melodic elements whose relations to one another had theretofore escaped reliable specification by classical Pythagorean harmonic analysis. Aristoxenus accomplished this by splitting up the mathematically indivisible whole-tone in the ratio of 9:8 into twelve separate and equal parts. This was in effect to split up the square root of two – the canonic measurement of the semi-tone ($3:2\sqrt{2}$) – into six equal parts. By intermingling these parts on the same line of pitch with the rational numbers derived from string-length proportions, Aristoxenus ingeniously defined each melodically determined point on the line of pitch with a rational number. Thus, for example, in the Enharmonic tetrachord, *parhypatē* was assigned the number 3 ($\frac{3}{12}$ of a whole-tone) and therefore a quarter-tone distant from *hypatē*; *lichanos* was accordingly assigned 6 ($\frac{6}{12}$) of a whole-tone, being a semi-tone distant from *hypatē*:[13]

[12] Cf. Solomon, *Ptolemy*, p. 41, n. 201.

[13] Da Rios, *Aristosseno*, p. 37, n. 2, thus explains: "The tetrachord, or fourth, is formed of two whole-tones and a half and, therefore, of thirty twelfths of a tone." This permitted the loci of the moveable notes, *lichanos* and *parhypatē*, to be ascertained with some degree of accuracy. According to West, *Ancient Greek Music*, p. 167, Aristoxenus' success in this enterprise was not the result of a rigorously thought-out theory: "In effect he [*sc.* Aristoxenus] is operating with a tempered tone of 200 cents and a tempered fourth of 500 cents. He does not understand that that is what he is doing; he is simply working by ear." On the contrary, as is being argued here, Aristoxenus was attempting, first, to reconstruct as far as was possible a precisely demarcated attunement which is given to, or realizable in, perception; and, second, to show that this attunement has a property that is mathematically consistent. Aristoxenus' was a *mental* achievement, therefore, and not simply a hit-or-miss "working by ear." It depended on the notion of tetrachordal continuity (*synecheia*) which, as

E	E+	F	A
	3	6	
hypatē	*parhypatē*	*lichanos*	

This system amounted to much more than just "doing something with number and reason," as Ptolemy had put it. It was, rather, a new way of dealing with an actual set of intellectual intuitions – the very backbone of human knowledge. The principle that lies at the root of this system is the natural law of melodic consecution, a rational synthesis wherein every note on the line of pitch is open to equivalent consideration and where every note on the line of pitch has a corresponding note a fourth or a fifth above it or below it. It is a rational synthesis that depends on treating the infinite collection of tonal elements as a whole, or as a totality; the result of this treatment is the imposition of consonantal bonds on the infinitude of the melodic *topos*. This gave Aristoxenus the clue to harmonic symmetry, which would guide him in all melodic circumstances.[14]

Ptolemy, despite his criticism of Aristoxenus' approach, actually tried to reconcile Aristoxenus' calculations with his own strictly canonic measurements. In fact, he went to great lengths to do so, as he apparently saw in Aristoxenus' system a serious attempt to preserve the musical phenomena, not to contradict them. But, as Ptolemy's efforts show, he failed at the outset to grasp the full implications of Aristoxenus' principle of rational synthesis. What he did is best described in his own words:[15]

Aristoxenus showed, is a function of equality in temperament. Cf. Vogel, *Die Enharmonik der Griechen*, I. pp. 43–44 on *Aristoxenus und die gleichschwebende Temperatur*. Aristoxenus' quantum, 12, can thus be grasped only in thought; on his reckoning, it is itself indivisible in virtue of its purely noetic character.

[14] To apprehend this symmetry, the ear alone was insufficient; the musical mind (*dianoia*) had also to come into play. For, as Aristoxenus understood, it is the mind that defines all the musical elements, not simply according to their positions on the line of pitch, but according to their functions (*dynameis*) in melodic contexts. Cf. Henderson, "Ancient Greek Music," pp. 343–44.

[15] Ptolemy *Harm*. II. 13 (Düring, 69.29–70.4). Barker, II, p. 345, n. 112, explains those respects in which Ptolemy's representations of Aristoxenus' divisions are misleading. As he points out, the numbers that Ptolemy assigned

> In order that the distance covered by the fourth below the disjunction [i.e., the fourth descending from A to E] may span thirty parts, the number proposed by Aristoxenus, and in order that when we take his divisions in the larger context we may still understand the segment consisting of a tetrachord through the same numbers, we have posited that the length from the common limit to the lowest note of the octave set out consists of one hundred and twenty segments, and the note higher than this by a fourth is ninety, in epitritic ratio [4:3], so that the note a fifth higher than the lowest is eighty, on the basis of hemiolic ratio [3:2], and the highest note of the octave is sixty, in duple ratio [2:1]. The intermediate, moveable notes take their numbers in accordance with the ratios of each genus. (trans. Barker)

The larger context within which Ptolemy adapted Aristoxenus' tetrachord of thirty units is the two-octave Greater Perfect System consisting of four tetrachords and hence computed to be an extent of 120 segments. The octave thus consists of sixty segments, the fifth of eighty segments, and the fourth of ninety segments. This means that all the consonantal relations established by string-length proportions are maintained as given on the canon:[16]

Octave: 120:60 = 2:1
Fifth: 120:80 = 3:2
Fourth: 120:90 = 4:3

In sum then, Ptolemy's calculations, which turn out to be like those of Aristides Quintilianus, completely bypass Aristoxenus' concept of melodic intervals as equal distances on the line of pitch. Instead, Ptolemy tried to make the results that Aristoxenus had obtained by ear and by

to the Aristoxenian interval-boundaries "either fail to capture Aristoxenus' intentions, or cannot be mapped directly on to the *kanonion* in the manner required."

[16] Solomon, *Ptolemy*, p. 97, n. 236, observes that Ptolemy's choice of the number 60, denoting a sixtieth part of a unit, may be attributed to the influence of Babylonian astronomy. He thus says: "This serves to corroborate what little evidence we have that Ptolemy came to the study of harmonics after his astronomical education was well established."

reason agree with those yielded by the string-length proportions on the canon.

Throughout his *Harmonics*, Ptolemy adhered to the Pythagorean principle that gave priority to the epimoric, or superparticular ratios such as 3:2 and 4:3. He did so on the basis that such ratios, by being the closest to the equality of the duple ratio (2:1), were bound to yield intervals that would sound melodic to the ear.[17] To be sure, this principle is contradicted by the ratio of the octave and a fourth, an interval which sounds melodic to the ear, but whose ratio is neither duple nor epimoric, and hence appears to break the rule of mathematically determined concordancy. Ptolemy circumvented this discrepancy neatly by arguing that 8:3, the ratio in question, is in reality put together from the epimoric and the duple (4:3 x 2:1) and therefore does not contradict the testimony of the ear.[18] In other words, as Ptolemy's analysis has it, if the numbers be true, the intervals produced will sound melodic to the ear. The proof that this is so comes from the canon; for if the moveable bridge divides the string as dictated by the numerical ratios, the sounds produced will be accepted by the ear as the consonances: fourth, fifth, octave, and the octave plus a fourth or a fifth. It is only when the moveable or intermediate notes of the tetrachord are introduced that mathematics and the ear become disputants. That the operations of mathematics are infinitely superior to anything that we can hear or even think about the moveable notes is the doctrine that lies behind Ptolemy's conclusion to the thirteenth chapter of Book II of the *Harmonics*:[19] "In the case of the moveable notes in between, they assume their numbers in accordance with the ratios of each genus."

[17] Ptolemy *Harm*. I. 7 (Düring, 15, 29–16. 2). As Barker, II, p. 289, n. 66, observes: "There are of course other pairs of ratios that divide 2:1 more nearly in half (e.g., 17: 12 x 24:17), but they are not epimorics." Ptolemy's Pythagorean principle is made explicit by Euclid in his *Sectio Canonis* (see Ch. 4). It holds that the number in question actually becomes the *being* (in this case, a consonance) of the thing to which it belongs.

[18] Ptolemy *Harm*. I. 7 (Düring, 16. 10–12). Solomon, *Ptolemy*, p. 23, captures Ptolemy's logicist's certainty on this point: "Now, that this interval is not superparticular nor multiple does not trouble us at all, since we proposed nothing of the sort beforehand."

[19] Ptolemy *Harm*. II. 13 (Düring, 70. 3–4).

In the next chapter, Ptolemy preserves the computations of all those mathematicians and theorists who agreed that musical intervals could be expressed only by numerical ratios (as of string-length proportions), who agreed also that the octave was less than the sum of six whole-tones, and that the whole-tone itself could not be divided into equal parts.[20] The computations preserved by Ptolemy are those of Archytas, the Pythagorean; of Didymus, a musician who lived at the time of Nero; of Eratosthenes of Cyrene, a man of immense learning who was appointed head of the library at Alexandria about 235 B.C. by Ptolemy III Euergetes; and of Ptolemy himself.[21] The numbers given by Ptolemy correspond to the lengths of string on the canon, the smallest number (60) assigned to the highest pitched note (*nētē Diezeugmenon*), the greatest number (120) assigned to the lowest pitched note (*hypatē Meson*). Each of Ptolemy's three tables or *kanonia* contains the ratios for the Enharmonic genus; the Chromatic genus (also called tonic Chromatic) and its nuances or shades (soft or flat Chromatic and hemiolic Chromatic); the Diatonic (also called tense or syntonic Diatonic) and the soft or flat Diatonic. To these *kanonia* of Archytas, Didymus, Eratosthenes, and those of his own making, Ptolemy adds the computations of Aristoxenus that are listed by Ptolemy in all of the same tetrachordal divisions as those named earlier.[22]

Outstanding among all these calculations are the differences in mathematical language between Aristoxenus' formulations and those of all the other theorists. The ratios of everyone but Aristoxenus are clearly and unambiguously defined in such a way that each term means the same

[20] Thus Ptolemy *Harm*. I, 11 (Düring, 25. 18–26. 2): "If, however, we construct six tones in succession by ratio [or 'reason,' *logos*], the extreme notes will make a magnitude slightly greater than the octave; and it will always be by the same degree of difference, that is, double the difference between the *leimma* and the half-tone, which, in accordance with the first of our postulates, comes very close to being in the ratio 65:64" (trans. Barker).

[21] On Ptolemy's sources here, see Solomon, *Ptolemy*, p. 8, n. 38, where all the pertinent references are brought together.

[22] The tables of divisions are set out by Ptolemy *Harm*. II. 14 (Düring, 70–73) and are translated by Barker, II, pp. 347–50 and by Solomon, *Ptolemy*, pp. 99–103. See also Mathiesen, *Apollo's Lyre*, pp. 468–72. As Mathiesen, *op. cit*., p. 467, observes, "The Aristoxenian parts cannot be accurately represented in this system of string lengths, although the intervals that result are close to those described by Aristoxenus."

thing to each theorist named – to Archytas, to Didymus, to Eratosthenes, and to Ptolemy himself.[23] And this thing, so clearly and unambiguously defined, is the meaning contained in the original Pythagorean *tetraktys*: 6 : 8 : 9 :12. Thus, 6, for example, in the language of the Pythagorean harmonicians, defines one limit of the fourth (6:8 = 3:4), of the fifth (6:9 = 2:3), and of the octave (6:12 = 1:2). These are the incontrovertible facts that are revealed on the canon, the instrument of acoustical precision, facts which Ptolemy and all the other theorists except Aristoxenus accepted as axiomatic. Aristoxenus, however, was not slow to detect the inadequacy of this mathematical language when it came to the imperatives of melody. As he stated most emphatically, these facts that were accepted as axiomatic by the mathematical harmonicians could never succeed in defining the infinite gradations of melodic change. Aristoxenus was thus compelled to find a new way to express the subtleties of melodic thought. This new way was a true blend of empiricism and reason, and it made for an explosive combination. It is reflected in Aristoxenus' use of the number 6; for as used by Aristoxenus, the number 6 comes from another world entirely – that of the finite within the infinite. In this world the number 6 means only one thing: the quantity of the semitone.

Because the observational values of intervals such as thirds of tones, which occur in the hemiolic Chromatic, could be measured by traditional mathematics with only limited accuracy, Aristoxenus thought it sufficient to give *approximately* the quantities to be calculated.[24] His conviction grew

[23] Eratosthenes' computations for the Enharmonic and the tonic (or intense) Chromatic stand out from all the rest for being identical to those of Aristoxenus, even though they are given in ratios. The inference is that Eratosthenes took Aristoxenes' quanta of twelve units seriously enough to rationalize them in terms of string lengths. Thus, Aristoxenus' Enharmonic tetrachord computed in units of twelve is: 3 + 3 + 24 = 30. This was worked out by Eratosthenes to be: 40:30 x 39:38 x 19:15 = 4:3. And Aristoxenus' Chromatic tetrachord computed in units of 12 is: 6 + 6 + 18 = 30. This was worked out by Eratosthenes to be: 20:19 x 19:18 x 6:5 = 4:3. Eratosthenes' rendition of the Diatonic, if he in fact succeeded in computing it, has not come down to us. As Barker, II, p. 346, n. 117, observed of Eratosthenes' computations: "there is no place in any known form of Greek harmonic theory for ratios of that sort."

[24] The terms hemiolic (*hemi* = half and *holos* = whole) refer to the most characteristic interval of the Chromatic genus: the tone-and-a-half (as between a *lichanos Meson*, Gb and a *mesē*, A).

that his results would be more exact the smaller the segments he used; and even that every preassigned accuracy – as for moveable notes like *lichanos* and *parhypatē* – could be achieved by using a sufficient number of segments. Aristoxenus was centuries ahead of his time. For his approach grasps the very essence of real numbers, the infinite decimal fractions of which form the domain of real or continuous numbers.[25] Drawing on the numbers from this domain, Aristoxenus computed *parhypatē* to be distant from *hypatē Meson* by $\frac{4}{12}$ or $\frac{1}{3}$ of a whole-tone; and he computed *lichanos Meson* to be twice that distance from *hypatē Meson* or $\frac{8}{12}$, namely, $\frac{1}{3}$ of a whole-tone. The remainder of the tetrachord is on this computation $\frac{21}{12}$, or, 1 and ¾ tones.[26] Speaking for all traditionally minded harmonicians, Winnington-Ingram observed of Aristoxenus' efforts in this instance:[27]

> There is in fact no musically probable interval that can be held to be represented by the top interval of this tetrachord and yet distinguished from that of the hemiolic chromatic. I suggest that Aristoxenus, favouring the equal division of pycna, and knowing his third-tone ($\frac{28}{27}$) to be a true musical interval, assumed that by doubling it he could obtain a satisfactory lichanos, and so produced a completely factitious nuance.

In an exactingly detailed and methodically argued passage, Aristoxenus determines the closest approximation to what the ear assumes as evident: the lowest chromatic *lichanos*, which is one-third

[25] As Aristoxenus has it, the function of the number 12 is not only the relationship it expresses, but also its single fixed value in the context of these relationships. The number 12 thus forms a well-ordered set and as such can be regarded as what mathematicians call a "real" number. It is Aristotle's thought, particularly his concept of the infinite that is set forth in the third book of his *Physics*, that lies behind Aristoxenus' vision here. For, as Aristoxenus had learned from Aristotle, the only way to deal with an infinite structure, such as obtains within the limits of an octave, is to treat of a finite portion of it. On Aristotle's doctrine of the infinite, see White, *The Continuous and the Discrete*, pp. 133–37.

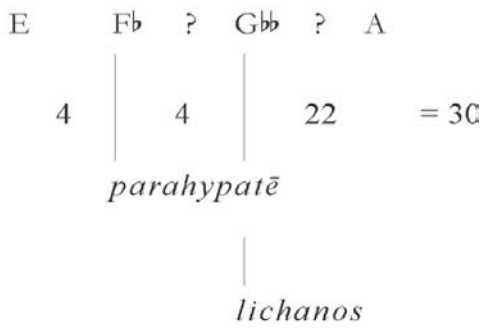
[26] These computations define the loci of the moveable notes, *parhypatē* and *lichanos*, in the *malakos*, or soft Chromatic genus, as detailed by Aristoxenus *Harm. El.* II. 50 (Da Rios, 63. 4–7). Such loci are virtually impossible to ascertain by traditional mathematics. For example, the calculations of Archytas turn out to be: 32:37 x 243:224 x 28:27 = 4:3. Cf. Barker, II, p. 348; Solomon, *Ptolemy*, p. 101.

[27] Winnington-Ingram, "Aristoxenus and the Intervals of Greek Music," 204.

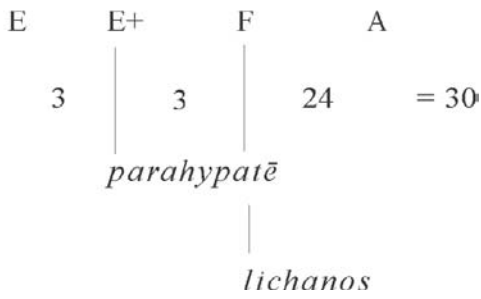
of a whole-tone above *parhypatē*, or $\frac{4}{12}$ of a whole-tone, is itself higher by $\frac{1}{12}$ of a whole-tone than the enharmonic *lichanos*, which is $\frac{1}{4}$ of a whole-tone above *parhypatē*, or $3\frac{3}{12}$ of a whole-tone.[28] This is in effect to isolate the quantum, or unit of measure, as $\frac{1}{2}$ of a whole-tone. In

[28] The lowest chromatic *lichanos* occurs in the Soft Chromatic, whose fine tunings of thirds of tones cannot be rendered in musical notation. Computed in Aristoxenian units of twelve, the intervals in question may be compared in tetrachord *Meson* (E–A):

Soft Chromatic:



Enharmonic:



Cf. Chapter V. Thus, West, *Ancient Greek Music*, p. 168: "According to him [*sc*. Aristoxenus] the two inner notes of the tetrachord can be pitched anywhere within a continuous band, and it is necessary to lay down boundaries to demarcate one genus from another." See Fig. 7, a presentation in graph form.

establishing this element as a unit of measure rather than as an interval in its own right, Aristoxenus says:[29] "Such intervals do not exist melodically; for we mean by the words, 'not exist melodically,' an interval that is not assigned a place in its own right in a scale."

What Aristoxenus intends here is an element so unheard-of theretofore that he had to invent a word for it, a word newly minted by him for this specific purpose: to designate a unit-interval by means of which intervals are counted, a unit-interval used solely for counting intervals. Aristoxenus' word for this unit-interval is ἀμελῴδητον. Thus, Aristoxenus was putting into practice where music is concerned the distinction that Aristotle had drawn between number and what is numerable. For as Aristotle saw it, a number is not *what* is being counted, but is used, rather, to count what is countable. As he said:[30] "As we ascertain the number by using as a unit the thing that is to be counted, e.g., the number of a group of horses by using the single horse." And since Aristoxenus was counting intervals, he used as a unit a single interval: one twelfth of a whole-tone.[31]

[29] *Harm. El.* I. 33 (Da Rios, 33. 3–4). The infiniteness and homogeneity of the melodic domain allowed Aristoxenus to combine units into assemblages of monads, or numbers of units, in whatever way he thought practical and musically logical. The mathematical rationale of such a procedure is lucidly explained by Jacob Kline, *Greek Mathematical Thought and the Origin of Algebra*, pp. 51–55.

[30] Aristotle *Physics* 220b19–20 (trans. Ross). And Richard Sorabji, *Time, Creation and the Continuum*, p. 89, refers to something about time that is applicable to Aristoxenus' mode of setting limits within continuous magnitudes: "Time is continuous, but number (whole number) is discrete. How, then, can time be a number? . . . it [*sc.* time] is infinitely divisible, in the sense that we can divide it at stages as close together as we please, and its infinite divisibility is precisely a mark of its continuity." It is this, namely, the Aristotelian concept of infinite by division, that underlies Aristoxenus' method of defining the sum of an infinite series of pitches as a limit within a continuous magnitude.

[31] As Aristoxenus apparently understood it, the whole-tone, like the circle, is a finite magnitude, and, as such, may be conceived as the sum of its parts: third, quarter, eighth, sixteenth, and so on. In other words, Aristoxenus intuited the contemporary notion of a limit as a mathematical sum that actually identifies a limit with the sum of an infinite series. On this notion as applied to the circle, see White, *The Continuous and the Discrete*, pp. 142–44.

To drive home his point about the value of a permanent and unchanging quantum, Aristoxenus refers to the need for a fixed unit of measure when dealing with rhythm:[32]

> Again, in matters of rhythm we find many similar examples. Without any change in the characteristic proportion constituting any one genus of rhythm, the lengths of the feet vary in obedience to the general rate of movement; and while the magnitudes are constant, the quality of the feet undergoes a change; and the same magnitude serves as a foot, and as a combination of feet. Plainly, too, unless there was a permanent quantum to deal with, there could be no distinction as to the methods of dividing it and arranging its parts. (trans. Macran)

Aristoxenus thus took great pains to demonstrate that his hypothesis of a twelfth part of a whole-tone is not refuted by the fact that we do not actually hear such micro-intervals as a twelfth of a whole-tone. Strictly speaking, such a demonstration is impossible; but it is in the struggle with this problem that Aristoxenus' originality is demonstrated with particular force.

Aristoxenus was not concerned with logic in general, but only with the logic of melody, that is, with logic in the sense of a formulation of the

[32] *Harm. El.* II. 34 (Da Rios, 43. 16–44. 1). The permanent quantum (lit. "the magnitude that remains fixed") is in Aristoxenus' theory of rhythmics the *protos chronos*, and like the harmonic quantum of $\frac{1}{12}$ of a whole-tone, is itself non-composite (*asynthetos*) or indivisible. In a fragment from Aristoxenus' work, *On the Primary Chronos*, preserved by Porphyry *Commentary on Ptolemy's Harmonics* (Düring, 79. 21–28), for which, see Pearson, *Aristoxenus. Elementa Rhythmica*, pp. 34–35, Aristoxenus says: "We must understand that the same reasoning obtains in the case of harmonic science. For this also has become clear to us: that as regards all intervals, their magnitudes happen to be infinite; but of those densely-packed infinite magnitudes one particular magnitude will be selected when singing in this scale in this nuance (*chroa*); in the same way, also, from the infinite magnitudes that come after that one, a particular magnitude will be selected, this one being commensurate with the *pyknon* that was assumed; I am referring to the interval that comes after that, for example, between *mesē* [A] and *lichanos* [F]." The magnitude selected by Aristoxenus – $\frac{1}{12}$ of a whole-tone – thus measures the interval in question – F-A – by the whole number, 24.

principles employed in the activity of melodic construction. His first act was to perceive that the sequences of pitch involved in the melodic *topos* could be continued *ad infinitum*. His next move was to discover by ear that such a continuous image or *continuum* lacked only the logical connectedness that comes with organized structures of melodic consecution, structures that form a closed world of relations. These relations were fixed by the limits of the tetrachord. Aristoxenus thereupon put his hypothesis to the test by breaking up the epitritic interval of a fourth into thirty parts, thereby neutralizing the discontinuity between what is numerable (*plēthē*) and what is measurable (*megethē*). Aristoxenus thus arithmetized what is measurable and in the process mapped onto the melodic *topos* the unit-interval $\frac{1}{12}$ of a whole-tone. In this way, he facilitated the treatment of continuous magnitudes as constituted of indivisible elements that are intuited to be discrete.

But Aristoxenus' arithmetization of the epitritic interval of a fourth was what made it impossible for Ptolemy and the other harmonicians to accommodate his melodic divisions into their system of ratios in all the genera. For Aristoxenus, through his strictly musical hypothesis, had in effect ended up treating the square root of two as if it obeyed all the laws of mathematics that are applicable to rational numbers. This was to violate the very principle upon which Ptolemy had built up his harmonic system: the priority of epimoric or superparticular ratios and their proper treatment – proper, that is, mathematically speaking. Ptolemy states the case in these words:[33]

> To find the positions and orders of the quantities, we adopt as our primary postulate and rational principle the thesis that all the genera have the following feature in common: that in the tetrachords too, the successive notes always make those epimoric ratios in relation to one another which amount to divisions into two or three that are nearly equal. (trans. Barker)

[33] Ptolemy *Harm*. I. 15 (Düring, 33. 5–9). Like Euclid, Ptolemy subscribes here to the view that "equality" between notes corresponds to equality or "near-equality" between the terms of the ratios involved. Thus, as Barker, II, p. 285, n. 49, points out: "Then one (epimoric) ratio is 'nearer to equality' than another where the difference between the terms is a larger simple part of each."

As Ptolemy goes on to explain, the proper division of the epitritic fourth is dictated not by the assumptions of the musicians, but by the accepted norms of mathematics:[34]

> With these principles laid down, then, we first divide the epitritic ratio of the concord of the fourth, as many times as is possible, into two epimoric ratios: such a thing, once again, occurs only three times, when we adopt in addition the three epimoric ratios in succession below it, the ratios 5:4, 6:5, and 7:6. For the ratio 16:15 added to the ratio 5:4 fills out the epitritic, as does the ratio 10:9 added to the ratio 6:5, and the ratio 8:7 with the ratio 7:6; and after these we cannot find the ratio 4:3 put together from just two epimorics. (trans. Barker)

Implicit in Ptolemy's reckonings is his acceptance of the inescapable and intractable fact of harmonic theory: mathematical equality can never be achieved on the division of epimoric ratios. Therefore, if intervals between moveable notes of the tetrachord are to be melodically acceptable to the ear, the divisions of the tetrachord into two or three parts can only be *parisos*, or "nearly equal." They will never be truly equal then, no matter what sort of mathematical divisions are used, so long as the principle of epimoric ratios obtains. With mathematical equality out of reach, Ptolemy nonetheless improved upon the traditional Pythagorean division of the tetrachord – 9:8, 9:8, 256:243 – by allowing that the whole-tones be unequal, there now being major whole-tones (9:8) and minor whole-tones (10:9). The remaining semitone is thus the epimoric 16:15, and the ratio added to that which will yield a perfect fourth is another epimoric 5:4, a ditone which evidently sounded more melodious to the ear than the Pythagorean 81:64. Ptolemy's efforts to divide the

[34] Ptolemy *Harm*. I. 15 (Düring, 33. 28–34. 4). The anatomy of just intonation has been set forth clearly by Stuart Isacoff, *Temperament*, pp. 97–100 and graphically represented on a keyboard spanning one octave (pp. 98–100). As he notes (p. 97): "the idea behind it [*sc.* just intonation] is at least as old as the second century A.D., where it appeared in the writings of the astronomer and philosopher Claudius Ptolemy." Taking note of its shortcomings, Isacoff observes (p. 100): "When all the proportions are calculated, it turns out that the distance between *do* and *re*, for example, is not the same as that between *re* and *mi*." This is all evident in Ptolemy's computations.

epitritic fourth into smaller intervals, while maintaining the epimoric principle is a tour de force; it produced the true or "just" scale whose intonation is still considered by many musicians to be most harmonious and especially receptive to modulations. The two disjunctive tetrachords in Ptolemy's tuning ascend as follows (8:9 = major whole-tone; 9:10 = minor whole-tone; 15:16 = semitone):

15:16	8:9	9:10	8:9	15:16	8:9	9:10
E-F	F-G	G-A	A-B	B-C	C-D	D-E
			|			
			disjunctive whole-tone			

The *kanonia* that are preserved by Ptolemy all evince the same regard for the rules of mathematical rationality as those of Ptolemy himself. To take one example – the diatonic tetrachord – the computations of the harmonicians and of Ptolemy himself appear on comparison as follows:[35]

Archytas:	9:8,	8:7,	28:27
Didymus:	9:8,	9:8,	256:243
Eratosthenes:	9:8,	10:9,	16:15
Ptolemy:	10:9,	9:8,	16:15

When to these sets of ratios the reckonings of Aristoxenus are added, the difference in mathematical idiom becomes striking indeed, for Aristoxenus' divisions of the same diatonic tetrachord are:

$$12 + 12 + 6 = 30$$

As is evident, not only does Aristoxenus express himself here in a different mathematical language, but his thoughts themselves appear to

[35] Ptolemy *Harm*. II. 14 (Düring, 73). Cf. Solomon, *Ptolemy*, p. 102. Eratosthenes' calculations turn out to be identical to the archetypal Pythagorean scale as configured by Plato in *Timaeus* 36A2–B5 and as echoed by various other sources. As Barker, II, p. 349, n. 125, remarks: "No doubt the simplicity of its construction recommended it to Eratosthenes." Yet, the arch-Pythagorean, Nicomachus, singled out Eratosthenes for unspecified criticism and even compared him with Thrasyllus (Tiberius' astrologer), whose twelve part canonic division specifically did contradict the Pythagorean method (Nicomachus, *Manual of Harmonics*, 11; Jan 260. 12–17). On Nicomachus' criticism, see Levin, *Manual*, pp. 165–67.

run on different rail lines from those of Ptolemy and the other harmonicians. This is because the harmonicians such as Ptolemy, Archytas, and the others describe musical intervals in terms of their measurements on the canon, whereas Aristoxenus describes them in terms of their effect on the ear. To put it another way, Ptolemy speaks and thinks in the objective terms of the canon; Aristoxenus thinks and speaks in terms of the assumptions of musicians.

One of the more obvious results of this is that Aristoxenus' computations cannot be successfully cast in the mathematical language of the mathematically objective harmonicians. For the truth is that no equation involving an unknown (x – especially if the x stands for the moveable note, *lichanos*) admits of a solution unless the class of numbers to which x belongs is stated first. The class of numbers with which Aristoxenus was dealing do not derive, as do the ratios of the harmonicians, from measurements on the canon; they derive, rather, from his reduction of the geometrical idea of a magnitude to the arithmetical idea of a collection of discrete points. The number that represents the quantity of discrete points in these collections is for Aristoxenus the number 12, the number which denotes the quantity of a whole-tone. Thus, 12 calls to mind one thing only: the whole-tone.[36] But in the language of the mathematical theorists, 12 denotes one limit of the octave in the ratio 12:6 = 2:1. In order to compare Aristoxenus' divisions of the tetrachord with those of Archytas, Didymus, Eratosthenes, and his own, Ptolemy resorted to the sort of mathematical heroics of

[36] The problem he faced is described by Aristoxenus in these words (*Harm. El.* III. 69; Da Rios, 86.19–87.2): "It is clear from what has been said, and from the situation itself, that if one were to try to ascertain the melodic routes of the intervals, not with reference to one single nuance of one single genus, but with reference to all the nuances of all the genera together, he would fall down into infinity (*apeirian*)." To deal with this situation, Aristoxenus did not choose the number 12 arbitrarily. Rather, he found it to function as the upper boundary for a finite sum of magnitudes and, in this way, it acted as a replacement for what cannot be known: infinity itself. In other words, Aristoxenus eliminated from his analysis the infinitely small magnitudes of the shades and their genera and replaced them with limits. His is thus the sort of logistic that is concerned with a permanent quantum that is grasped only in thought and defies all partition.

which he alone was capable. He did so by arriving at a compromise of his own making that would account for the virtually limitless number of generic divisions that arise in melodic discourse. As he says:[37]

> And since it happens that the numbers comprising the differences common to the genera run into the tens of thousands, we have used the nearest divisions of whole, entire units right up to the first sixtieths of a single unit, so that our comparisons never differ by more than one-sixtieth of one part in the division of the *kanonion*.

Using the number 60 as his unit of measure, Ptolemy fashioned tables (*kanonia*) of numbers indicating those segmental boundaries on the canon that correspond to each set of ratios into which the octave is divided. In this way, Ptolemy made every effort to accommodate Aristoxenus' divisions to his own measurements, but as his *kanonia* demonstrate, the language of string-length proportions can never succeed in representing Aristoxenus' intervals as equal tonal distances. Barker has explained the situation in the clearest terms possible:[38]

> Ptolemy represents Aristoxenus' divisions in terms that require equal differences of number (within any one tetrachord) to represent equal intervals. It follows that the numbers he assigns to the boundaries of the Aristoxenian intervals either fail to capture Aristoxenus' intentions, or cannot be mapped directly on to the *kanonion* in the manner required.

The basic problem is, of course, that the octave, the perfect consonance, like the perfect geometric figure, the circle, does not yield up its mysteries to mathematical analysis.[39] In the case of the circle, such analysis

[37] Ptolemy *Harm*. II. 13 (Düring, 69. 24–29). Solomon, *Ptolemy*, p. 97, understands *synechontas* (which Barker translates as "comprising") to denote "continuous numbers" or "integers," (*synecheis?*). This provocative reading suggests that Ptolemy was treating the tens of thousands of numbers in question here as continuous numbers. If so, Ptolemy can be thought of as approaching Aristoxenus' intuition that such numbers could be well-ordered and, as such, uniformly continuous. Beyond Solomon's reading, however, nothing can be taken as certain.

[38] Barker, II, p. 345, n. 112.

[39] Isacoff, *Temperament*, pp. 16–17, has described the case in these dramatic terms: "Music's invisible building-blocks – the magic numbers defining

led to the transcendent infinity of *pi*; in that of the octave, it led to the infinite irrationality of the square root of two.[40] And just as Archimedes invented a method for bringing *pi* as close to equality as possible, so, too, did Aristoxenus pioneer a method for approximating equality in his system of tunings. Like Archimedes, he did this without trigonometry, without logarithms, and without decimals or any other positional notation. His method for approximating equality in his tuning system is his alone and, as such, is idiosyncratically "Aristoxenian." For this reason, neither Ptolemy, nor anyone else, whether it was Didymus, the musician, or Eratosthenes, the polymath, could ever arrive at an accurate representation of Aristoxenus' octave divisions in the language of string-length proportions. For Aristoxenus' tunings, which embody the Archimedean concept of "almost equal to" or "as closely as possible to," are calculated on wholly different bases from those of Ptolemy and the other harmonicians. They are calculated on a fourth, an epitritic ratio, divided into thirty equal parts. In other words, the constant on which Aristoxenus based his computations is the thirtieth root of the ratio, 4:3.[41]

sonic beauty – were increasingly like great, ethereal forms that had lost their bearings. It was as if the stately pyramids had been transplanted to hilly terrain, their bases toppling over helplessly, their points obtruding at odd, ugly angles."

[40] Irrational numbers, such as Pi, which cannot be roots of an algebraic equation, are called transcendental; but because the infinitely irrational square of two is a solution of the algebraic equation $x^2 - 2 = 0$, it does not qualify as a transcendental number. Cf. Chapter 3, note 2.

[41] No mere rough-and-ready method could have led Aristoxenus to this approximation, a device by which he replaced multiplication and division in geometry by addition and subtraction in arithmetic. His approximation here shows an intimate understanding of the correspondence between arithmetic and geometric progressions. The quantum at which he arrived is a minimal one in the sense that it is itself geometrically indivisible, but inseparable from other such quanta by the intuited melodic *topos*. Cf. Mathiesen, *Apollo's Lyre*, p. 467, n. 205, who cites this calculation from an unpublished document, "Ancient Greek Tunings in Cycles per Second," by Malcolm Litchfield. Aristoxenus was in effect seeking the smallest transfinite ordinal number, the smallest fixed number, as the limit to which the variable number, the $\sqrt{2}$, aspires. To achieve this limit, Aristoxenus saw the necessity of distinguishing between an interval such as the semi-tone as an object of sense and, hence, infinitely divisible, and a unit such as $\frac{1}{12}$ of a whole-tone as an object of thought (*dianoia*) and, hence,

Eratosthenes apparently took Aristoxenus' quantum units of 12 seriously enough to attempt their representation by equal distances on the string of the canon. What is more, he succeeded ingeniously in working out their ratios for the lower tetrachords of the Perfect System, but in the Enharmonic and Chromatic genera only. Thus, where Aristoxenus computes the Enharmonic tetrachord as 3 + 3 + 24 (= $\frac{1}{4}$ tone, $\frac{1}{4}$ tone, ditone, or Major Third), Eratosthenes translates these units into the ratios 40:39 + 39:38 + 19:15 = 4:3.[42] And where Aristoxenus computes the Chromatic tetrachord as 6 + 6 + 18 (= semitone, semitone, tone and a half, or minor third), Eratosthenes translates these units into the ratios 20:19 + 19:18 + 6:5 = 4:3.[43] That is as far as Eratosthenes could carry his representations. Most important, as Barker points out, Eratosthenes' divisions are not only ungainly, but they are also decidedly non-Pythagorean. As such, they are further evidence of Aristoxenus' unprecedented mathematical innovations.

Ptolemy drew freely on the work of his predecessors, but apart from citing the authors of the various *kanonia* that he preserved, he almost never mentioned his sources by name. He refers but once to Pythagoras and only twice to Aristoxenus (apart from his *kanonia*), whom he fails to distinguish by his usual epithet, "The Musician." When Ptolemy does use this epithet, it is to single out Didymus.[44] The work by Didymus

impartible and indivisible. On this critical distinction, see Klein (note 29), pp. 39–41.

42 Barker, II, p. 346, n. 117, observes accordingly: "Certainly he [Eratosthenes] was trying somehow to represent Aristoxenian intervals in the terminology of ratio theory, a fact that helps to explain the ungainly and un-Pythagorean character of his highest enharmonic interval, 19:15. Since the arithmetic differences between terms in Pythagorean ratios were quite different forms of quantity from the 'distances' between Aristoxenian pitches, the attempt is quite incoherent."

43 André Barbera, "Arithmetic and Geometric Divisions of the Tetrachord," *The Journal of Music Theory* 21 (1977), 302, points out, however, that Eratosthenes' lowest chromatic interval (20:19) is equivalent to the bottom two intervals or *pycnon* (40:39 x 39:38) of his Enharmonic and that these computations bring him within reach of Aristoxenus' $\frac{1}{4}$ tone + $\frac{1}{4}$ tone = $\frac{1}{2}$ tone. Cf. Solomon, *Ptolemy*, p. 100, n. 249.

44 Barker translates Ptolemy's epithet for Didymus as "the music-theorist"; but Solomon's "The Musician" seems more fitting, if only because Didymus' writings bespeak the knowledge of practicing musicians (as opposed to that

that won him Ptolemy's obvious esteem was entitled *On the Difference Between the Aristoxenians and the Pythagoreans*.[45] In the two fairly long extracts from this work that are preserved by Porphyry, Didymus makes a strong argument for the preservation of the phenomena by harmonicians, his authority on this critical point being Aristoxenus himself. With the text of Aristoxenus' *Harmonic Elements* clearly in evidence before him, Didymus observes:[46]

> For it will be possible for a geometer, who has treated the arc on his drawing-board as a straight line, to bring his theorem to a successful conclusion without compunction, since he is not concerned with persuading his eyes about the straightness of the line; the subject-matter of his inquiry is reason (*logos*).

Like Aristoxenus, Didymus was a consummate musician, an expert in vocal and instrumental music, as well as an authority on all aspects of Pythagorean harmonics. He was also a man of sufficient critical mind to undertake the refutation of the Pythagorean conclusions where music was concerned. The line he draws between the two doctrines – the Pythagorean and the Aristoxenian – is in fact so sharp as to allow for their having little theoretical thought in common. According to Didymus then, musicians rely solely on their perceptual instincts in their decision making; the Pythagoreans trust solely in the objectivity of reason. As Didymus argues, therefore, it is one

of musical theorists). When, however, Ptolemy refers in *Harm*. I. 12 (Düring, 25. 5) to the most skilled of musicians (*musikotatos*), the inference of the scholiast is that Ptolemy has in mind none other than Aristoxenus himself. Cf. Solomon, *Ptolemy*, p. 35, n. 178.

45 The details of Didymus' modifications of the canon, these focusing mainly on improvements in the placement of its moveable bridge (*hypagōgē*), are discussed by Ptolemy *Harm*. II. 13 (Düring, 67. 20ff.), and most likely came from this lost work by Didymus.

46 This is preserved by Porphyry *Commentary on Ptolemy's Harmonics* (Düring, 28. 12–15). Didymus' reference is clearly to Aristoxenus *Harm. El*. II. 33 (Da Rios, 42, 15ff.) where Aristoxenus compares the geometrician, who can dispense altogether with his faculty of sense-perception, with the harmonician, whose power of sense-perception is the origin of his knowledge.

thing to trust in reason, but it is quite another to allow reason to dictate, uncontrolled by perception, the acoustical preferences of melody. Thus, Didymus:[47]

> Generally speaking then, of those who came to the study of music, some paid attention solely to perception (*aisthēsis*), disregarding reason (*logos*) entirely. I do not mean that they treated the perceptive judgment as something completely divorced from reason and as something not in conformity with certain rational factors in musical practices, but that as far as was possible for them, they had no proofs to offer, nor did they refer anything to reason or show any concern at all for a coherent theory, but were content to rely exclusively on the perceptual method to which they had become accustomed. Such in particular were the instrumentalists and the voice trainers and, quite simply, all those who even today are commonly said to engage in a non-rational activity. On the other hand, however, those who took a path opposite to these musicians, championed reason as their arbiter and no longer paid attention to perception in that manner, but heeded it only as a sufficient point of departure from the objects of perception, so that reason might make its observations from that source. These latter are the Pythagoreans. For, adopting certain sparks of light in each circumstance and constructing theorems that have been put together from them by the office of reason acting on its own, they no longer pay attention to perception. That being the case, it sometimes befell them that when a logical consequence was sustained by reason alone and perception contradicted it, they were not discomfited in the least by such a discrepancy, but put their trust in reason and repudiated perception as aberrant. And they accept the facts that are favored by those who frame their thoughts on experience only when they do not contradict reason.

As Didymus has it then, the practicing musicians, whom he puts in a class of their own, were satisfied to use what Winnington-Ingram had charged against Aristoxenus: a "rough-and-ready" or "hit-and-miss" method of attunement that was not governed by any definite and comprehensible laws. This failure on the part of the instrumentalists and voice-teachers to discern what is rational in music, let alone to

[47] Porphyry, *Commentary on Ptolemy's Harmonics* (Düring, 26. 6–25).

demonstrate it with examples, is in fact what Aristoxenus had found so deplorable in the work of his predecessors. It is also why Didymus accused the musicians of his own day of engaging in a nonrational (*alogos*) enterprise. Didymus thus realized that it is not enough to have an innate sense of melodiousness, that is to say, an aesthetic instinct based on the simple act of hearing. The art of music requires much more. It required what Aristoxenus had made demonstrably clear: an intuitive sense of the melodic and a rational understanding of the musical. Didymus put it this way:[48]

> Music is not only a rational knowledge, but is at the same time perceptual and rational; hence it is necessary for the truly systematic student not to neglect either one of the two, even while having what is evident to perception take the lead, since it is from there that reason must make its beginning.

At the other extreme, Didymus explained, were the Pythagoreans, who dispensed with the ear's knowledge altogether once it fired the engine for the construction of abstract theorems of mathematics. Between these two extremes is the approach of Aristoxenus who, by according an equal status to perception and reason could treat music as inseparable from the special form in which it is presented to the ear. Didymus thus explains that whereas the Pythagoreans seemed always to be striving after independence from mere perception in order to arrive at a system of pure reason, Aristoxenus saw in music so close a union of reason and perception that a single effect made itself known to the complex faculty of perceptual reason, an effect akin to a feeling for music in thought itself.

In using Didymus, one of the few authorities on schools of harmonic theory whom he mentions by name, Ptolemy established a direct link between himself and Didymus' own source: Ptolemy's like-named musical theorist and philosopher, Ptolemaïs of Cyrene. It is Porphyry, however, to whom we owe the name of Didymus' source, as well as the preservation of four fragments from her book, which Didymus had

[48] Porphyry, *Commentary on Ptolemy's Harmonics* (Düring, 28. 9–12). Didymus' argument here is based upon that of his source, Ptolemaïs of Cyrene, whose identification of reason and perception as equal in power is referred explicitly by her to Aristoxenus himself. See Chapter 7.

consulted: *The Pythagorean Doctrine of the Elements of Music*.[49] By including in his *Commentary on the Harmonics of Ptolemy* those citations from the work of Ptolemaïs of Cyrene, Porphyry honored this otherwise unknown scholar with knowledge's most coveted reward: he perpetuated her words. It can only be regretted that Porphyry did not quote from her work at greater length. Nonetheless, Porphyry accorded her still another honor, one hardly less significant, by omitting to mention that the author of the few words he quoted was a woman. His reference to Ptolemaïs is matter-of-fact; he evinces neither surprise nor incredulity at finding the work of a woman in his library of authors. Yet he would have been entitled to register both, if only because female philosophers and musical theorists were as much a rarity in his day as they have been at all other times.[50] Were it not in fact for the grammatical gender required by the Greek language in which he wrote, we would not even notice that the author was a woman. What we do notice is that she was, to judge from Porphyry's citations, a woman of high social standing, for Porphyry's mode of address – Πτολεμαΐς ἡ Κυρηναία – is more befitting

[49] Porphyry introduced these fragments in connection with his commentary on Ptolemy *Harm*. I.2 (Düring. 5.11–6.13), where Ptolemy discusses the virtues of the precision instrument, the harmonic canon. In the chapter to follow, Porphyry's commentary, together with the fragments of Ptolemaïs, are translated and examined for the light they shed on the doctrine of Aristoxenus.

[50] To be sure, learned women were not unknown in antiquity. Indeed, such women as the brilliant Aspasia, and the celebrated poets, Sappho, Corinna, Erinna, and Aristodama, stand out for their exceptional accomplishments. For these women, and others, see Sarah B. Pomeroy, *Goddesses, Whores, Wives, and Slaves*, pp. 89ff. Moreover, a list of illustrious women associated with Pythagorean studies was compiled by Iamblichus *De Vita Pyth.* 267 (Deubner, 146.17–147.6), fragments of whose writings have been collected by Holger Thesleff, *The Pythagorean Texts of the Hellenistic Period*. Among these writings, Thesleff has included the fragments of Ptolemaïs with the observation (p. 229): "Of the other accounts of Pythagorean matters which may, or may not, be dated in the Hellenistic age, only the fragments of Ptolemaïs have been printed here, because they have so far received little attention." Since the writing of these words, Ptolemaïs has been accorded considerable attention by Barker, II, pp. 239–42. To these names of female scholars must be added that of the celebrated mathematician and philosopher, Hypatia, for whom, see Maria Dzielska, *Hypatia of Alexandria*.

a princess of the Egyptian royal house than merely a member of the aristocracy.[51] Equally noticeable is what her writings demonstrate: the mind of a scholar who was learned in Greek philosophy as well as in Aristoxenian theory and Pythagorean harmonics. She is unique in the annals of ancient intellectual history for three reasons: hers was a pursuit that was otherwise unexampled among women of antiquity; her name is a dynastic one that cannot have been given to anyone before the time of Ptolemy I Soter (c. 367/8–283 B.C.); though she wrote on Pythagorean theory, she favored the approach of Aristoxenus.

All that we know of Ptolemaïs has been summed up by Andrew Barker in these words:[52]

> About Ptolemaïs of Cyrene ... we have no information at all outside Porphyry's work, and he says nothing about her, not even remarking on the fact that she is a woman. (The fact is striking: few female scholars, and no other female musicologists, are known to us from classical antiquity.) Even her date is a matter of conjecture, and might conceivably lie anywhere between the third century B.C. and the first century A.D.

If anything, Barker's words invite one to attempt an identification of Ptolemaïs or, if not that, then a reasonable time in which she may be thought to have lived and worked. One might begin by saying of Ptolemaïs that, like the eponymous goddess of her Cyrenian land, Hypseus' fair-armed daughter, she "loved neither the pacings back and forth before the loom, nor the pleasures of dining with her hearth-bound companions." Rather, she would wrestle alone with the abstractions of aesthetic theory even as Cyrene, the huntress-maiden, once

[51] Unlike Ptolemaïs, the other writers cited by Porphyry (and they are numerous) are identified according to their specialty, as, for example, Adrastus the Peripatetic, Aelian the Platonist, or Euclid the teacher of the Elements; other celebrated authorities are identified by Porphyry according to their national origin, these being Dionysius of Halicarnassus and Archytas the Tarentine. Porphyry's conferral of the national appellation – Cyrenian – on the otherwise unknown Ptolemaïs suggests that she, too, is a celebrated personage whom Porphyry expects his readers to know or recognize as such. On the types of address indicating people of high estate or royal standing, see Basil L. Gildersleeve, *Syntax of Classical Greek*, I, p. 29, para. 58.

[52] Barker, II, p. 230.

did with a lion. We might well ask then, as Apollo did of Cheiron: "What mortal begot her? From what manner of race was she wrenched away to live in the hollows of Cyrene's overshadowing mountains?"[53] History has thus far offered no answer; but it does provide a *misē en scène* that could confer on Ptolemaïs of Cyrene an appropriately royal antecedent in the person of the like-named mother of Demetrius the Fair, King of Cyrene and grandson of Ptolemy I Soter: Ptolemaïs, Princess of Egypt.[54]

The name Ptolemaïs is truly operatic: like the more celebrated name, Thaïs, it rings with enough romance and political intrigue to inspire the sensibilities of any composer of operas.[55] It is also a very rare name, the princesses of the royal house having been more commonly called Berenice, Arsinoë, and Cleopatra. And it specifies legitimacy, for the Ptolemies never gave dynastic names like Ptolemaïs to illegitimate children.[56] Ptolemaïs, the daughter of Ptolemy I Soter and Eurydice of Macedonia, was, of course, no match for the Cleopatras, the Arsinoës, and the Berenices, those princesses and queens who rivalled their consorts and siblings in political sagacity or, as the case may be, in political crimes.[57] Hers

[53] See Pindar *Pythian* IX. 17ff.

[54] The Princess Ptolemaïs was born perhaps about 318 B.C. into the house of Ptolemy that included an older brother, Ptolemy, later celebrated for his viciousness as Keraunos (the "Thunderbolt"), and a sister, Lysandra. Their mother was Eurydice, the daughter of Antipater, Regent of Macedonia, her marriage to Ptolemy I in 322 B.C. or thereabouts having served the mutual interests of Macedonia and Egypt against the ambitions of Perdiccas, a senior officer under Alexander. On the offspring of this political marriage, see Edwyn R. Bevan, *The House of Ptolemy*, pp. 52–54.

[55] Massenet's *Meditation* from Act II of his opera is in and of the name, Thaïs, italicizing it, as it were, upon the mind. On the famous *hetaira* of Ptolemy I, see, for example, Pomeroy, *Women in Hellenistic Egypt*, pp. 13f ; pp. 53–55.

[56] See Grace H. Macurdy, *Hellenistic Queens*, p. 102.

[57] The women of the three chief Hellenistic dynasties – Macedonian, Seleucid Syrian, and Ptolemaic Egypt – are notable for their strength of character and political intelligence. As is observed by W. W. Tarn, *Hellenistic Civilization*, p. 98: "If Macedonia produced perhaps the most competent group of men the world had yet seen, the women were in all respects the men's counterparts;

is but a shadowy figure on the historical stage, and when she does appear, ever so briefly, it is more as a victim than as an influence on the circumstances of her life. Yet something of her life can be recovered that allows for her having been touched by developments in Cyrene through her marriage to the incredibly handsome war-lord and adventurer, Demetrius Poliorketes, the "Besieger of Cities."[58] It was, however, his son by Ptolemaïs who was to win an epithet for his incomparable looks. He came to be known as Demetrius the Fair and was, as the ruling monarch of Cyrene (mid-third century B.C.), the central figure in a drama of passion and political intrigue that ended in his assassination through the machinations of the Cyrenian Princess Berenice. This contact between Ptolemaïs of Egypt, the mother of Demetrius the Fair, and the court of Cyrene is enough to evoke the name that is so imposing in the text of Porphyry: Ptolemaïs of Cyrene.

In addition to being even more handsome than his father, Demetrius Poliorketes, Demetrius the Fair seems to have been a man of developed intellect. For he is found as a youth studying in Athens with no less than Arcesilaos, the head of Plato's Academy, the scholar to whom Eratosthenes himself was drawn. Demetrius would probably have continued in the near-total obscurity that surrounds the first thirty-odd years of his life had not his half-brother, Antigonos II Gonatas, selected him as a political pawn against the growing power of Ptolemy II Philadelphus of Egypt. Demetrius was to be king of

they played a large part in affairs, received envoys and obtained concessions for them from their husbands, built temples, founded cities, engaged mercenaries, commanded armies, held fortresses, and acted on occasion as regents or even co-rulers."

[58] Demetrius was the son of Antigonous, the "One-eyed," one of the three continually warring successors of Alexander (the others being Seleuceus and Ptolemy). From 312 B.C. on (after suffering a major setback at the battle of Gaza), Demetrius went on to establish his credentials among his father's allies and chieftains as Poliorcetes. Plutarch pairs Demetrius with Antony, since, as he says (*Dem*. I. 7): "Both were similarly concupiscent, fond of drink, warlike, liberal spenders, extravagant and vainglorious." Like Antony, Demetrius was also handsome beyond the capacity of painters to achieve his likeness (Plutarch *Dem*. II. 2).

Cyrene and the instrument of Gonatas' will.[59] With the throne would come the hand of the Princess of Cyrene, Berenice, daughter of Magas, King of Cyrene, and of his now-widowed queen, the Seleucid Apama.[60] This union made for a three-way alliance of power that would contain the world in a triangle of steel: Berenice representing Egypt as grand-daughter of Ptolemy I; Demetrius representing Macedonia as half-brother of Gonatus; Apama inclining toward Syria as the daughter of the formidable Antiochos I.[61]

The plan might have worked out as intended by Gonatas, had Demetrius not fallen in love with his mother-in-law to be, Apama. This could not but incur the wrath – in truth, a Medea-like vengeance – on the part of Demetrius' intended bride, Berenice. For what apparently began with the consciousness of a common point of view between two people of the same age (Apama being perhaps only a few years older than Demetrius), developed into the sort of commitment that asks neither forgiveness nor seeks condonation. The third party to the affair, Berenice, the bride-to-be of Demetrius, although still quite young, must have loved the worst. Indeed, nothing could have been worse than the punishment which Berenice had premeditated for Demetrius and Apama: he was to die and she was to live on with the image of his death forever before her. It is a story worthy of grand opera: the libretto has been provided by Justin, according to whom Berenice was not as yet wed to Demetrius. And she was so outraged at Demetrius' preference for her

[59] On the history of Demetrius the Fair, see W. W. Tarn, *Antigonos Gonatas*, p. 290, note 36; 293; 334. Cf. H .J. W. Tillyard and A. J. B. Wace, "The History of Demetrius the Fair," *Annual of the British School at Athens*, pp. 113–19.

[60] On these developments in Cyrene and the role of Demetrius the Fair, see Bevan, *The House of Ptolemy*, pp. 73–75.

[61] The lives that interact within this political coalition have produced some of the most dizzyingly complex genealogies in the Hellenistic era. Thus, for example, Apama, the widow of Magas of Cyrene (he being the son of Berenice, the second wife of Ptolemy I) was named after her grandmother, Apama, the wife of Seleucis I. Her mother, the wife of Antiochos I, was Stratonice, the daughter of Demetrius, "the Besieger," and Phila, the sister of the first wife of Ptolemy I, Eurydice. This means that Ptolemaïs, Eurydice's daughter, whom Demetrius married around 286 B.C., was his own niece. See Macurdy, *Hellenistic Queens*, p. 103. Cf. Fig. 8.

mother over herself that she sought help from the pro-Egyptian party who had favored her marriage to Ptolemy III Euergetes in the first place. As Justin tells it:[62]

> She [*sc*. Berenice] devised a treacherous entrapment of Demetrius. After he had retired to the bed of his *socrus* [mother-in-law to be], assassins were despatched there to attack him. But when Apama heard the voice of her own daughter, who was standing at the door to the bed-chamber, give orders to the assassins to spare her mother, Apama threw her own body over that of the man she loved and managed to shield him for a brief while. By having Demetrius killed, Berenice, without suffering her own piety to be sullied, not only exacted vengeance against her mother for her sin, but also followed her father's wishes in her selection of a husband.

The inconsistencies in Justin's account – both internal and external – are many. For one thing, he refers to Berenice as a *virgo* at the time of Demetrius' assassination, this implying that the marriage between the two had not yet taken place. If so, there would be no basis for Justin to call Apama *socrus*, when she was not yet a mother-in-law, but still only a mother-in-law-to-be. In the second place, Demetrius was reported to have ruled over Cyrene for ten years, a position he could not have sustained without a legal and binding marriage. If Apama had been his queen, Demetrius would have been secure on his throne, as he seems in fact to have been.[63] And, as king, he would have become a

[62] When it comes to trustworthiness, Justin (c. 3rd century A.D.), the primary source for these proceedings, is far from the equal of a Plutarch. According to Tarn, *Hellenistic Civilization*, p. 292, Justin is a prime example of those writers who worsened the state of historical writing by boiling down from the greater writers and repeating from one another. Justin seems also to have hated the people (particularly the women) about whom he was writing. The worse the story, the better he liked the telling of it. Cf. Macurdy, *Hellenistic Queens*, p. 2. In the present case, it is Justin's own bias that lends credibility to the story of Berenice, Demetrius the Fair, and Apama.

[63] Demetrius' position as king of Cyrene is established by an inscription from Mantinea. This is discussed by H.J.W. Tillyard, *Athens* 11 (1904–05), 111–12. To this evidence may be added that of the great chronographer, Eusebius *Chron*.

substantial obstacle to the rising ambitions of the princess, Berenice – a king who had to be removed from power. Unfortunately, the accounts of Demetrius' tragic history and its consequences are so confused and muddled in the details that a case can be made for almost any fairly reasonable conjecture. Only one thing stands out as certain from the welter of deranged evidence: Demetrius was betrothed to Berenice, but it was Apama whom he truly loved.

To continue a story based upon appearances, Demetrius may be seen ascending the throne of Cyrene in the vicinity of 259/58 B.C. His marriage to Apama (if there was one) would have been in the same year. And the child born of this marriage would have entered the stage in 257 B.C. at the earliest. But the story, as we know it to have been played out, has Berenice succeeding in her plan by marrying Ptolemy III Euergetes and becoming Berenice II, Queen of Egypt. Ironically, by her deed of murder, she became celebrated as a heroine, the subject of a famous poem by Callimachus, *Berenikes Plokamos*, which has come down in a Latin version by Catullus: *Coma Berenices*.[64] Thus the death of Demetrius left Berenice free to marry her cousin, Ptolemy III, as her father Magas had desired and, after what must have been an uncomfortable delay in the marriage ceremony, to become Ptolemy's queen.[65] Gonatas in turn lost his diplomatic hold on the Cyrenaica which, through Berenice's marriage, passed once again into Egypt's territory. And the once-proud Apama, condemned by the cruel mercy of Berenice to live on bereft and

I. 237, who says that Demetrius not only consolidated his power over all of Libya and Cyrene, but also ruled as monarch for ten years.

[64] Catullus 66. 23–28 expresses the deepest sympathy for Berenice: Quam penitus maestas exedit cura medullas!/ Ut tibi tunc toto pectore sollicitae/ Sensibus eruptis mens excidit! At te ego certe/ Cognoram a parva virgine magnaniman./ Anne bonum onlita es facinus, quo regium adepta es/ Conjugium, quod non fortior ausit alis? "How deeply the sorrow wore away at the grief in the very marrow of your being! As then in your anguish, your mind wrenched apart, your feelings ripped away from all your heart! Yet I have known you to be heroic from the time when you were a little girl. Have you forgotten the noble deed by which you won a king for a husband? A braver deed no other would dare."

[65] The marriage of Berenice to Ptolemy III Euergetes (her first cousin) did not take place until 245; thus the marriage was delayed some thirteen or fourteen years. Cf. Bevan, *The House of Ptolemy*, p. 74.

abased, was never to be heard from again. Yet the silence into which the vagaries of historiography have cast the persons of Demetrius and Apama makes all the more significant the single and uncontested fact of a Ptolemaïs of Cyrene.

Ptolemaïs of Cyrene ought to have been the legitimate daughter of Demetrius the Fair and Apama of Syria. That is to say, in order to have become the scholar she indeed was, she should have been all that her name aspires to and, on that account alone, qualified by birth for the training to which her writings bear ample testimony. Had she in fact been a half-sister to Berenice, or even someone of comparable estate, her admittance to the royal court of Alexandria would have been guaranteed. Once there – her life spared, let us suppose, by the same clemency that Berenice granted to Apama – she would have been brought up in company with the children of the king's household, being numbered, appropriately, among the *paides basilikoi*, those well-connected boys and girls who were privileged to be reared with the princes and princesses of the court.[66] Indeed, if anything was to make her intellectual deliverance complete, it would have been present there. For there the palace of Ptolemy III gave easy access to the most brilliant center of learning outside of Athens – the celebrated Museum and great Library of Alexandria.[67]

The cultivated woman of Hellenistic Alexandria is exemplified in the writings of Ptolemaïs of Cyrene.[68] She is also the only woman known thus far to have dealt with harmonic theory.[69] The all-too-few

[66] On the education of the nobility, see Bevan, *The House of Ptolemy*, p. 123. On the education of upper-class women, see Pomeroy, *Women in Hellenistic Egypt*, pp. 59ff.

[67] This remarkable institution was in reality a complete university, not unlike the schools of philosophy at Athens. It came into being under Ptolemy I Soter (perhaps at the suggestion of Demetrius of Phalerum, who was familiar with Aristotle's own great library at Athens), and it attracted the most brilliant minds of the Hellenistic era. See Bevan, *The House of Ptolemy*, pp. 124–27. For a most recent study of the library and its history, see Lionel Casson, *Libraries in the Ancient World*, pp. 31ff.

[68] On the cultivated women of Egyptian society, see Pomeroy, *Women in Hellenistic Egypt*, pp. 41ff.

[69] Ptolemaïs' name has been mentioned by numerous scholars, but thus far, her writings have occasioned little commentary. To be sure, Lukas Richter, *Zur Wissenschaftslehre von der Musik bei Platon und Aristoteles*, pp. 178ff, and Ingmar

lines of her work that survive, thanks to Porphyry, offer more than faint glimpses of a sober intelligence in easy command of an intricate and a divisive field. But however complex and technical the subject-matter, her style remains simple, direct, and intelligible. Her words bristle with authority, and she speaks with a mind of her own. She seems in fact to have inherited not a little of the indomitable spirit that provoked the ambitious Macedonian princesses of Egypt to such different purposes.[70] She could therefore attack Pythagorean doctrine at its very core on grounds that would have satisfied Aristotle himself. However few then are the words that Porphyry chose to transmit from her writings, they do tell us something of what she knew and how she was disposed to think. This much can be ascertained: she was sufficiently trained in music to understand the assumptions of musicians; her knowledge of canonic theory, for which she had to have had mathematics, was such that Porphyry could consult her as an authority; she was thoroughly grounded in the principles upon which the Pythagoreans based their doctrine of harmonics; her understanding of Aristoxenus' theory of music exceeded that of many specialists whose writings have survived more or less intact; she had a firm grasp of the theoretical priorities that divided the Pythagoreans and the Aristoxenians; she knew how the principle of "saving the phenomena," which underlies all of Greek natural research, applied to harmonic theory. Most important, she seems to

Düring, *Ptolemaios und Porphyrios Über Die Musik*, pp. 143–45, have drawn some important insights from her position with respect to the Pythagoreans. See also Mathiesen, *Apollo's Lyre*, pp. 514–17, who summarizes the content of her work. Apart from these contributions, the most significant work on Ptolemaïs is that of Barker, II, pp. 239–44, who has translated her writings into English for the first time and assessed their meaning.

70 In sum, if Demetrius the Fair, King of Cyrene and grandson of the resplendent Ptolemy I Soter, had had a daughter by Apama, herself the daughter of the Seleucid Antiochos I Soter and Stratonice (daughter of Demetrius I and Phila), such a daughter would conceivably have been called Ptolemaïs of Cyrene. On the conventions of naming children of the royal house, see W.W. Tarn, "Queen Ptolemais and Apama," *CQ* 23 (1929), pp. 138–41. Because the weight of evidence favors 259/58 B.C. for the accession of Demetrius the Fair to the throne of Cyrene, the princess, Ptolemaïs, would have been born at the earliest in 258/57. Pomeroy, *Women in Hellenistic Egypt*, p. 61, thus has Ptolemaïs coming from Cyrene to Alexandria some time around 250 B.C.

have had access to more of the writings of Aristoxenus than is available to us today, for she adds a fragment of his thought that has not as yet found its way into his extant writings.[71]

If the name Ptolemaïs of Cyrene urges thoughts of past intrigues and endeavors too mad for understanding, her words rouse a far different world: one that persists undisturbed by the dreary intercourse of political foes or society's self-infatuated degradations. Thus, while Berenice of Cyrene, instrument of her own mother's ruination, was living only to be poisoned by her own son, Ptolemy IV Philopater (c. 244–205 B.C), Ptolemaïs of Cyrene was delving into things that will never end. And while it was Philopater, the murderous voluptuary and vicious dilettante, who had as his private tutor the towering scholar, Eratosthenes of Cyrene (c. 275–194 B.C), it took a Ptolemaïs of Cyrene (c. 257–c. 211?) to do justice to the teachings of such a master. An acknowledged expert in the principles of harmonics, Eratosthenes could conceivably have taught his countrywoman, Ptolemaïs, to penetrate as deeply as she in fact does into the intricacies of harmonic theory. More than anything else, however, it is what she has to say on the subject that makes such a possibility thinkable. Quite apart from Eratosthenes, who may indeed have been her mentor, it was Ptolemaïs herself who placed the knowledge she had absorbed into the service of reason. In this regard, she made herself exceptional. For not only did she prove Aristotle wrong in his estimation of women's deliberative and rational faculties, she did so on his own grounds: she reasoned critically, dialectically and, what is more, as a true Aristotelian.[72] As such, she pitted herself against Pythagorean and Platonic mathematical conceptions the better to champion Aristoxenus' Peripatetic philosophy of music.

In his *Commentary on the Harmonics of Ptolemy*, Porphyry makes four citations from *The Pythagorean Doctrine of the Elements of Music* by Ptolemaïs

[71] See Chapter 7, note 45.

[72] Interestingly enough, there were no female peripatetic scholars at all. For Aristotle made it quite clear in the *Politics* 126a25ff. that women lacked the necessary rational element for deliberative thinking. Aristotle did concede that women formed half of the human race, but he could not help quoting Sophocles *Ajax* 293 to the effect that "for a woman, silence is an ornament." On Aristotle's position with respect to women, see Pomeroy, "TECHNIKAI KAI MOUSIKAI: The Education of Women in the Fourth Century and in the Hellenistic Period," *American Journal of Ancient History* 2 (1977), 58.

of Cyrene. Taken as a whole, these passages are of sufficient length and continuity to communicate something of the writer's intellectual energy and range of knowledge. Porphyry characterizes the work as an *Isagōgē*, or *Introduction*, and takes note also of the question-and-answer method in one of the passages. He informs us further that the work was a source for Didymus. For Didymus, he says, not only followed Ptolemaïs in several places of his own work; he also elaborated on various of her statements. Inasmuch as Didymus was himself a source for Ptolemy, Ptolemaïs must have antedated them both. Porphyry quite possibly had the work of Ptolemaïs before him in its entirety. For apart from the context for which he extracted her words, this being his commentary on Ptolemy's *Harmonics* I. 2, he had occasion to refer once again to her as an authority on a different subject: the classification of consonances. Here, he singles out Ptolemaïs as a representative example of what he says was an ancient tradition, one that Ptolemy had altered by basing his classification of consonances on kinds of sounds and not, as in Aristoxenus, on vocal movements.[73] The inference is that Ptolemaïs not only antedated Ptolemy, but did so to a considerable degree. On the other hand, however, Ptolemaïs' reference to the canon, which was invented around 300 B.C., and to the theory based upon it as one of long-standing institution with the Pythagoreans, suggests that even if she lived well before Ptolemy, she also lived some time after 300 B.C. That being the case, she might well have studied with Eratosthenes, who was an acknowledged expert in the principles of canonic theory.[74] For what she has to say on the subject suggests that her training in the field was more than adequate to her purpose. Whoever her mentor was and whatever the circumstances that helped her to extend her knowledge, it was Ptolemaïs herself who used this knowledge well to explain Aristoxenian theory.[75] She began by introducing a new term into the field, one by which the field has since been designated: *Kanonikē*.

[73] Porphyry *Commentary on the Harmonics of Ptolemy* (Düring, 114. 5–7) includes Ptolemäis here with "other ancients" who classified consonances as she did.

[74] As West, *Ancient Greek Music*, p. 239, has observed: "It would be interesting to know the relationship between Eratosthenes and Ptolemäis, a female musicologist of uncertain date who came from Cyrene."

[75] Cf. Barker, II, p. 239, n. 134.

7 *Aisthēsis* and *Logos*: A Single Continent

The heart is the capital of the mind,
The Mind is a single State.
The heart and the Mind together make
A Single continent. Emily Dickinson

IN THE FOURTEENTH CENTURY, A VIENNESE SCRIBE, THE COPYIST OF Porphyry's *Commentary on Ptolemy's Harmonics*, known to us only as T, referred to Ptolemaïs' definition of the canon as that of Ptolemaïs the Cyrenian Musician (ὅρος κανόνος παρὰ πτολεμαΐδος τῆς κυρηναίας μουσικῆς).[1] The knowledge and mastery of the material displayed throughout by T in his various interpretive corrections and refinements of Porphyry's text lend an uncommon authority to his characterization of Ptolemaïs as *mousikē*.[2] To be sure, T may have been using *mousikē* here in the general sense of "cultivated," thus to designate a person educated "under the auspices of the Muses."[3] And this reading – "the Cyrenian Savante" or "woman

[1] Düring, *Porphyry Commentary on Ptolemy's Harmonics*, p. 22, *criticus apparatus*, line 25.

[2] In his introduction to Porphyry's text, p. xv, Düring identifies T as the sixty-ninth of the seventy manuscripts that have come down to us, its provenance being *Vindobenensis*, and its production being that of a learned man (p. xix).

[3] A "musical" man was not understood in all cases to be a musician, but, as often as not, an especially cultivated and educated member of society – one trained to be a philosopher and a leader of the people. For, as Plato has it, philosophical knowledge presupposes a musical education. A "musical" man is, according to Plato *Rep*. 401D8-E, one who has attained the grace of body and mind that comes from a properly "musical," or well-attuned, upbringing.

of letters" – would comport with one other rare mention of Ptolemaïs, that by Gilles Ménage, in whose *Historia Mulierum Philosopharum* of 1690 Ptolemaïs of Cyrene is acknowledged as a scholar and philosopher.[4] Observing Ptolemaïs at work, however, and watching the results of her constant venture into the "Battle of the Criteria"[5] – *Aisthēsis* and *Logos*– lead to a certain conclusion: she was *mousikē*, a musician in the narrower sense of the word. When, therefore, T called Ptolemaïs *mousikē*, he must have had the same thing in mind as Porphyry did when he called Didymus *mousikos*, namely, a musician.[6] Indeed, T was evidently too clear-sighted not to perceive that in the debate between the Aristoxenians and the Pythagoreans, Ptolemaïs spoke as a musician.

The debate between the Aristoxenians and the Pythagoreans concerned the criteria – *aisthēsis* and *logos* – and their respective roles in the acquisition of musical knowledge and the formulation of a musical theory. This debate had been set into motion long before by Aristoxenus when he broke with the Pythagoreans over the position that mathematics was to occupy in the science of harmonics. Both he and the Pythagoreans were agreed in starting from the same point: the phenomena perceived by the ear (*aisthēsis*), these representing what was to be interpreted by

[4] Of all the scholars who studied her words, only Ménage expressed doubt as to Ptolemaïs' being an orthodox Pythagorean. For, as he said, however extensive her knowledge of Pythagorean mathematical theory, he had to admit that she did not adhere to Pythagorean doctrine in all respects: *Quare cum Ptolemaïda Cyrenaeam Sectae Pythagoricae adscripsimus, non omnibus Pythagoricam fuisse dicere voluimus* (*Historia Mulierum Philosopharum*, p. 123). This work was translated into English by Beatrice H. Zedler, *The History of Women Philosophers*. Ménage's reference to Ptolemaïs appears on page 62 of this translation. Zedler places Ptolemaïs in the second–third century A.D. and suggests that her high level of learning would have made her eligible to participate in the erudite circle surrounding Empress Julia Domna.

[5] The metaphor, "Der Kritierienstreit," was introduced by Lukas Richter, *Zur Wissenschaftslehre von der Musik bei Platon und Aristoteles*, p. 184.

[6] This is not to suggest that a woman of Ptolemaïs' high estate would ever have performed in public. Hers was doubtless a private, but no less accomplished, artistry. Her type of musical activity is portrayed on a grave *stēlē* from Alexandria, c. 250 B.C., showing a woman being handed her lyre by her maid. See Pomeroy, *Women in Hellenistic Egypt*, Plate 13, p. 167. See also Pomeroy, "Technikai kai Mousikai; The Education of Women in the Ancient World," AJAH 2 (1977), 51–68.

reason (*logos*), and how mathematics was to assist in this interpretive process. Aristoxenus took issue with the mathematical theorists, but at the same time, he never countenanced any approach to music that was less sound in its hypotheses or less accurate in its computations than that of mathematical science. As he argued, the "miraculous order" belonging to the nature of music merited a science of its own – one that could classify the genera and species of melody, define their respective functions, and formulate the laws determining their manifold connections.[7] To prosecute this goal, Aristoxenus introduced a new factor into the proceedings, one which he maintained could only be cognized deep in the soul. The organ of this cognition was understood by him to be *synesis*, or musical intuition.[8]

Musical intuition in Aristoxenus' theory implies a new conception of knowledge as something different from sense-perception. To him, it was a knowledge that penetrated beyond the manifold properties of melody into the unity of true musical expression. He thus believed that it was in the unity he intuited, and its unchanging attributes, that music's true nature could be grasped. The Pythagoreans were convinced, however, that music's true nature lay in the discontinuity of musical space,

[7] That Aristoxenus succeeded in differentiating between the genera and species of melody may be referred to Aristotle's teachings. For just as there seems to be no satisfactory evidence that genus and species were used in a technical sense before Aristotle, so, too, there is no evidence to suggest that they were applied in a technical sense before Aristoxenus. To be sure, *eidē* (species) and *genos* (genus) were sometimes used loosely to denote "style," as by Ps. Plutarch *De. Mus.* 114OE5 (the "style" or *eidos* of decadence) and 1142C4–5, where Telesias is said to have tried, but failed, to compose in the Pindaric or Philoxenian "style (*genos*). In Aristoxenian theory, however, these classifications were abstracted from sense-perception and were applied, as did Aristotle in his biology, absolutely. On the evolution of genus and species in the technical sense, see D. M. Balme, "*GENOS* and *EIDOS* in Aristotle's Biology," *CQ*, n.s. XII (1962), 81–92.

[8] Levin, "*Synesis* in Aristoxenian Theory," TAPA 103 (1972), 211ff. thus construes synesis as an inherent mental capacity comprising one's implicit musical understanding. Commenting on this line of argumentation, Solomon, *Ptolemy*, p. 55, n. 269 generously observes: "[Levin] restores Aristoxenus to a position of some respect as a musical theorist after the lambasting he has taken from the Pythagoreans, Ptolemaians, and their modern annotators and advocates." Cf. also A. Neubecker, *Altgriechische Musik*, p. 24, n. 79.

that discontinuity wherein the gaps between rational numbers could be accurately defined on the canon, the instrument of measure. Thus, whereas the Pythagoreans kept their focus on the discontinuity between the rational numbers defined on the canon, Aristoxenus shifted his focus to the perceived continuity of melody as it is experienced by the ear and is expressed by the combination of rational and irrational numbers in infinite quantities between any two integers. In other words, Aristoxenus found evidence of melodic continuity in a true mathematical *continuum*.[9]

Aristoxenus appears consistently to have underemphasized the quantitative aspects of music's nature. Yet, in the final analysis, it was in the properties of a mathematical *continuum* that he found the true synthesis of melodic construction.[10] As he explained, this synthesis was framed on sets of consonantal correspondences. His line of thought can be followed in this statement:[11]

> Melody that is in attunement must not only be composed of intervals and notes, but must also be constituted of a certain kind of synthesis, indeed, a synthesis of no haphazard sort; for it is obvious that being composed of intervals and notes is a common property [of melody] since it belongs also to melody that is out of

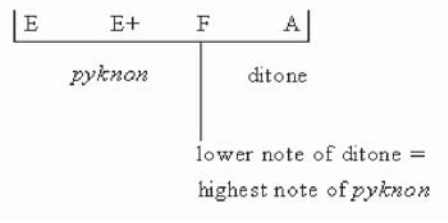
[9] This was a great achievement for which Aristoxenus has never received adequate credit. He arrived at it by positing a unit for measuring musical intervals, one that was not an absolute unit in the Platonic sense, but something of the same kind that was being counted – a melodic distance. Aristoxenus' selection of this unit of measure was like that of Aristotle for measuring time: by numbering "nows." For a "now" is a part of time, or as Aristotle says in *Physics* 220b22–24: "We measure motion by time and we measure time by motion." The number Aristotle selected for measuring time was of the same kind as what was being measured: the "nows" that were being counted. Cf. Sorabji, *Time, Creation and the Continuum*, pp. 88–89.

[10] The property of the mathematical *continuum* that answered Aristoxenus' purpose inheres in its formation of a system of elements wherein one can pass from any one of them to any other by a series of consecutive elements such that each cannot be distinguished from its predecessor. This linear series is to the musician what the isolated point is to the mathematician. But for such a series to be uniform, there is required the addition of irrational numbers to the rational numbers that define the consonances. This is what allows every note in the series to be treated equivalently and thus to assume the function assigned to it by the laws of melodic consecution.

[11] *Harm*. El. I. 18–19 (Da Rios, 23. 16–24.11).

> attunement. This being so, we must take it that the most important element, and, in fact, the most critical factor by far on which the right constitution of melody turns, is that which has to do with its overall synthesis and the particular form this synthesis takes. Indeed, it is all but obvious that musical melody differs from the melody occurring in speech by executing intervals in the motion of the voice; while musical melody differs also from unattuned and faulty melody by the difference in its synthesis of indivisible intervals. What this difference is will be demonstrated in our analysis to follow. For the present, let it be said in general that while attunement admits of many different possibilities in its synthesis of intervals, there is nevertheless something of such sort which we shall assert to be one and the same in every attunement, something that embodies so important a function that when it is taken away, the attunement disappears also.

The orderly arrangement of intervals – their proper synthesis – is, according to Aristoxenus, characteristic of all good melodies. This synthesis, he explains, is a function of tetrachordal continuity. At the same time, it is contingent on the sort of underlying *continuum* framed by Aristoxenus. For there can be no synthesis of any sort, whether it be well- or ill-attuned, without there being a well-defined melodic *continuum* in place to accommodate it. To put it another way, there can be many types of continuity in melody, but there is only one melodic *continuum*. For example, in *Harm. El.* III. 63, Aristoxenus describes a type of continuity from which there results a unity in virtue of the contact (*synaphē*) between certain notes.[12] The case he describes here is that in which the lower of the notes containing the ditone is itself the highest note of a *pyknon*, and the higher of the notes containing the ditone is the lowest note of the *pyknon*. The unity resulting from this contact can be mapped on the *Meson* tetrachord (E–A) as follows:



[12] *Harm.* El. III. 63 (Da Rios, 79. 14–80.2).

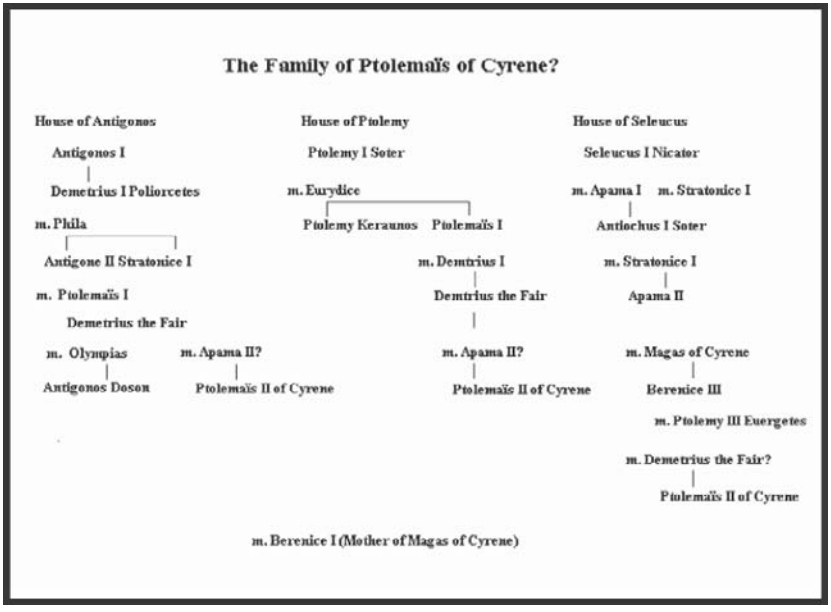


FIGURE 8. The Family of Ptolemaïs of Cyrene (?)

As Aristoxenus intuited, a synthesis of this sort would be impossible, if the distance between E and A (a fourth) were not conceptually divisible into thirty equal parts.

Aristoxenus' conceptual division of the tetrachord into thirty equal parts was as close as he could come in approximating the ear's own knowledge: that there is perceptible in this synthesis (as given above) a divided (*synthetos*) semitone between E and F, and an undivided (*asynthetos*) ditone between F and A.[13] And F is recognized to be the point of union. To Ptolemy and the mathematical theorists, Aristoxenus appeared to base his synthesis of intervals on the assumption of the whole-tone as the difference between the fourth and the fifth and on the acceptance of the octave as compounded of six equal whole-tones. And this, they argued, ran counter to the axioms of mathematics. Their proof of this lay in the size of the semitone,

[13] *Synthetos* and *asynthetos* are translated also as "compounded" and "uncompounded" intervals, respectively. The ditone, F–A, cannot contain any intervening notes in the Enharmonic genus, but in the Diatonic genus it is divided by the note G and, as such, it is thought of as "compounded" of two whole-tone intervals.

the interval remaining on the subtraction of two whole-tones from the fourth. For this subtraction left an interval computed to be 256:243, an interval that is slightly smaller than a true semitone if, on their construction, such an interval could be said to exist. Moreover, they showed that adding one such semitone to another, or 256:243 x 256:243, results in an interval that is somewhat smaller than a whole-tone computed to be 9:8. In sum then, the assumptions of Aristoxenus contradicted everything that mathematicians accepted as self-evident knowledge.[14]

Ptolemy and the mathematical theorists were right in charging Aristoxenus with violating the fundamental laws of mathematical harmonic theory. But their grounds for indicting him on these charges were misjudged. For when Aristoxenus computed the whole-tone and the octave in the manner charged against him, it was not as the basis for his theory but, rather, the result of his altogether original procedure. As argued earlier, Aristoxenus' is a generalizing procedure by which one can calculate the distance between points on the *continuum* only when one knows the quantities that fix their positions. Knowing Aristoxenus' formula – the whole-tone of twelve equal units – one can discover all the intrinsic properties of the *continuum*, that is to say, all those properties which do not depend upon its relation to points outside of the *continuum*. Thus, one can discover that two semitones (6 + 6) do in fact equal a whole-tone (12). The differences between the Pythagorean theory of music and the actual practices of musicians, which Aristoxenus' theory represents, are, in short, as great as the differences between the geometry on a plane and the geometry on a sphere.[15]

[14] This is part of the classical argument against the Aristoxenian method in which the focus is kept on the distances, or empty spaces, between notes, rather than on the intrinsic qualities of the notes themselves. Cf. Barker, II, p. 295, n. 86. To argue this point, Ptolemy *Harm*. I. 10 (Düring, 21. 21ff.) actually parodies Aristoxenus *Harm. El*. II. 56 (Da Rios, 69. 12ff.), in which Aristoxenus describes his method of tuning by consonances and referring his conclusions to the ear alone. Ptolemy's parody is so apposite that Barker suggests that he may have had Aristoxenus' text before him. See Barker, II, p. 295, n. 88. For a mathematical analysis of the fourth, see Solomon, *Ptolemy*, pp. 34–35.

[15] Thus, where the rational hypothesis of metrical geometry, as implemented by the Pythagoreans and the canonicians, eventually replaced the ear with the eye, the algebraic properties of the geometric circle may be said to have replaced

If Ptolemy had much to say against the line of investigation that Aristoxenus projected in his theory of harmonics, it was in part because he could not envisage a system based on any activities other than those assigned to perception and reason. To arrive at a positive understanding of *what is* in the world of the ear was possible in his view provided only that the proper methods be used. And these derived from sensation and rational thought – the only sources of knowledge. That the nature of empirically given things could be accounted for by acts of intuition – which are often attended by feelings of absolute certainty – could not be countenanced by Ptolemy. Thus, whereas Aristoxenus was aiming at the liberation of music from traditional mathematics through the mediation of musical intuition, Ptolemy was committed to a different purpose entirely: the reconciliation of music with traditional mathematics through the application of logic. How far Ptolemy was ready to proceed along this route is the question to which the opening chapters of his monumental treatise, *Harmonics*, bear testimony.

Ptolemy begins his *Harmonics* by defining the province of the field and identifying the criteria it employs in its investigations. The proper focus of harmonics being sound and its attributes of pitch, its goals, according to Ptolemy, are not only to account for the noticeable differences between the pitches of the various musical scales but also to provide reliable means for measuring these differences. These goals, Ptolemy explained, required a close collaboration between the ear and rational thought – an alliance, as it were, in which the abilities peculiar to the one would assist the other to an accurate determination of attunement, or *harmonia*. Andrew Barker has stated Ptolemy's position with respect to the criteria in these unambiguous terms:[16]

the eye with the mind. Cf. Kline, *Mathematics in Western Culture*, p. 177. In Aristoxenian theory, the mind is added to the ear so as to back up the proposition that a series of notes is a collection of intervals having no minimum distance. But the mind can posit an arbitrary distance of the kind represented by Aristoxenus' quantum to make the series numerically measurable. This requires that the *quanta* used have a common (*koinon*) character such that each number belongs only to the things being counted: musical intervals. Cf. Klein, *Greek Mathematical Thought*, p. 81.

[16] Barker, *Ptolemy*, p. 15.

> It is clearly Ptolemy's view . . . that reason and perception are not competitors for the scientist's allegiance, as some harmonic theorists – and others – had supposed. Properly understood, they are allies, and the scientist cannot afford to ignore either.

Thus Ptolemy:[17]

> The science of harmonics is a function of making perceptible the differences between sounds with respect to highness and lowness, sound being an affection of air that has undergone percussion – the primary and most general cause of things that are heard. The criteria of *harmonia* – hearing and reason – do not operate in the same way, insofar as hearing occupies itself with matter and its affection,[18] while reason is concerned with form and cause. In general, this is because the peculiar province of the senses is to discover what is approximate and to be apprised of its accuracy; while the peculiar province of reason is to be apprised of what is approximate and to discover of its accuracy.

Ptolemy goes on to argue that because the domain in which perception operates is unprocessed matter, whose properties are unstable, fluctuating, and multifarious, perception cannot but be influenced by the same unreliability that afflicts the data it records. But because it is the capacity of reason not only to impose form on these data obtained by perception in its rough-and-ready fashion but also to discover their cause, reason operates in conjunction with that which is uniform and, like itself, inherently stable. On this basis, reason, being dictated to by all that is orderly and cohesive, must support perception by providing it

[17] Ptolemy *Harm*. I. 1 (Düring, 3. 1–8). On the difficulty of translating Ptolemy's *dynamis* (function), see Solomon, *Ptolemy*, p. 2, n. 3.

[18] Ptolemy's word here is *pathos*, which is translated by Barker, II, p. 276, as "modification," by Solomon, *Ptolemy*, p. 3 as "condition." The problem with *pathos*, as explained by Barker, *Scientific Method in Ptolemy's Harmonics*, p. 16, n. 16, is that it is often used to refer to a pain or to a disease. Here, however, Ptolemy seems to mean the "impression" made on the ear. As Barker adds: "More broadly, a person's *pathē* may be his experiences, not necessarily ones of a distressing sort. Hence, in connection with perception a *pathos* may be the content of a sensory experience, the impression made on a person's consciousness by an external object, through the channels of the senses."

with a corrective for its innate deficiencies. Perception needs, therefore, a crutch of some sort (*baktēria*) that will function as a monitor. And it is only reason itself that can supply this necessary assistance.[19] Ptolemy proceeds thereupon to demonstrate that what may appear to the senses to be identical will be proved by reason to be in fact different. This, he explains, especially holds true in the case of minimal relations, the capacity of perception to make judgments being far more reliable when confronted by maximum proportions. In saying this much, Ptolemy appears to be referring to Aristoxenus' own method of arriving by ear at such minimal relations as ditones and semitones, namely, by using the larger and more easily perceptible intervals – fourths and fifths – as his standards of judgment.[20] But the consonances are used by Aristoxenus not solely as aids for the ear to make correct judgments; rather, they are enlisted by the ear to confirm what it already knows to be true in advance of the tuning process. Because he is speaking to musicians, Aristoxenus makes this crucial point in such casual terms that its significance is easily missed. Thus, Aristoxenus:[21]

> It is obvious to those who are not inexperienced in instruments that by increasing the tension on the string, we raise its pitch; and in decreasing the tension, we lower its pitch. But during the time in which we are raising the pitch and changing the tension on the string, it is not possible that the height of the pitch which is going to result through the increase in tension is as yet in existence.

In other words, if the pitch to be arrived at by these means exists anywhere, it is potentially in the mind's ear. For, as Aristoxenus implies, the mind's ear knows in advance what pitch it is seeking. It is this knowledge that guides the instrumentalist's tuning processes. Thus, even while the ear is doing the perceiving, it is the mind that is doing the judging. And all this complex activity is taking place simultaneously, without outside assistance or intervention.

[19] See Barker (note 16). According to Solomon, *Ptolemy*, p. 4, n. 13, however: "One of the premises of the *Harmonics* is that reason generally surpasses the senses and that, insofar as music and harmonics are concerned, reason surpasses the hearing."

[20] Cf. Chapter 3, note 58.

[21] *Harm. El.* I. 11 (Da Rios, 16. 3–7).

To Ptolemy, however, neither the eye nor the ear can be trusted to make correct estimations, especially in the case of minimal differences. He argues, for example, that the eye will assess a circle to be a perfect one until another is drawn by a compass. Comparison between the two will then reveal to the eye the imperfections in the circle which it had judged to be perfect. So, too, when certain differences between sounds are accepted by the ear to be correctly distributed, measurements in the form of appropriate mathematical ratios will often prove the ear to be wrong. As he says:[22]

> The ear, being provided with a basis for comparison, will then recognize which is the more accurate, distinguishing the genuine, as it were, from the spurious. And since it is generally the case that the judging of something is easier than the creating of that same thing, it being easier, for example, to judge a wrestling match than to engage in one, easier to judge a dance than to dance oneself, easier to judge a performance on the aulos than to play it oneself, easier to judge a song than to sing it oneself; then the deficiency of perception is such that when it comes to recognizing merely the difference or the lack of it between things, it would not depart appreciably from the truth.

Whereas Ptolemy had argued earlier for the close alliance of perception and reason, here, for reasons not immediately apparent, he so effectively severs them, one from the other, that they end up becoming combatants in the never-ending war between critics and performers. For in assigning the role of judging (*krinai*) to perception and that of performing to reason, Ptolemy not only undermined his own argument, but revealed the philosophical chasm that existed between himself and Aristoxenus.[23] To Aristoxenus, the performer is nothing if not his own best judge. In the case of an aulete or citharode, certainly,

[22] Ptolemy *Harm*. I. 1 (Düring, 4. 6–12).

[23] Barker, II, p. 277, note 9, defends Ptolemy's thesis by saying "our senses are better equipped to judge such things than to construct them. Hence, we can detect our mistakes through the same channels that we relied on when we made them." Cf. Solomon, *Ptolemy*, p. 5, note 19, who observes of this passage: "The entire passage has harmonic implications foreshadowing Ptolemy's refutation of the Aristoxenian assertion that six whole tones equal a diapason."

the collaboration between sensation and reason is complete. It is so complete, as Aristoxenus makes explicit,[24] that reason seems not only to surmise, but actually to experience its union with something, which Ptolemy calls matter (*hylē*), that is foreign to its own nature, which Ptolemy calls form (*eidos*). If anything, Ptolemy seems here to be putting the act of attunement – a preliminary to performance on an instrument – on a par with the far more difficult act of performance itself.

Leaving aside the problems raised concerning performers and critics, Ptolemy goes on to point out how noticeably inaccurate perception can be when it comes to comparing minimal distances. Taking the division of a straight line as an example, Ptolemy proceeds to explain that the eye needs greater assistance from the tools of measurement as the relative dimensions it attempts to compute grow progressively smaller, dissimilar, or more numerous. He then concludes the opening chapter of the treatise with the following statement:[25]

> Just as the eyes require for that purpose [the judging of minimal, dissimilar, and numerous distances] a rational criterion in the form of appropriate instruments, as for example, a carpenter's line (*stathmē*), let us say, for estimating the straightness itself, and a compass (*karkinos*) for measuring off a circle and its parts, the same obtains for sounds and hearing. In the same way, the ears, which together with the eyes are the chief servants of the contemplative and rational part of the soul, need some sort of approach derived from reason for dealing with those things of which the senses are by nature not given to judge accurately – an approach against which the senses will not bear contradictory testimony, but with which they will be in agreement.

In the second chapter of Book I, Ptolemy discusses the instrument derived from reason to assist the ear – the harmonic canon. Its name, he explains, testifies to the collaboration between perception

[24] See note 43.

[25] Ptolemy *Harm*. I. 1 (Düring, 5. 2–10). Ptolemy has set the stage here for the *Harmonics* in its entirety by treating vision as the primary source of reliable knowledge. For it is always vision, as he argues the case, that aids hearing, and not the other way round. In this, Ptolemy is of a mind with Aristotle *Met*. 980a 1–7, who prizes vision above the other senses as the bringer of knowledge. Solomon, *Ptolemy*, p. 6, n. 28 thus observes: "Ptolemy uses the sense of sight as a visible analogy to the sense of hearing."

and reason described in the first chapter, inasmuch as its harmonic component preserves the integrity of the observed data which its canon – literally, ruler – attempts to reconcile rationally with the laws of mathematics. To pursue his purpose, then, the harmonician must turn away from the deficiency and inaccuracy of the senses, away from all verbal solutions, away from all poor a priori reasons and pretended absolutes, and direct himself toward the concreteness and sufficiency of the facts supplied by the precision instrument, the harmonic-canon. His purpose in so doing is to arrive at the truth. With this in mind, his approach to the phenomena must be no less scientific than that of the astronomer. That is, he must discover what the observed facts mean without contradicting what they say. What the harmonician should care about above all else are the inferences as to the sensible phenomena which his hypotheses enable him to make. His hypotheses should therefore be workable insofar as their results will conform with the observed phenomena.[26] Ptolemy then goes on to explain in what respects the proponents of the leading schools of harmonic theory – the Pythagorean and the Aristoxenian – depart from this scientific method.

Porphyry's aim is to capture the essence of Ptolemy's dialectic. Thus, to read Porphyry's commentary along with Ptolemy's text and Porphyry's citations from the work of Ptolemaïs is to listen in, as it were, on something much like a Platonic dialogue – Platonic, in the sense that the three participants in this case are engaged in the same sort of pursuit as that which occupied Plato's discussants: a search for pure knowledge. The knowledge that these three are seeking concerns *harmonia* – the type of knowledge it exemplifies and the best way in which it can be represented. The dialogue derived from these texts (*mutatis mutandis*) commences with a discussion of the canon.[27]

[26] Thus Barker, *Ptolemy*, p. 26: "Ptolemy's exhortation to 'save the hupotheseis,' is evidently related to the more familiar project of 'saving the phenomena.' The difference is one of perspective and emphasis. In both cases the goal is to show that the truths accessible to reason and the phenomena presented to the senses are in harmony with one another, and that if their evidence is judiciously considered one can consistently accept both."

[27] The changes in Porphyry's text will be noted where required.

Ptolemy:

> What is the purpose of the harmonician? The instrument for such an approach is called "harmonic-canon," a name derived from its common category and from its measuring-out, literally, "canonizing," those things which the senses lack for arriving at the truth.[28]

Porphyry:

> You are saying, I gather,[29] that the instrument for the approach in question is that which reason invented and gave to the senses for the purpose of "canonizing" those things that the senses lack in themselves for arriving at the truth; and that the instrument under discussion is called "harmonic-canon," so-called from the common term for the instrument – which is called canon – that discovers what the senses lack in their ability for attaining accuracy. Of course, the name, "canon," and the approach for the senses called canonic in harmonic theory, do not come from the *canon*, so-called, namely, the cross-bar of the cithara on which its strings are stretched. Rather, the Pythagoreans, who were, above all, the inventors of this approach, called it canonic, in the same sense in which we today call our theory harmonic; but some of them define the canon, which is the measure of accuracy of proportions, in this way: "A canon accurately measures

[28] An extensive, but late, tradition assigns the invention of the canon to Pythagoras himself. Cf. above, Chapter 4, n. 34. Ptolemy *Harm*. I. 8 (Düring, 18) describes its construction, he being the only theorist to have done so, and offers a drawing of it. According to his description of it in *Harm*. II. 12 (Düring, 66–67), it was a lutelike instrument consisting of a long neck, which terminated in a hollow resonating chamber over which a single string was stretched from a peg fixed at the top of the neck to a fixed bridge (*batēr*) at the base. Between the neck and the base there were moveable bridges (*magadia*) at each end, these operated by the left hand, while the right hand plucked that part of the string which was stretched over the resonating chamber. It was an excellent tool for scientific study, but very difficult to use for musical purposes. As Ptolemy reports in *Harm*. II. 13 (Düring, 67. 21ff.), Didymus, the musician, was the first person to introduce some improvements to make the instrument more amenable to musical performance, making the bridge easier to manipulate, for example. Cf. Ruelle, "Le Monocorde, Instrument de Musique," *REG*, 311–12. See also, Barker, II, p. 292.

[29] Porphyry, *Commentary* (Düring, 22. 10) wrote, literally: "He [sc. Ptolemy] says that the instrument for the approach . . ."

the different attunements amongst musical notes, those differences which are studied in the form of numerical ratios."[30] But here is Ptolemaïs of Cyrene,[31] who writes about this in her work, *The Pythagorean Doctrine of the Elements of Music*.

Ptolemaïs:

For whom, then, is the canonic discipline of greater importance? In general, for the Pythagoreans. For the discipline which we speak of today as harmonic, they used to call "canonic." For what reason do we speak of it as canonic? Not, as some people think, from its having been named after the instrument, the canon, but from its [quality of] straightness, since it is on the basis of this science that reason discovers what is right and what the fixed rules of attunement are.[32]

Porphyry:

The term "canonic" is in fact applied also to the study of syrinxes, auloi, and other instruments, even though these are not canonic instruments.[33] But since ratios and theorems are applicable to them, even these instruments are called canonic. It is rather the case, then, that the instrument called canon was named after the canonic discipline. But in general, a canonician is a harmonician who deals with the ratios that concern attunement. There is, however, a difference between musicians and canonicians. For musicians

30 Porphyry does not identify the source of this statement other than that it comes from the Pythagoreans, who invented and practiced canonic science.

31 Porphyry, *Commentary* (Düring, 22. 22–24) wrote, literally: "Indeed, Ptolemaïs the Cyrenian writes about this also in her *Pythagorean Doctrine of the Elements of Music* as follows: . . . "

32 The science of straightness to which Ptolemaïs refers here is the sort of metrical geometry that is exemplified in the *Sectio Canonis* of Euclid; cf. Chapter 4, n. 6.

33 Porphyry means by 'canonic' the mathematical laws according to which all musical instruments – whether they be stringed, like lutes, lyres, and harps, or winds, like syrinxes (reedless panpipes) or auloi (reeds) are constructed. See, for example, K. Schlesinger, *The Greek Aulos*, pp. 3ff., Mathiesen, *Apollo's Lyre*, pp. 159ff. And K. Coates, *Geometry, Proportion and the Art of Lutherie*, pp. 106ff. Cf. R. A. Higgins and R. P. Winnington-Ingram, "Lute Players in Greek Art," *JHS* 85 (1965), 62–71.

who proceed from the evidence of the senses are called harmonicians; but canonicians are *Pythagorean* harmonicians. Both are, of course, musicians in the generic sense. But Ptolemaïs has some question to raise here.[34]

Ptolemaïs:

Of what things are the theory of the canon constituted? Of the general assumptions that are postulated by musicians and that are adopted by mathematicians.[35]

Porphyry:[36]

The assumptions that are postulated by musicians are those data derived from the senses that the canonicians accept, as, for example, that certain intervals are concordant and discordant, and that the octave is composed of the fourth and the fifth and that the whole-tone is the excess of the fifth over the fourth, and similar assumptions. But the facts accepted by mathematicians are those that the canonicians study in their own way in terms of proportion, only after they have been moved to do so by the points of departure provided by the senses. They base their theory on the fact, for example, that the musical intervals consist in numerical proportions and that musical pitch derives from numbers of percussions, and matters of a similar sort. One might determine, then, that the postulates of canonic science belong to the science of music as well as to that of numbers and geometry.

34 Porphyry *Commentary* (Düring, 23. 10) wrote, literally, "She [sc. Ptolemaïs] adverts to these matters again in question and answer form."

35 In this concise statement, Ptolemaïs is using the technical language of philosophy. The general assumptions postulated (ὑποτιθεμένων) by musicians such as Aristoxenus are made after countless observations and much experience. Her reference here is clearly to Aristotle *Met*. 981a5: "Art is born when out of many bits of information derived from experience there emerges a grasp of those similarities in view of which they are a unified whole" (trans. Hope). See note 40.

36 Barker, II, p. 240, has assigned this entire passage to Ptolemaïs. But in n. 138, he adds: "This sentence and the remainder of the paragraph may be from Porphyry's pen, rather than that of Ptolemaïs."

Ptolemy:

> The purpose of the harmonician should be the preservation by all possible means of the rational postulates of the canon,[37] the postulates that in no way whatever conflict with the senses as they pertain to the judgment of the greatest number of people, just as the purpose of the astronomer should be to preserve the postulates concerning the heavenly movements that accord with their courses as observed by us [on earth], seeing to it that the postulates themselves have been derived from the manifest and general outline of the phenomena, while discovering by rational means their particulars as far as is accurately possible.[38] For it is in all cases the special province of the theoretician and the scientist to demonstrate that the works of nature have been wrought with a certain logic and fixed cause, there being nothing in nature that is without plan, nor anything accomplished by nature in random fashion. This applies especially to all those constructions of such surpassing beauty as those belonging to the more rational senses, sight and hearing.

[37] In translating *hypotheseis* as "postulates," I am following the rationale of Barker, II, 278, n. 15. In his *Ptolemy*, pp. 23–24, he says, however: "I have rendered the word [*hypotheseis*] as 'postulate': but as a translation it is hardly adequate . . . [*hypotheseis*] are fundamental propositions which are not formally derived from others that the discipline has already established, but which form the basis for the derivation or explication of subordinate propositions." Solomon, *Ptolemy*, p. 7, translates the phrase as: "the reasoned hypotheses of the canon." As he points out, p. 7, n. 34, "Preserving the hypotheses" was an important concept in ancient Greek science." Cf. note 38.

[38] Barker, *Ptolemy*, p. 26, observes: "Ptolemy's exhortation, to 'save the *hypotheseis*,' is evidently related to the more familiar project of 'saving the phenomena'." According to P. Duhem, *To Save the Phenomena: An Essay on the Idea of Physical Theory from Plato to Galileo*, pp. 5–6, the ancient astronomers had the same purpose in mind as that expressed here by Ptolemy: "the object of astronomy is here defined [by Simplicius in *In Aristotelis quator libros de Caelo commentaria* 2. 43. 46] with utmost clarity: astronomy is the science that so combines circular and uniform motion as to yield a resultant motion like the stars. When its geometric constructions have assigned each planet a path which conforms to its visible path, astronomy has attained its goal, because *its hypotheses have then saved the appearances*."

Porphyry:[39]

> You write of these matters, Ptolemaïs, in your introduction which I mentioned earlier.

Ptolemaïs:

> Pythagoras and his followers are pleased to accept perception to begin with as a guide for reason, so as to offer reason some sparks, as it were; but reason, after it has been stimulated by these sparks, is treated by the Pythagoreans as something that works independently, standing quite apart from perception. And even if their system, though discovered by the diligent prosecution of reason, is hence no longer in tune with perception, they do not repudiate their position, but charge perception with being aberrant and assert that it is reason which discovered by itself what is correct and that it is reason which convicts perception of being wrong. Standing in opposition to these Pythagoreans are some musicians of the school of Aristoxenus who adopted a theory based on conceptual thinking (*ennoia*), while developing it from their training in instrumental practices. For these musicians viewed perception as their leading authority, while treating reason as a follow-up to be used only as needed. According to these musicians, as may be expected, the rational postulates of the canon[40] were not in every case in accord with perception.

[39] Porphyry *Commentary* (Düring, 23. 24–25) wrote, literally: "Ptolemaïs writes about these matters in her aforementioned *Introduction*."

[40] The phrase, "the rational postulates of the canon" was introduced here by Ptolemaïs, in whose writings it was found by Didymus, who passed it on to Ptolemy *Harm*. I. 2 (Düring, 5. 14). It is repeated verbatim just below by Porphyry. As Barker, *Ptolemy*, p. 25 sees it, the expression "is not altogether clear. The parallel phrase used here [by Ptolemy], 'the *hupotheseis* of the movements in the heavens' . . . refers plainly to the principles governing those movements, or to expressions of those principles as scientific propositions." As applied to the canon, however, Barker finds the expression "ambiguous." Ptolemaïs, as is argued (see note 44), understood these rational hypotheses or postulates of the canon as axioms accepted by mathematicians, because they were necessarily imposed on them, not as affirmations

Porphyry:

> But Ptolemy sought to show that the rational postulates of the canon do not in any way conflict in any respect with the senses, as regards the judgments of these musicians; and, in fact, it is the special feature of his harmonic treatment that accounts, too, for his departure from the calculation of divisions made by the former theorists [the Pythagoreans]. He made the same point in his astronomical writings also in holding that the astronomer's purpose must be to preserve the postulates concerning the heavenly motions that accord with their courses as observed by us [on earth]; and that the postulates themselves must be derived from the manifest and general outline of the phenomena, not only discovering by rational means their particulars as far as is accurately possible.[41] He in fact speaks of this at the beginning of his *Treatise on Mathematics* [*Mathematikē Syntaxis* = *Almagest* I. 2; Heiberg, 9. 11] in these words: "We shall try to show each of these things, using as principles and foundations for our investigations the manifest phenomena and the undisputed observations of the ancients and of our contemporaries, reconciling with their observations the resulting judgments through the use of proofs involving the geometrically linear approach." It is correct to say that the task proper to the observer and to the scientist is demonstrating that the works of nature have been wrought with a certain logic and fixed cause, there being nothing in nature that is without plan, nor is anything accomplished by nature in random fashion. For the Pythagoreans also were given to saying and promulgating the view that it is essential to banish randomness from all quarters, both from our way of life and from our theory and practice, and to realize that in no way does randomness exist in nature, seeing that nature is the finished product of Mind (*nous*), the why and the whence existing in Mind. Almost all philosophers agree that the fairest of the senses are sight and hearing and some, such as he [*sc*. Ptolemy], call them rational because they render a special service to reason in its pursuit of an appropriate theory.

of the musical mind, but as affirmations of the properties of the stretched string of the canon.

41 It may be said in this connection that Aristoxenus' hypothesis of equal measure is a statement of principle by which he proposed to "save the phenomena" that are presented audibly to the ear, namely, the phenomena, or appearances, of the melodic *continua*.

Ptolemy:

> Some theorists appear to have paid no attention at all to this purpose, devoting their attention to the technical craft alone and to the bare and irrational exercise of perception; while others have been more theoretically inclined in approaching their goal. These would be the Pythagoreans and the Aristoxenians in particular – both of whom were wrong. For the Pythagoreans, because they did not defer to the impression of the ear in all those cases in which it is essential to do so, applied ratios to the differences between sounds, ratios which were often incompatible with the phenomena, the result being that they created a basis for calumny against their sort of criterion amongst the theorists who held a different opinion. On the other hand, the Aristoxenians, because they accorded the greatest importance to the facts derived from the resources of perception, treated ratios as extraneous to the subject, being in violation of the ratios as well as of the phenomena. They [*sc*. the Aristoxenians] were in violation of the ratios in that they adapt numbers, that is, the semblances of ratios, not to the differences between sounds, but to the spaces between them; they were in violation of the phenomena in that they also match up these numbers with divisions which are extrinsic to the testimony of the senses. Each of these violations will become clear from the facts that will be introduced, provided that the preliminaries to what follows are first defined.

Porphyry:[42]

> I believe that you wrote concisely about these issues in your introduction, Ptolemaïs of Cyrene, and that Didymus, the musician, followed up on them at greater length in his work, *On the Difference Between the Aristoxenians and the Pythagoreans*. What is it that you had to say?

[42] Porphyry *Commentary* (Düring, 25. 3–8) wrote, literally: "The Cyrenian Ptolemaïs wrote concisely about these issues in her *Introduction*, and Didymus the musician followed up on them at greater length in his work, *On the Difference Between the Aristoxenians and the Pythagoreans*. We shall record what was said by both of them, making a few alterations for the sake of brevity. Ptolemaïs writes thus as follows."

Ptolemaïs:

> What is the difference between those that have been preeminent in music? For some of them preferred reason alone, while others preferred perception alone, and still others preferred both together. Reason alone was preferred by the Pythagoreans, those Pythagoreans who rather enjoyed contending with the musicians for the purpose of casting out perception altogether and introducing reason as an independent criterion that works by itself. But these Pythagoreans are completely refuted by their accepting something that is perceived by the senses at the very beginning and then disregarding it altogether. Instrumentalists, on the other hand, preferred perception, but for them it did not in any sense become the conceptual basis for a theory, or if it did, it was a trivial one.[43] What is the difference between those who preferred both in combination? Some accepted both perception and reason on equal terms as being equivalent in power, while others took the one as their guide and the other as a follow-up. Aristoxenus the Tarentine took them both on equal terms. For [as he saw it] an object of perception cannot exist on its own apart from rational thought, nor is rational thought sufficiently strong to prove anything without accepting its first principles (*archai*) from perception and then paying back its debt to perception in the form of a theory, the conclusion of which theory agrees in turn with perception.[44] What is the basis for his willingness to grant perception the lead over rational thought? On the basis that perception comes first in the order of events, but not on the

[43] Ptolemaïs speaks here of instrumentalists (*organikoi*) in much the same way as Aristoxenus did of his predecessors, the *harmonikoi*. In fact, Barker, II, p. 241, n. 145, suggests that this entire statement by Ptolemaïs is based on Aristoxenus *Harm. El*. II. 32 (Da Rios, 41. 19ff.), where Aristoxenus criticizes his predecessors for rejecting the senses as inaccurate. Her words also closely echo those of Aristoxenus *Harm. El*. II. 41 (Da Rios, 52. 4ff.), where he dismisses any theory that is based on instruments as *atopos*, or "out of place."

[44] Cf. Barker, II, p. 241, n. 146, who points out that what Ptolemaïs says here as to Aristoxenus' principles (*archai*) and their dependence on perception is a theme that is pervasive throughout Aristoxenus' *Harmonic Elements*. It should be noted also that she has in this statement concisely defined the true character of science: the preservation of the phenomena within the connective links of true generalizations. That certain scientific theories never die is thus owing to their consistent expression of true relations. Cf. H. Poincaré, *Science and Hypothesis*, pp. 164–65.

> basis of its power. For, he says,[45] "Whenever a perceptible object, whatever it may be, makes contact with this [*sc*. perception], then we must promote rational thought to the forefront, in order to get a theoretical understanding of this [*sc*. perceptible object]." Who regard them both in the same way? Pythagoras and his followers.[46] For they are willing to accept perception initially as a guide for reason so as to provide reason with sparks, as it were; but reason is treated by them as something which, though set into motion by perception, works by itself, standing quite apart from perception. And even if, as a result, their system which was discovered by the application of reason no longer is in tune with perception, they do not retract their opinion, but charge perception with being deviant and assert that it is reason which discovered on its own what is correct and that it is reason which refutes perception. Who are opposed to these theorists? Some musicians of the school of Aristoxenus, all those who accepted a theory based on conceptual thinking, while promoting it from their training in instrumental practices. For these musicians considered perception as their leading authority, while treating reason as a follow-up to be used only as needed.

It is at this dialectic juncture that Porphyry introduced into his commentary his citations from the work of Didymus, the musician, most of which have been represented at the end of Chapter VI. And, as becomes obvious on comparison between Didymus' statements and those of Ptolemaïs, it was indeed Ptolemaïs herself who was the primary source for Didymus. This is especially true in the case of Didymus' words, which bear repeating here:[47]

> Music is not only a rational knowledge, but it is at [one and] the same time perceptual and rational; hence it is necessary for the truly systematic

[45] It is not uncommon for direct quotations to be introduced, as Ptolemaïs does here, by φησί ("he says"). Since quotation marks were unknown, other devices such as that used by Ptolemaïs were routinely employed. Numerous examples of a similar sort may be found in Ps.-Plutarch *De mus*. quoting from Aristoxenus. See. e.g., *Frs*. 80, 81, 82 (Wehrli) where the convention, "he [*sc*. Aristoxenus] says," is as Ptolemaïs has it.

[46] In making a few changes for the sake of brevity, Porphyry manages to misattribute this strictly Aristoxenian position to the Pythagoreans. The rest of this passage repeats almost verbatim what was recorded earlier. On these problems with Porphyry's text, see Barker, II, p. 242, n. 148 and n. 149.

[47] See Chapter 6, note 48.

> student not to neglect either one of the two, even while having what is evident to perception take the lead, since it is from there that reason must make its beginning.

Didymus' words bear repeating for the additional reason that they appear to contradict his earlier statement about those *organikoi* (instrumentalists) and *phonaskikoi* (voice coaches) who, he said, are habitually engaged in an irrational or non-rational (*alogos*) activity. Didymus was wise enough to know, of course, that no irrational activity deserved to be called a *technē*, an "art," such as *iatrikē* (the art of medicine), or *rhetorikē* (the art of public speaking).[48] Yet here he agrees with Ptolemaïs in characterizing music as a "rational knowledge" (*logikon mathēma*) and, hence, qualified to be included among the various fields of knowledge as a genuine *technē*, or *mousikē*. The inference to be drawn is that instrumentalists and voice experts were more concerned with performing techniques than with the status of *mousikē* as a rational knowledge. The question is, however, in what respect is music a rational knowledge: Is it abstract and intuitive or is it experimental and scientific? To her credit, Ptolemaïs saw that this problem is linked to the meaning of *logos*, or, more properly, the meanings of *logos*. For *logos* and the forms to which it gives rise in these contexts – *logikos* (rational) and *logikoi hypotheses* (rational hypotheses or rational postulates) – meant one thing to Ptolemy, but quite another to Aristoxenus.

To Ptolemy, the rational postulates of the canon[49] – a phrase he may have picked up from Ptolemaïs by way of Didymus – are those mathematical formulae which interpret or "rationalize" the intervallic relations that are classified to begin with by the ear. They are judged by the ear to be concordant or discordant, but it is the task of the canonician to prove them to be such. To Aristoxenus, however, the question of rationality and irrationality belongs to a different line of inquiry entirely, one that has nothing to do with the ratios (*logoi*) of canonic science. For in his theory of music, intervals

[48] Indeed, as Socrates said in Plato *Gorgias* 465A5–6: "I will not call any irrational endeavor (*alogon pragma*) an art (*technē*)."

[49] Cf. note 40.

are considered *rhēta* (rational) or *aloga* (irrational) in virtue of the relation that they bear to the melodic genera in which they participate. Intervals are rational, therefore, if they can be played or sung in any of the genera and nuances of melody. To be sure, Aristoxenus' quantum – $\frac{1}{12}$ of a whole-tone – is, if taken in isolation, *amelōdētos*, or "unsingable," as he specified; but taken as a natural unit of enumeration, it broke through the obstreperous bonds of the canonic ratios and freed Aristoxenus to define rationality and irrationality in strictly melodic terms. With 12 as his basic unitary and indivisible measure, Aristoxenus could treat any interval, no matter how small it might be, as rational, if it was commensurate with his quantum. Thus, intervals such as the enharmonic *diesis* ($\frac{1}{4}$ tone) or the chromatic *diesis* ($\frac{1}{3}$ tone), which escape reliable measure on the canon, could be established to the ear's satisfaction as consisting of $\frac{3}{12}$ and $\frac{4}{12}$, respectively.

The debate over the criteria – *Aisthtēsis* and *Logos* – thus belongs to a larger topic: the proper scientific method for dealing with the subject, Harmonics. As practiced today, Harmonics is a strictly mathematical discipline that is limited to analyzing the complex periodic motions in sound production (called Fourier analysis), and to determining the complex periodic motion from a given set of harmonic components (called Fourier synthesis).[50] In antiquity, Harmonics had a far greater reach. For Ptolemy, it had to do with the absolute and organic unity implicit in the notion of *harmonia*: the correct fitting together of parts. Ptolemy's goal, therefore, was to discover the correct attunement (*hērmosmenon*) for melody. As Aristoxenus saw it, however, Harmonics was a branch of a larger discipline, which he called the *Epistēmē of Melos*, the Science of Melody. His goal, therefore, was to discover the correct constitution (*synthesis*) of melody.[51]

[50] So, too, in projective geometry, the relation between two points is thought of as 'harmonic' when it involves a reference to two other fixed points. Thus, a relation of four terms may serve to divide a straight line into two parts with respect to any two of its points. See Russell, *Principles of Mathematics*, pp. 384–85 on quadrilateral constructions and harmonic ranges.

[51] This *synthesis* began with the topological limits or boundaries formed by the proper arrangement of fourths and fifths in the *continuum* of melodic space.

In Ptolemy's lexicon, Harmonics was the very counterpart of Astronomy. Indeed, as he argues the case, the difference between the two is essentially one of language and not in what is being described. In the course of arguing this position, Ptolemy uses the words *logos* and *logikos* to mean three very different things: rational thought, the spoken word, formula:[52]

> No one would predicate beauty or ugliness of things touched, tasted, or smelled, but only of things seen and heard, as when we speak of form and melody, or again, of heavenly motions and human actions; for which reason sight and hearing are the only ones of all the senses that quite often offer their mutual services to the rational (*logikos*) part of the soul in the form of their reciprocal perceptions, as though they two were actually sisters. Indeed, hearing explains visible things solely through its interpretations, while vision reports on audible things solely by means of written illustrations.[53] And it is often the case that a clearer understanding of each of these

This method of dealing with the infinite divisibility of the *continuum* made it possible for Aristoxenus to speak, for example, of two tetrachords of the same genus as being consecutive (*ephexēs*), or of two tetrachords being in contact (*synaphē*) when the highest note of one tetrachord coincided with the bass note of another tetrachord. That all such types of *syntheses* or constitutive elements in Aristoxenian theory are based on the teachings of Aristotle has been detailed by Bélis, *Aristoxène*, pp. 153–55. The Aristotelian model for these concepts in Aristoxenian theory are examined (independently of Aristoxenian theory) by White, *The Continuous and the Discrete*, pp. 23–25.

52 Ptolemy Harm. III. 3 (Düring, 93. 20–94. 20). According to Plato *Theaetetus* 206D1–2, *logos and dianoia* have a very close relationship for, as Theatetus points out, a *logos* is the making apparent one's thought (*dianoia*) through speech (literally, through nouns and verbs).

53 Ptolemy does not explain how the sense of hearing interprets what the eye sees. Barker, II, p. 373, in translating this passage, supplies the words "by means of [spoken] explanations." When it comes to the priority of the visual sense where music is concerned, Aristoxenus has a significantly opposing view. As he argues in *Harm. El.* II. 39 (Da Rios, 49. 12–14), the written, or notated, form of a Phrygian melody is of little or no help in conveying the mode (*tropos*), or "Phrygianness" of such a melody. As it happens, the Aristoxenian Aristides Quintilianus *De mus.* I. 9 (Winnington-Ingram, 19)

> facts of perception is attained than if the same facts are interpreted by the one to which they belong. Such is the case when things transmitted by the spoken word (*logos*) are rendered more intelligible to us and are more easily remembered by us when accompanied by diagrams or symbols; while things recognizable by sight are revealed more imitatively through poetic interpretation – the appearance of sea-waves, for example, and scenes and battles and the circumstances of emotions, so that people's souls are made to sympathize with the forms of the things that have been communicated to it, as though these things had been seen with one's own eyes.[54] It is, therefore, not only by the senses each apprehending what is proper to it alone, but by both senses somehow assisting one another to learn and to understand everything that is accomplished by each in accordance with its own appropriate formula (*logos*), that these senses themselves, and the most rational of the sciences that are based on them, reach more deeply into beauty as well as into usefulness. Of these most rational sciences, the one pertaining to vision and to the motions in place of things that are only seen – that is, the motions of the heavenly bodies – is Astronomy; and the science pertaining to the motions in place, again, of things that are only heard – that is, the motions of sounds – is Harmonics. Since

gives us a glimpse of the ancient Phrygian mode, or *harmonia* (together with the Lydian, and Iastian) to which he said Plato referred in *Rep.* 399A. Because Aristoxenus himself may have been Aristides' source here, the Phrygian scale which Aristoxenus may have had in mind would look something like this:

D | E E+ F A | B B+ C¹ D¹ |

The enharmonic character, or ethos, of this scale springs instantly into view. But, according to Aristoxenus *Harm. El.* I. 23 (Da Rios. 29. 15ff.), the ear has only to hear the ditone (as between F and A) to recognize the beauty of a melody composed in this style – the beauty which cannot be conveyed by viewing the entire scale. On this evidence, see Levin, "The Hendecachord of Ion of Chios," *TAPA* 92 (1961), 304–05; Barker, II, pp. 420–21; Mathiesen, *Aristides*, p. 86. Cf. Henderson, "The Growth of the Greek ἁρμονίαι," 98–99.

54 Following Aristoxenus' line of argument, however (note 53), poetry, not even with all its mimetic powers, can succeed in interpreting a strain of melody.

> both of these sciences use the incontrovertible instruments, arithmetic and geometry, for judging the quantitative and qualitative attributes of the primary motions, they are as first cousins, born of the sister senses, sight and hearing, and are reared so closely as is generically possible by arithmetic and geometry.

These are the words of a confirmed Pythagorean – one who was not only imbued with Platonic doctrine, but one who had also assimilated the teachings of Archytas, the Pythagorean par excellence. For it was Archytas who wrote at the beginning of his *Harmonics*:[55]

> The mathematicians seem to me to have acquired superb knowledge; and it is not at all unusual that they think correctly about individual things and their nature. For by having acquired exceptional knowledge about the nature of the universe, they must have come to see its parts and their nature very well, also. They have, in fact, handed down to us a precise understanding about the velocity of the stars and their risings and settings, as well as about geometry, numbers, and spherics, and not least, about music. For these seem to me to be sister sciences, since their concern is with the sister subjects: the first two forms of being [things heard and things seen].

Even though Ptolemy criticized the Pythagorean method for dealing with consonant intervals, such as the octave and a fourth, and judged their system for grading ratios as "utterly ridiculous" (*panu geloios*), his sympathy with the Pythagorean worldview was fundamental.[56] If this

[55] This is quoted by Nicomachus *Introduction to Arithmetic* I. 3 (Hoche 6. 17–7. 4). Cf. *Vors*. 47B, *Fr*. 1. Diels – Kranz, I, p. 432, understand Archytas' first two forms of being here as "Zahl und Grosse" (number and size).

[56] On Ptolemy's criticism of the Pythagoreans, see Barker, *Ptolemy*, p. 71, who says of Ptolemy's criticisms of the Pythagoreans' principle that all consonances have to be either multiple or epimoric ratios: "It is one of the most savage passages of critical writing in the *Harmonics* – Ptolemy positively devours the unfortunate theorists, spicing the funeral meats with peppery sarcasm." Barker adds to this highly seasoned account this important observation: "Ptolemy's

licensed him to chide them – sometimes mercilessly – on points of detail, it was only because he wanted the Pythagorean method to be perfect in all respects, even if he had to make it so himself. That he succeeded in his undertaking is an effect of his mathematical genius; but it is no less an effect of the very nature of mathematics itself. For Harmonics, as Ptolemy conceived it, had to become a branch of universal mathematics if it were to be truly scientific. Ptolemy's success in this purely scientific endeavor is in fact reflected in the mathematically balanced structural design of his *Harmonics*, the three books of which are each neatly composed of sixteen chapters.[57] Even more remarkable is the supersymmetry he achieved in his system of seven *tonos*-scales in whose central octave (as from E to E') the seven species of a revolving set of octave scales are uniformly located.[58]

unmerciful assault should not be allowed to mislead us. In one important respect their approach is closely parallel to his own." The point to be made (and which has been made fully by Barker in these pages) is that Ptolemy shares the assumptions of the Pythagoreans when it comes to distinguishing the consonant intervals.

[57] See Solomon, *Ptolemy*, pp. xxxi–xxxii.

[58] Centuries before Ptolemy, Aristoxenus had revolutionized the system of cithara tunings by fixing the relative pitch of the mode-bearing keys of transposition at equal semi-tones from one another. These tunings, or raisings and lowerings of the entire cithara (the principle being much like that of the modern pedal-harp), were thought of as *tonoi*. Each tuning brought the desired modal segment of the Greater Perfect System into a practical vocal range. See Fig. 4 and Fig. 9. To take one example: the diatonic sequence of intervals marking the Phrygian mode is: D, E, F, G, A, B, C, D (tone, semi-tone, tone, tone, tone, semi-tone, tone). This sequence is preserved intact in the F to F octave of the tuning: C, D, E♭, F, G, A♭, B♭, C, D, E♭, F, G, A♭, B♭, C (the Greater Perfect System of the Phrygian *tonos*). Ptolemy *Harm*. II. 5 (Düring, 51–53) effected an innovation on this centuries-old system, first by reducing the number of keys from 15 to 7, and second, by distinguishing notes in respect to their position (*kata thesin*) in the individual mode and in respect to their function (*kata dynamin*) in the tonos. Thus, for example, in the octave segment F–F (earlier), the note C is *mesē* by function, but B♭ is *mesē* by position. Cf. Macran, *Aristoxenus*, pp. 63–64; Barker, II, p. 325, n. 37; Solomon, *Ptolemy*, p. 73, n. 68; Henderson, "Ancient Greek Music," 352–56. Reinach, *La musique grecque*, p. 58, speaks of Ptolemy's *tonoi* as "malentendus" and even more Aristoxenian

In sum, the more mathematically logical his method, as he developed it, the farther Ptolemy removed himself from *Mousikē*, the art of music. Jon Solomon has aptly stated Ptolemy's aims in these words:[59]

> than Aristoxenus' own system of 13 *tonoi*. Ptolemy's schēmata, for which see Fig. 9, seem to have found little success with practicing musicians.

Mixolydian	F	G^b	A^b	B^b	C^b	D^b	E^b	F
							Mesē	
Lydian	F	G	A	Bb	C	D	E	F
						Mesē		
Phrygian	F	G	A^b	B^b	C	D	E	F
					Mesē			
Dorian	F	G^b	A^b	B^b	C	D^b	E^b	F
				Mesē				
Hypolydian	F	G	A	B	C	D	E	F
			Mesē					
Hypophrygian	F	G	A	B^b	C	D	E	F
		Mesē						
Hypodorian	F	G	A^b	B^b	C	D^b	E^b	F
	Mesē							

FIGURE 9. The Seven Tonoi of Ptolemy

[59] Solomon, *Ptolemy*, xxxv.

But Ptolemy had no intention of writing a book about music. His work is on harmonics, and despite our familiarity with the modern derivative of that term, and despite that "harmony" has quintessentially a musical meaning and only [a] secondarily a metaphorical meaning. . . . Ptolemy's conception of harmonics was one that was self-contained, well-established, highly scientific, and technical.

Harmonics, as practiced by Aristoxenus, however, was a different field entirely, and no one saw the root of the difference more clearly than Ptolemy. In his criticisms of the Aristoxenians, by whom he meant Aristoxenus himself, he draws our attention to Aristoxenus' radicalism by contrasting it with the Pythagoreans' rationalism. The Pythagoreans focused on individual notes and, in accordance with Ptolemy's views, defined the differences between these notes by numerical ratios, even though perception contradicted them on occasion.[60] But Aristoxenus, by concentrating on the intervals or spaces between the notes, not only contradicted the numerical ratios but also arrived at results that Ptolemy found truly impossible to describe. He called them *eikonēs* of the ratios,[61] which Barker translates as "the images" of the ratios and Solomon as "the symbols" of the ratios. *Eikonēs* is, however, not a complimentary term. In using it, Ptolemy had in mind those units of measure by which Aristoxenus divided the whole-tone into twelve equal parts. Divisions of this sort were no more rigorous to Ptolemy than the Aristoxenian method of computing the whole-tone as the difference between the fourth and the fifth. The whole-tone must be defined instead by its proper ratio, 9:8. Nor could Ptolemy refer Aristoxenus' divisions of the melodic *topos* to the notes themselves, for there would in

[60] Ptolemy's rationale for the Pythagorean position with regard to pitch and quantity is best expressed by Barker, *Ptolemy*, p. 35 (perhaps better than by Ptolemy's own representation): "All differences whose causes differ quantitatively, however, are themselves quantitative, no matter how they are perceived; and hence the question whether a distinction that seems qualitative actually is so raises quite recondite issues that fall into the province of the scientist. It is not to be answered on the basis of appearances alone." No statement can be more *contra Aristoxenum* than that.

[61] Ptolemy, *Harm*. I. 2 (Düring, 6. 8–9).

this case turn out "to be infinitely many of them in each ratio."[62] Worst of all, Ptolemy could not submit Aristoxenus' divisions to verification.

As Ptolemy argued, therefore, the Pythagoreans may have gone astray by relying too heavily on the ratios, thereby contradicting the ear at times; but Aristoxenus did far worse: not only did he violate the integrity of the ratios, but he had to have offended the ear by postulating *eikonēs*, or "simulacra" of the true facts: the canonic ratios. To Ptolemy, these *eikonēs* of Aristoxenian theory were as abstract wraiths, rooted in imagination and bearing no relation to reality, for they could not be verified objectively. When it came, therefore, to his theory of melodic quanta, Aristoxenus was in as difficult a position as that of present-day cosmologists; for cosmologists today are subject to the same sort of criticism as that leveled against Aristoxenus by Ptolemy: there are no objective ways to verify their proposals for a quantum theory of gravity.[63]

Aristoxenus, had he been present to hear Ptolemy's criticism, would have been little troubled by it, because he had no doubts about the convenience of his method for the study of melody. He knew something fundamental: that if we study melodic consecution, it is in order to apply it; and we can only apply it if it remains objective. For Aristoxenus, then, the source of melodic truth was not solely perception, as his critics charged, but experiment with the concurrence of the ear (*akoē*) and musical reason (*dianoia*). Ptolemy and the Pythagoreans had of course conceded that the matter (*hylē*) of harmonics is provided by the ear. But, as Ptolemy argued, harmonic truths can only be verified by measurements on the canon. In order to arrive at harmonic truths,

[62] Ptolemy *Harm*. I. 9 (Düring, 20. 11–14).

[63] Aristoxenus was engaged in a pursuit not unlike that of present-day scientists. For scientists today are seeking a single theory that will unify all the insights derived from relative and quantum theory and link them to the real world (that is, to the world outside their minds). So, too, Aristoxenus was seeking a unified theory of melody that would link together the delicate networks of scales, keys, and modes and anchor them to musical reality. To Ptolemy, Aristoxenus' was apparently just an imaginative theory that was rooted in the more unscientific realms of metaphysics. Thus, Ptolemy had to have failed when it came to translating Aristoxenus' representations into the language of mathematics. Cf. Barker, *Ptolemy*, pp. 115–17.

the harmonician or canonician must believe that the numerical ratios derived from these canonic measurements are themselves true; and if his belief in these ratios is unconditional, then the ratios become his incontestably true facts. As Aristoxenus argued, however, these ratios, derived as they were from the straight line of the canon, had nothing to do with his subject, because his subject had no connection with a straight line. His subject, as he defined it, was the nature of musical melody, and the facts with which he had to deal were not simply notes and intervals, but phenomena that are much harder to interpret: melodic genera, melodic species and, most important, the nature and origin of continuity in the musical scales. That being the case, a new science was required to deal with these and related phenomena and, as Aristoxenus reminds us, he had to invent this new science himself.[64]

Aristoxenus' science was far from being universally received; but, as Ptolemy's criticisms (given earlier) show, it was a science that could not be ignored. Ptolemaïs neither ignored it nor did she criticize it; on the contrary, she is one of the few authorities, if not the only one, to champion it. She began by indicating that the science of canonics has the closest kinship with the eye, whereas that of Aristoxenus belongs to the ear alone. It follows from this, as she implies, that Aristoxenus chose to give unity to all his melodic constructions not within ratios abstracted from the canon, but within the vivid and audible forms of melody. Setting Ptolemy, Porphyry, and Ptolemaïs in the framework of a discussion is admittedly rather unfair to Porphyry. For in contrast to the single-mindedness of Ptolemy, arguing for the virtues of canonic science, and Ptolemaïs, offering a philosophical basis for Aristoxenus' theory, Porphyry comes across as verbose and digressive. If he is not repeating the words of his sources almost verbatim, he is introducing irrelevancies, as, for example, the cross-bar of the cithara, only to reject it as a derivative of the term "canon"; similarly gratuitous is his observation about syrinxes and auloi being "canonic" for being subject to

[64] Aristoxenus *Harm. El.* I. 4 (Da Rios, 8. 16–9. 4): "Hitherto these questions [continuity of species and genera in *musical* melody] have been absolutely ignored, and in dealing with them we shall be compelled to break new ground, as there is in existence no previous treatment of them worth mentioning" (trans. Macran).

mathematical laws, even though they are not, strictly speaking, canonic instruments. Porphyry is obviously a polymath and, just as obviously, he wants the reader to know it.

Porphyry's most important contribution to the discussion is his categorization of musicians into two kinds of harmonicians: those who proceed from the evidence of the senses, by whom he probably means the Aristoxenians; those who base their theory on the canon, these being, he says, the Pythagoreans. What lies behind these categorizations is the notion that things perceived by the ear can be accounted for in the same way as things perceived by the eye. Assuming, then, that the world of the ear and that of the eye consist somehow of parallel phenomena, the path to their respective realities must lie in the application of what Ptolemaïs, in open disapproval, calls "the rational postulates of the canon," a phrase which Ptolemy picks up with intentional approbation. For, as Ptolemy argues, the harmonician is engaged in the same sort of task as that of the astronomer: to preserve the phenomena – those heard as well as those seen – while adhering to the principles invented by reason, that is, mathematical reason. To violate the one, mathematical logic, is therefore just as unscientific as contradicting the other, the evidence of the senses. Barker is especially helpful on this point:[65]

> To "save the *hupotheseis*," in this context [that of accounting for beauty in the phenomena] at least, is not just to show that certain complexes of mathematical propositions are true, but to show that they are indeed the principles which underlie the facts to be coordinated and explained – that is, that certain perceptible patterns of sound relations are admirable and beautiful, while others are not.

There are, of course, serious limitations to the similarities between visible and audible phenomena, and it is to these that Ptolemaïs directs us in all of her statements. She begins by calling attention to the quality of straightness that is the basis for all the computations of canonic theory. As she explains, the canon, by virtue of its straight edge, gave title to the branch of mathematics called *Kanonikē*. This term – *Kanonikē* – was in fact introduced into the language by Ptolemaïs herself to designate

[65] Barker, *Ptolemy*, p. 27.

a discipline – the canonic method, or *pragmateia* – that was intended to bridge the gap between the world of audition and that of vision. The question and answer form she adopts here is reminiscent of the Aristotelian *Problems*; what is more, the noun she uses – *euthytēs* – to designate the quality of straightness that belongs to the canon, is uniquely Aristotelian. In using *euthytēs* in this context, she directs us, perhaps intentionally, to Aristotle's *Categories*, where Aristotle explains that sometimes the quality of a thing (as, in this case, straightness) does not give its name to the science that is based on it.[66] But, as Aristotle says elsewhere, the very quality that enables us to judge both it and its opposite is present in the soul:[67]

> For one part of the soul is sufficiently able to judge itself from its opposite and to judge what is opposite to it. For indeed it is by means of a straight line that we recognize both straightness itself and its opposite, crookedness. For the straight-edge (*Kanon*) is a judge of both. But what is crooked is neither a judge of itself nor of that which is straight.

As Ptolemaïs argues, therefore, "canonic" denotes straightness – the essential visual property that allows for the representation of a stretched string by a mathematically straight line. Little wonder, then, that the Pythagoreans looked to the canonic method for arriving at harmonic

[66] See Aristotle *Categories* 10a10 on *euthutos* and its opposite, *kampulotēs*, crookedness. See also, *Categories* 10b5: "Sometimes, moreover, the quality possesses a well-defined name, but the thing that partakes of its nature does not also take its name from it."

[67] Aristotle *De anima* 411a 3–7. The notion of recognizing the "rightness" of things through a knowledge of its opposite, "wrongness," is one that goes back to Plato and the pre-Socratics. Thus, for example, in *Gorgias*, Plato explains how a doctor recognizes illness because he has a thorough knowledge of its opposite, health. The principle is adhered to by Aristoxenus in his *Harmonic Elements*, where he emphasizes that the truly scientific musical theorist is one who recognizes an unmusical (*ekmelēs*) melody because of his knowledge of its opposite: a musical (*emmelēs*) melody. Cf. *Harm. El*. I. 18 (Da Rios, 23. 19–24. 4). As Aristoxenus argued, there is a fixed principle underlying all musical melodies, a principle which ordains that a correspondence of fourths and fifths be maintained between every note of the scale on which a good melody is framed. Cf. Barker, II, p. 129, n. 24.

truths since, as Ptolemaïs observed, it is only on the straight line that mathematical reason (*logos*) can discover, without the assistance of the ear, what is right (*orthos*). Implicit in Ptolemaïs' assessment of the Pythagorean doctrine is this important fact: there are no terms in the world of melody that are parallel in meaning to "straightness" and "rightness." We may speak metaphorically of "high" and "low" on the vertical dimension of pitch, and we may refer to an *orthios* melody, by which term we do not mean in ancient Greek music a "right" or "correct" song but a high-pitched one.[68] And although it is a commonplace to speak of a melodic "line," be it ancient or modern, it is never in application to a straight line, but to the thematic "contour" of a melody. As Ptolemaïs impresses on us, then, the language of canonic theory is strictly visual in its references, whereas the spatiality of the melodic world is in no way amenable to measurement on the straight line of the canon. She emphasizes the distinction between the visual and the aural world additionally, in the most inventive way possible, by the use of a metaphor: *parapēgmata*. The translation of this word, as here, by the phrase "fixed rules of attunement," does not do full justice to the metaphor, and Barker has perhaps more wisely left the word to stand transliterated rather than translated. *Parapēgmata* is of the world of vision in its most conspicuous form: it is an astronomical term that refers to the pegs inserted into stone to mark the predictable events of an astronomical and meteorological nature.[69] By applying *parapēgmata* to the proportional divisions on the canon, Ptolemaïs has brought to our notice the unalterable status of the numerical ratios of canonic theory.

In her next intervention, Ptolemaïs links musicians and mathematicians to the same starting point of harmonic knowledge: sensory perception. Here, her carefully chosen words are intended to mark the differences between the two theories: that of the musicians, such as Aristoxenus and his followers and that of the Pythagoreans and their

[68] Thus, Talthybios in Euripides' *Trojan Women* 1265–66 refers to the high-pitched (*orthian*) sound of the *salpinx* (trumpet). And Herodotus I. 24 tells of Arion's singing an *orthion nomon*, a song perhaps so plaintive that it brought about his rescue from the sea by a musically astute dolphin.

[69] Diogenes *Laertius* 9. 48 speaks of a work by Democritus entitled *The Great Year or Astronomy*, Calendar (*parapēgma*).

exemplars, the canonicians. As she has it, the musicians have their hypotheses, by which term she means those undemonstrable principles that are arrived at by induction; the canonicians adopt these same principles as matters of fact that are accepted, provided that they be demonstrably true. Ptolemaïs thus framed the difference between the two theoretical schools on the subtle distinction between those melodic presentations which the minds of musicians construct for themselves – consonance and dissonance, for example – and the numerical ratios by which the canonicians give these presentations a concrete and demonstrable reality.

Porphyry's commentary on these points is welcomely instructive and doubtless was drawn from the text of Ptolemaïs herself. He thus defines those general assumptions of musicians that lend themselves to mathematical documentation: the composition of the octave, the dimension of the whole-tone, and the fact of concordancy and discordancy. He might have added the important fact that the canonicians would accept musicians' assumptions so long as the logical aesthetic of the Pythagoreans was maintained, this being that only two types of ratios were deemed admissible into the canonic system: multiples and superparticulars.[70] Instead, Porphyry introduced something that is totally irrelevant in the present context: that musical pitch is produced by numbers of percussion on the air by vibrating bodies. It is an issue that was thoroughly explored by Euclid in his *Sectio Canonis* and falls beyond the range of Ptolemaïs' discussion.[71]

Ptolemaïs' range of discussion was not limited to the similarity of the evidence treated by the musicians and the canonicians, but, rather, to the different modes of reasoning which ruled on the admission of this evidence into their respective theories. All things considered, Ptolemaïs

[70] Ptolemy *Harm.* I. 6 (Düring, 13. 23–14.2). Cf. Barker, II, p. 287, n. 59, who directs us to Plato's criticism of the Pythagorean harmonicians in *Rep.* 531B8–C4: "They are just like the astronomers – intent upon the numerical properties embodied in these audible consonances; they do not rise to the level of formulating problems and inquiring which numbers are inherently consonant and which are not, and for what reasons" (trans. Cornford).

[71] Cf. note 36. On the role of percussion in the production of pitch, see Chapter 4. Cf. Barker, II, p. 240, n. 139. See also Levin, "*Plēgē* and *Tasis* in the HARMONIKA of Klaudios Ptolemaios," *Hermes* 108 (1980), 212–19.

was in effect stating that musicians were concerned primarily with *what is*, while the canonicians were concerned primarily to demonstrate in the most scientific terms possible *that it is so*. Thus, for example, a musician's knowledge of what a consonance is may not in the strict sense be a thought or a proposition (*logos*); it is more like an intimation, or a prompting of the musical mind. To a canonician, however, it is the effect of a numerical relation of a certain type. The kinds of ratios involved in the articulation of consonances and dissonances thus make up the constituents of canonic science. In stating these points. Ptolemaïs draws our attention once again to Aristotle, even to adopting his mode of discourse (question and answer) as well as his terminology. Thus, Aristotle:[72]

> What is a consonance?
> A numerical ratio of high and low pitch.
> Why is the high note concordant with the low one?
> Because a numerical ratio obtains between the high note and the low one.
> Can high and low be concordant? Does their relationship consist in numbers?
> Accepting that it does, what then is the ratio?

Ptolemaïs' response to questions such as these is an implicit "yes." Yes, relationships such as octaves, fifths, fourths, and whole-tones can be represented in "the rational postulates of the canon," these being some of the elements of which melody is constituted. But melody contains much more than these, much that cannot be obtained from mathematics. As her few words suggest, then, harmonic knowledge proceeds from perception through induction to the first principles of musicians, and back through deduction to the conclusions of mathematicians from these first principles. Ptolemaïs saw that the end-point of musicians' induction is the starting-point of the canonicians' demonstration.[73]

[72] Aristotle *Post An*. 90a19–23.

[73] As Ptolemaïs suggests here, canonics is a deductive science, and every deductive science, canonics in particular, rests on a certain number of indemonstrable axioms. But there is a distinction to be drawn between the intuitions of musicians and the axioms of canonic science. Thus, for example, when Aristoxenus asserts in *Harm. El*. II. 45 (Da Rios, 56. 10–12) that if any consonance be added to an

She also saw that, owing to their differences in structure, the two doctrines – that of the canonicians and that of the Aristoxenians – were headed for a philosophical collision. On the one side, that of musicians, there was limitless resourcefulness; on the other, there were the inherent limitations of canonic theory. For, as Ptolemaïs perceived, the hypotheses of musicians comprehend only those things that can be expressed melodically, while the canonicians will accept only those things that can be expressed in multiple or superparticular ratios. But, as any musician would see immediately, these, "the rational postulates of the canon," are too limited in their scope to provide a theoretical understanding of the complexities of melodic phenomena. Taking one example – the ditone: to a canonician, mathematical reasoning (*logos*) defines the ditone as 9:8 x 9: 8 = 81:64 (the Pythagorean major third); but to a musician, it is musical logic (Aristoxenus' *dianoia*) that defines the ditone as the outstandingly characteristic interval of the Enharmonic genus.

Ptolemaïs was scrupulous, therefore, in distinguishing between the criteria employed by *Kanonikē* and *Mousikē*. For, as she saw it, because the objects of each science were decidedly different from one another, the criteria, being specific to these objects, had also to differ from one another. Pushing her analysis of these criteria further, Ptolemaïs ended up treating the rational postulates of canonic theory as just so many toilsome justifications of a doctrine that was, as Aristoxenus charged, irrelevant to the subject: *Mousikē*. But Ptolemaïs did not stop there. She charged *Kanonikē*, and the Pythagoreans who practiced it, with the most egregious crime against science: the facts being treated were forced to fit the theory being postulated.[74] It is not coincidental that the

octave, the sum is a consonance, this is not a proposition in canonic science, but one that is proper to musical analysis specifically.

[74] In charging the Pythagoreans with having violated the most fundamental rule of scientific inquiry – saving the phenomena – Ptolemaïs again directs us to Aristotle *Met*. 986a3–8, where he charged the Pythagoreans with altering the evidence in the interests of their theory. Cf. Harold Cherniss, *Aristotle's Criticism of Pre-Socratic Philosophy*, pp. 223–25. Ptolemaïs echoes Aristotle even more closely, her words carrying the same content as his in *De caelo* 293a25–27, where he says of the Pythagorean invention of a counter-earth to make the planets add up to the perfect number, ten: "They did this not by seeking arguments and reasons to explain the phenomena, but by

basis for her criticism is detailed by a musician for, according to Paul Hindemith, even though he spoke centuries later, nothing about music has changed in this respect:[75]

> Generally, a musician is not too fond of the sciences, especially of those that in his opinion have no connection with music. Physics he perhaps allows to have its say, since he is well aware of the acoustical conditions of his art. At mathematics, however, he looks with scorn, because in his opinion the obvious exactitude of this science cannot be reconciled with the artistic liberty of musical creation.

To reconcile the artistic liberty of musicians with the exactitude of musical logic, Aristoxenus had to invent a new system, one that would be the least artificial in harmonic structure, but, at the same time, the most convenient for melodic composition. This system had, therefore, to be as unlike the canonic theory of the Pythagoreans as Aristotle's mathematics was unlike that of Plato's. For the Pythagoreans, pure mathematics – including the science of canonics – described strictly mathematical forms and the relations between them. In contrast, Aristoxenus' system would be an applied form of mathematics, one that described empirical objects and their relations to the extent, at least, that they approximated the mathematical form on the canon. One might say, then, that Aristoxenus' system of approximation is the converse of the canonician's idealization of intervallic reality. As Ptolemaïs evidently realized, the propositions of canonic science were regarded by canonicians as necessarily true, because they described unchangeable relations (intervals) between unchanging objects (notes). Moreover, as she explains, the necessity of these truths was treated as independent of their apprehension by the ear and independent, in fact, of any preliminary acts of canonic measurement. Her implication is that construction of intervals on the canon was not essential to prove, say, that the octave is indivisible; such constructions served merely a practical need of the canonician to guide him in his progress toward harmonic discovery.

forcing the phenomena and trying to make them fit their own arguments and opinions."

[75] Hindemith, *A Composer's World*, p. 29.

Despite her criticisms of Pythagorean doctrine, Ptolemaïs could not but concede that their theory of harmonics had a powerful backing from pure mathematics. But, as she made clear, this mathematical method applied in her view only to what the Pythagoreans kept in of the melodic phenomena, not to what they left out. For what they left out is the fact that perception denies those very items which reason, independent of perception, accepts as true. Some of the more obvious examples of such discrepancies between mathematical imperatives and the ear's judgment occur in the case of the multiple ratio, 5:1. This is a consonance according to mathematical standards, but it registers on the ear as a double octave and a major third – a dissonance. So, too, the superparticular ratio, 5:4, which should be a mathematically legitimate consonance, is apprehended by the ear as a major third – a dissonance. But Ptolemaïs had much more on her mind than discrepancies such as these. She was bent, rather, on comparing Aristoxenus' theory with that of the Pythagoreans, not so much to discredit theirs where music is concerned, as to promote his. To judge from the few words of hers that Porphyry chose to quote, Ptolemaïs was trying to explain in what respects Aristoxenus' doctrine was theoretically sound.[76]

To begin with, she points out that Aristoxenus differed from the Pythagorean canonicians in placing perception and rational thought on an equal basis. To be sure, as she acknowledges, perception, on Aristoxenus' standard, comes first in the order (*taxis*) of events, but it is not first in terms of its function or power (*dynamis*). For perception, operating on its own, is incapable of providing a rational basis for a genuine theory. Inasmuch as her quote from Aristoxenus to this effect is not to be found in his extant writings, it may be assumed that Ptolemaïs had more of his writings before her than have come down to us. Those of his words that she quotes refer specifically to the

[76] Cf. Richter, *Zur Wissenschaftslehre von der Musik bei Platon und Aristoteles*, pp. 178–79, who sees Ptolemaïs' words and those of Didymus (who followed her) as based on the arguments of Aristoxenus against the Pythagorean method in *Harm. El.* II. 32–33.

phenomena of melody, these being the perceptible objects of which he speaks:[77]

> Whenever a perceptible object, whatever it may be, makes contact with this [*sc*. perception], then we must promote rational thought to the forefront, in order to get a theoretical understanding of this [*sc*. perceptible object].

The rational thought, or *logos*, as Ptolemaïs quotes it here, has nothing to do with the logical postulates of the canon. On the contrary, it is a strictly musical type of reasoning which comes into play when the question of a note's or an interval's function (*dynamis*) arises. For the life of every note or every interval takes its meaning and its justification from the function that it performs as a member of the melodic whole. Thus, what is omitted in Ptolemaïs' representation of Aristoxenus' theory is complemented by Aristoxenus himself:[78]

> As a whole, our theory is about all musical melody, be it in song or on instruments. Our treatment is referred to two things: to the ear and to reason (*dianoia*). We judge the sizes of intervals by ear, but we get a theoretical understanding of their functions by our reason.

Dianoia – musical reasoning – is pivotal to Aristoxenus' theory. And somewhere in her writings Ptolemaïs may have explained how it operates in identifying the various functions of notes and intervals.[79] Without that explicit information, we are left to infer

[77] On this quotation, see note 45. Cf. Barker, "Music and Perception: A Study in Aristoxenus," 10–12, on the nature of the phenomena under analysis.

[78] *Harm. El*. II. 33 (Da Rios, 42. 8–13).

[79] Ptolemaïs seems to have understood far better than other Aristoxenians that the agreement between knowledge and its object where music is concerned consists in an identity between the act of thinking and the object of thought. To put it another way, it is an identification of an intelligible form with its defining characteristics, these latter being in the soul itself. Cf. Aristoxenus *Harm. El*. II (Da Rios, 51. 16–52. 4). These notions are made explicit by Aristotle, who says of sound in *De anima* 425b26–28: "The action of the perceptible object and that of perception is one and the same. I refer to the case of sound

from the writings of the master himself that *dianoia* comprehends an immense range of human musical sensibility, as immense a range as that required to see, for example, into the life of a poem. Where melody is concerned, *dianoia* is a triangulation of intellectual forces whose combined effect produces magnetic fields of understanding that react now to one note and now to another. *Dianoia* responds to the way in which notes in musical space attract, repel, and interact with one another, these activities depending on their respective functions.[80] It is for such reasons that Aristoxenus insisted that a melody is no mere jumble of notes, but a statement of logic as comprehensive as that of a sentence. The logic of melody depended in Aristoxenus' theoretical scheme on the continuity of musical space, the rationality of melodic functions, and the permanent consonantal moorings of melodic correspondences. The logic of melodic continuity being paramount in Aristoxenus' estimation, he stipulated that despite everything asserted by mathematicians as to the divisibility

in its activity and hearing in its activity." These activities are one, on the basis that the defining characteristics of melody and the intelligible form it takes in the mind of the listener are identical. See Sorabji, *Time, Creation and the Continuum*, pp. 144–46.

80 In her criticism of instrumentalists, Ptolemaïs argued that their interests were practical and not intellectual or conceptually theoretical (*ennoia theorias*). Cf. note 43. Of the various types of musical knowledge, *ennoia*, on Ptolemaïs' construction, would correspond to that faculty of the mind that retains concepts, while *dianoia*, as in Aristoxenus' lexicon, corresponds to the deliberative, or noetic, intellect, which is capable of discursive or syllogistic reasoning. Ptolemy *Harm*. III. 5 (Düring, 96. 21–27) actually distinguished seven types of musical knowledge to match the seven species of the octave: imagination (*phantasia*), which is concerned with the reception of sensible objects; intellect (*nous*), which registers the first impression of things; conceptual (*ennoia*), which retains the form of a thing in its totality; rational thought (*dianoia*), which is designed for inquiry and research; opinion (*doxa*), which is designed for superficial conjecturing; abstract reasoning (*logos*), which is concerned with correct judging of things; scientific knowledge (*epistēmē*), which is concerned with truth and the comprehension of things (*katalēpsis*). All of these faculties come into play where music is concerned. Cf. Barker, II, p. 376, n. 44; Solomon, *Ptolemy*, p. 146, n. 114. Cf. Johnson, "The Motion of the Voice," *TAPA* 30 (1899), 55.

or indivisibility of any musical interval, it is melody alone which divides musical space into the maximum number of parts it can occupy so as to be intelligible to the musically cognizant ear (another term for *dianoia*). As he maintained, melody, if it is truly musical, is nothing if not logical in the way it divides into parts the space that it occupies:[81]

> We can trust in the fact that there is no interval which we play or sing that we cut up into infinity. On the contrary, it is on the basis of melody that we determine a certain maximum number into which each of the intervals is divided. If we assert that this not only merits our trust, but is in fact a necessity, it is obviously from the fact that musical notes that succeed one another consecutively comprehend fractions of the aforesaid number. Examples of such notes are those we have been playing since antiquity, as, for example, the *nētē*, the *paranētē*, and the notes that succeed them.

If Aristoxenus had been as fortunate as Pythagoras in having had an Archytas or a Philolaus to offer an account of his doctrine, the profundity of his insight into the nature of melody would be better appreciated today than is currently the case. As matters stand, we can recover this much: Aristoxenus' method for arriving at a proper attunement of melody involved a process of dividing continuous magnitudes by using as a natural unit of enumeration a quantum which was itself indivisible. This process was motivated by his views on the distinguishability of melodic function (*dynamis*). Unfortunately, Aristoxenus" own account of melodic function and its bearing on the arithmetization of melody's continuous magnitudes has not come down to us. But, to judge from certain relevant and provocative statements that appear in his *Harmonic Elements*, his concept of the infinite with respect to the division of melodic space must have come to him the moment he unhinged his attunement from the bonds of the canonic ratios. Under this impetus, he contrasted his own method with that of the Pythagorean canonicians in these words – words that

[81] *Harm. El.* II. 53 (Da Rios, 66. 14–67.3).

betray his frustration with the reception that was being accorded his theory of attunement:[82]

> First of all, we must be quite aware of this: many have misunderstood us in supposing us to say that melody admits the division of the whole-tone into three or four equal parts. This misunderstanding is due to their not observing that to employ the third part of the whole-tone is a very different thing from dividing the whole-tone into three parts and singing them. Secondly, taken in the abstract, we assume no interval to be the smallest.

As Aristoxenus had explained earlier, all of his statements regarding maximum and minimum limits on the line of pitch should be construed with reference to these factors only: the capabilities of the human voice and ear. He had these factors very much in mind in the statement quoted earlier. For there he was concerned to contrast the division of the whole-tone into three or four equal parts by mathematical means – an impossibility – with the employment by the voice of thirds and fourths of whole-tones – a melodic necessity. He brings this contrast into sharp focus by suggesting that it is one thing to compute the fourth part of a whole-tone as a mathematician would, as 28:27, for example; but it is quite another matter to employ the fourth part of a whole-tone in song or on an instrument in the Enharmonic genus by having it approximate three twelfths of a whole-tone, an approximation which the voice is capable of making. So, too, it is one thing to compute a third of a whole-tone mathematically at 15:14, say, but it is quite another thing to employ a third of a whole-tone in song or on an instrument in the soft or flat chromatic nuance by having it approximate four twelfths of a whole-tone, a micro-interval that can be recognized by the ear. Aristoxenus' point, then, is that, taken in the abstract, there can be no smallest (or greatest) interval, melodically speaking, save that which the voice cannot sing or the ear cannot identify.

On the one standard, then, that of melody, all intervals smaller than the whole-tone can be spoken of (*rhēta*) because they are thinkable

[82] *Harm. El.* II. 46 (Da Rios, 57. 6–12).

and, in that sense, rational.[83] On the other standard, however, that of mathematics, all intervals smaller than the whole-tone cannot be thought or spoken of (*aloga*), because there is no whole-number ratio by which they can be expressed. The inference is that in music, rationality has to do with the kind of reasoning called *dianoia* by Aristoxenus, a level of thought that not only generates phenomena such as quarter-tones and thirds of whole-tones, phenomena which are connected with its own cognition, but also accepts the effects of these phenomena as they are given in sensory perception. Thus, where music is concerned, what is rational is of human origin; and what is irrational must be the work of nature. This is, of course, to reverse the universal order of things because, as Ptolemy had expressed it, there is nothing in the works of nature that is irrational or done at random.

Once musical space was shown by the Pythagoreans to be *alogos* by being mathematically indivisible, the fault, as they saw it, could not be assigned to nature, but to man's own disposition of the melodic elements. To pursue Aristoxenus' doctrine, however, this state of things had in reality to derive from man's own intuition of musical space as homogeneous and from his construal of the melodic functions as melodically imperative. As if to prove as much, Aristoxenus' intention was to chart every specific route and generic artery of melody's topography directly onto a symmetrical grid which he conceived to represent melody's ear-borne topology.[84] It was there that he located the very center of music's emotional gravity. This mysterious force, or gravitational pull, issues forth on melody's contact with the rational topography of

[83] Thus, Macran, *Aristoxenus*, p. 238: "An interval [in Aristoxenian terms] must be rational or irrational in virtue of the relation it bears to some quantum outside itself."

[84] Such a grid is worked out by Macran, *Aristoxenus*, p. 249. As this grid shows, Aristoxenus' quanta form a domain which is reached by thought (*dianoia*), whose acts find fulfillment in giving an account (*logos*) of its operations and in reckoning up (*arithmein*) their totality. Comparing Aristoxenus' method to that of the mathematical theorists, West, *Ancient Greek Music*, p. 168, makes this important observation: "Aristoxenus' approach is very different. According to him, the two inner notes of the tetrachord can be pitched anywhere within a continuous band, and it is necessary to lay down boundaries to demarcate one genus from another."

human origin – the evenly distributed attunement of mathematical continuity – and the irrational topology of physical origin – the harmonic series[85] that is habituated in mathematical discontinuity.[86]

According to Aristoxenus, then, intervals are rational if they have a legitimate function to discharge in melody; and the only arena in which this function can work in a musically perceptible way is the melodic continuity of a mathematical *continuum*. Musical reasoning, or *dianoia*, can detect any violation of this most fundamental rule of melodic continuity (*hexēs kata melos*) that was laid down by Aristoxenus: the correspondence of the interval of a fourth between every note and the fourth note distant from it; and the correspondence of the interval of a fifth

[85] There is no certain evidence that the ancients were aware of the harmonic series – the set of notes (called 'harmonics') that issue from pipes and strings on the sounding of a "fundamental," the notes produced by an entire air-column or an entire string. There is a reference by Aristoxenus *Harm. El.* I. 21 (Da Rios, 27. 1–3) to what may be a 'harmonic" produced by the over-blowing technique of a virtuoso on the *aulos*. The word syrinx used here by Aristoxenus is interpreted by Schlesinger, *The Greek Aulos*, p. 54, to signify the mouthpiece of the single-reed *aulos*. But see West, *Ancient Greek Music*, p. 102. On the harmonics of strings, see Roederer, *Introduction to the Physics and Psychphysics of Music*, pp. 90ff.; on those of winds, pp. 117ff. The Pythagoreans did seem to intuit that the closer the relations are to the fundamental note (isotone?), the more consonant the intervals produced.

[86] Just as Superstring theory seems to resolve the conflict between general relativity and quantum mechanics, the juxtaposition of equal temperament on the harmonic series of physical nature appears to create a conflict between the notes of melody, a conflict that is as hard to explain as the workings of the universe. Thus, harmony, in the modern sense, and the tensions and resolutions between the notes of melody (*epitasis* and *anesis* in Aristoxenian theory) derive from the conflict between equalized intervals and those of the harmonic series. Cf. Cooke, *The Language of Music*, pp. 40–41. The most remarkable thing about the harmonic series is that its numbered sequence gives us the ratios of frequencies of all the intervals produced from a fundamental tone. Thus, if one numbers the notes of the series from one to sixteen, one finds that the upper note of any octave always doubles the frequency of the lower note. Accordingly, the notes represented by 1, 2, 4, 8, 16 are octaves, while 4 and 5 gives the ratio of the major third, and 8, 9 give the whole-tone. See Fig. 10.

between every note and the fifth note distant from it.[87] Any deviation from this rule would not only upset the attuned *continuum*, but would have necessarily to result in an unmusical (*ekmelos*) melody. Aristoxenus gives a vivid example of such a violation against the *continuum*, where he

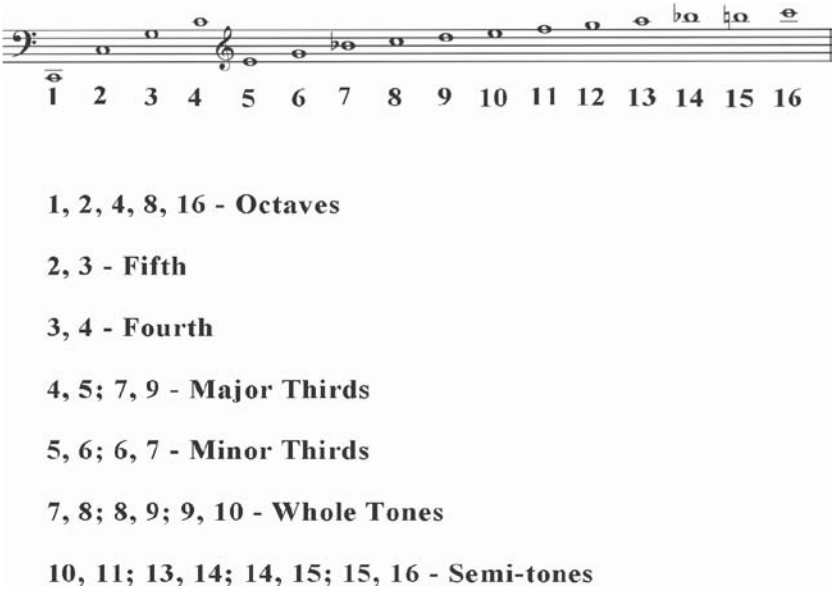


FIGURE 10. The Harmonic Series

At the same time, harmonics from 4 to 10 contain two kinds of major thirds (4:5 and 7:9), two kinds of minor thirds (5:6 and 6:7), and three kinds of whole-tones (7:8, 8:9, and 9:10). See Maconie, *The Concept of Music*, pp. 110–11. These, the facts of acoustical nature, are met by anyone building a simple panpipe or playing a complex *aulos*, and they challenge the musical mind to seek a solution. It is in fact possible that Archytas came upon the harmonic series in the course of his experiments with the panpipe, for as his computations show, he had located the major third in the ratio of 5:4, the 4th and 5th partials of the series. Cf. Barbera, "Arithmetic and Geometric Divisions of the Tetrachord," *Journal of Music Theory* 21 (1977), 296. Cf. Barker, II, pp. 43–45. Fokker, "On the Expansion of the Musician's Realm of Harmony," *Acta Musicologica* 38 (1966), 197, speaks of the duodecimal temperament as man-made: "It is a compromise between the finite technical power of man and his inmost desire to realise the highest musical truths attainable in his songs and instrumental playing."

[87] See Chapter 5, note 96.

describes a sequence in which one *pycnum* (= two quarter-tones) directly follows another, in other words, a sequence of two *pycna* would result in the unmelodic: E E + F F+ Gb. Aristoxenus observes:[88]

> A *pycnum*, neither a whole one nor a part of one is to be played or sung next to another one. For the result will be that notes that are fourth from one another will not form the consonance of a fourth, nor will notes that are a fifth from one another form the consonance of a fifth. Notes thus situated have been deemed unmelodic.

In confronting a sequence such as that described above by Aristoxenus, the ear and *dianoia* do work as one: for the ear, the sequence would sound utterly displeasing and would accordingly be interpreted by *dianoia* as having violated the limiting law of tetrachords, that of the consonantal fourth, or E–A. The four notes in this particular case must strike the ear as unmelodic since, as *dianoia* would explain, they occupy an ill-defined interval: E–Gb. An even closer alliance of the ear and *dianoia* is most urgently required on the advent of *metabolē*, or modulation, this being a change that is as essential to melody as metaphor is to language, to poetic language, especially. Poetic metaphors have to be rationalized, because they are expressed typically in language that is semantically deviant. So, too, melodic modulations must be rationalized (by *dianoia*) against an established harmonic, or well-attuned structure to which the ear has become thoroughly trained. In processing a metaphor, we experience and require ourselves to interpret one thing in terms of another. Similarly, modulation, in its simplest form, requires us to experience and to process the function of a melodic element in one setting in terms of its function in a different setting. The settings in question may be scale structures, pitch ranges of the voice, modes, or keys.[89] Speaking

[88] *Harm. El*. III. (Da Rios, 78. 13–16).

[89] Modulation was evidently as important to the ancients as it is to all composers of great and lasting melodies (one thinks instantly of Schubert, one of the special masters of the technique). According to Ptolemy *Harm*. II, 6 (Düring, 55. 3–6), there is modulation by change of key (what we today call transposition) and modulation in which "the melody is turned aside from its proper ordering, while the pitch [*tasis*] is not altered as such, but as having an effect on the melody" (trans. Barker). For discussion, see Barker, *Ptolemy*, pp. 169–72.

of modulation in the modern diatonic system, Richard Norton has observed:[90]

> Modulation, true modulation, means a complete change from one key to another, which happens in three ways: 1) modulation by changing the meaning of a harmony; 2) modulation by chromatic change; and 3) modulation by enharmonic change.

Metabolē and *metaphora* both mean "change"; the one, *metabolē*, signifying a change in melody, the other, *metaphora*, a change in meaning. Aristoxenus' all-too-few references to *metabolē*, there being only four such in his *Harmonic Elements*,[91] teach us something well beyond the isolated facts of the canon; true melodic change (*metabolē*) can only be

As Solomon, *Ptolemy*, p. 77, n. 92 explains: "Ptolemy means that the change in pitch does not simply carry the old succession of notes at a different pitch; in fact, the pitch is almost irrelevant, especially compared to the importance of pitch in modulation by tonos." To state it another way, modulation has to do with an identity of pitch and the change of its function in melodic formation, for every modulation, whatever its type, demands some common element.

90 Norton, *Tonality in Western Culture*, p. 42.

91 In his first reference to the subject of modulation in *Harm. El.* I. 7–8 (Da Rios, 12. 15–18), Aristoxenus speaks of the most critical requirement for modulation, a requirement that not one of his predecessors saw fit to discuss or, as he suggests, were too ignorant to consider worthy of discussion: the symbiosis or affinity (*oikeiotēs*) between scales, regions of the voice, and keys. Although Aristoxenus' own examination of this critical factor in modulation has not come down to us, his implication is clear enough: without such an affinity or intimacy between scales, regions of the voice, and keys, there is no possibility for modulation to occur. In his second reference to modulation in *Harm. El.* II. 34 (Da Rios, 43. 13–15), he speaks of melodic intervals that are identical, but whose limiting notes differ in their melodic functions, these changes in the functions of common elements leading to modulation. In his third reference in *Harm. El.* II. 38 (Da Rios, 47. 17–48.3), Aristoxenus promises to define modulation, to explain how it arises, and to tell how many kinds there are, none of these topics ever having been discussed by his predecessors. Aristoxenus' own discussion of these subjects does not survive. His fourth mention in *Harm. El.* II. 40 (Da Rios, 50. 14–51. 3) concerns the differences between modulating scales and non-modulating scales, and their relevance to different melodic styles (*tropoi*) of composition. Aristoxenus concludes his remarks here by referring to the profound and powerful ignorance of his predecessors.

explained by true melodic permanence.[92] The two things that remain constant and permanent in his harmonic doctrine are the *Systēma Ametabolon*, the unchangeable or immutable scale, and the permanent quantum (*menon ti megethos*) that makes the construction of the *Systēma Ametabolon* possible. As he explains, unless there is a permanent quantum to deal with the division and arrangements of the various elements involved in melodic composition, no distinctions can be made between the ways in which these elements may function. He says accordingly:[93]

> It is obvious that the distinctions between the divisions and their forms (*schēmata*) are predicated on a certain permanent quantum.

On the advent of modulation, the ear – especially the trained ear – perceives that a note's function must be intrinsically different from what it would be if it were not modulating. But the construal of this advent is a conceptual exercise of the musically cognizant mind (*dianoia*), a construal that has been induced by a novel melodic transition. Aristoxenus thus realized, where other theorists did not, that the ear and reason must operate as one. Moreover, as he saw it, the ear and reason must deal simultaneously with permanent as well as with changeable elements, for modulation involves both at the same time.

As Ptolemaïs of Cyrene evidently understood, then, there are certain melodic situations that cannot be explained by the detached facts of the canon, and modulation is one of the most vivid of them. When Ptolemaïs pointed out, therefore, that Aristoxenus employed perception and reason on equal terms for the interpretation of melodic phenomena, it was in recognition of a fundamental fact at the heart of Aristoxenian theory: he did not work with experiment alone; he worked also with method. It was this method that demanded the inseparable alliance of ear and mind which enabled him not only to make melodic generalizations, but also

[92] *Harm. El.* II. 33 (Da Rios, 53. 3–6).

[93] *Harm. El.* II. 34 (Da Rios, 43. 19–44. 1). There is more than a hint of Aristotle's thoughts on change in this statement, especially in Physics 190a13, where he speaks of the necessity for something always to underlie change. Cf. Barker, II, p. 152, n. 19.

to make melodic predictions. Ptolemaïs' quotation from Aristoxenus thus bears repeating if only to indicate that the conflicts between the facts of the canon and the percepts of the ear forced Aristoxenus to look at the melodic phenomena under different aspects, for he refused to be a slave to the canonic ratios:[94]

> Whenever a perceptible object, whatever it may be, makes contact with this [*sc*. perception], then we must promote rational thought to the forefront, in order to get a theoretical understanding of this [*sc*. perceptible object].

Without the ear and musical cognition working together in the manner described by Aristoxenus (via Ptolemaïs), it is altogether impossible to deal with something as subtle as modulation. Of the four references to modulation by Aristoxenus, the saddest is the one in which he promises to deal with modulation in the abstract, to explain how it comes about, and to specify how many types there are in the composer's musical lexicon.[95] This promise is not fulfilled in Aristoxenus' extant writings. But in a discussion about intervals, he refers to the fact that beyond the ear's ability to judge the sizes of intervals, it is unable in itself to deal with certain melodic distinctions, as, for example, between composite and incomposite intervals. For, as he explains further, knowing the difference between the sizes of intervals is not sufficient for distinguishing between a simple melody and one that contains a modulation, or between one mode (*tropos*) and another. The reference that has provoked the greatest dispute amongst scholars is the one in which Aristoxenus says:[96]

> We must speak of the symbiosis (*oikeiosis*) of scales (*systēmata*) and regions of the voice (*topoi*) and keys (*tonoi*), not by focusing on the tables of micro-intervals (*katapyknoseis*) as the harmonicians do, but on the melody as

[94] Porphyry *Commentary* (Düring, 25. 23–26).
[95] See note 91.
[96] *Harm. El*. I. 7–8 (Da Rios, 12. 8–18). See Winnington-Ingram, *Mode in Ancient Greek Music*, pp. 74ff.

> it moves from one system to another, and on what the keys are in which it lies for the melody to move from one to another.

After criticizing his predecessors for barely touching upon these critical issues, Aristoxenus concludes:[97]

> But, to speak in general, this is the part of the study of modulation that pertains to the theory of melody.

Aristoxenus' theory of the symbiosis between scales, modes, and keys depends ultimately on the permanent quantum which he extrapolated from the ear's perception of continuity in melody. It is true, as he acknowledged, that his permanent quantum is itself not an immediate fact of perception. But, as has been argued here, it can be viewed as an appropriately selected unit of measure for approximating the primary fact of perception: the melodic *continuum*. Even though it is mathematically disconnected from the canonic ratios, it can nonetheless be used effectively and interchangeably with the empirical propositions of melodic theory. It is the purpose, therefore, which Aristoxenus' permanent quantum served that makes it acceptable. This purpose was to represent melodic concepts by ordered classes of quanta (twelfths), their instances in melodic structures by elements of these classes, and their relations in melodic forms by an arithmetization of continuous magnitudes. It was this process – in reality, a replacement of the purely exact mathematics of the canon with the approximately exact arithmetic functions deduced from empirical premises – that enabled Aristoxenus to do what he said had to be done:[98]

> It is necessary to the science of music to assign and to arrange everyone of the elements in music in accordance with the place to which it is limited, and if it is infinite, to let it be.[99] As regards the magnitudes of the

[97] *Harm. El.* I. 8 (Da Rios, 12. 17–18).

[98] *Harm. El.* III. 69 (Da Rios, 86. 6–12).

[99] Macran and Barker translate Aristoxenus' word *apeiron* as "indefinite" and "indeterminate," respectively. As argued here, however, the problem that Aristoxenus faced was to reconcile those elements that are fixed by the laws of melodic consecution – the functions (*dynameis*) of notes, the species (*eidē*) of consonances, the positions (*thēseis*) of the moveable or interior notes of the

> intervals and the pitches of the notes, there seem somehow to be elements pertaining to melody that are infinite; but as regards their functions and their species and their positions, they are both limited and prescribed.

The domain of music is "illimité."[100] As Aristoxenus explained, it is potentially infinite in both directions. – *in magnitudinem* and *in parvitatem*. To deal with the infiniteness of music's domain, Aristoxenus focused on the process that generates it: the infinite divisibility of musical intervals. He began by accepting the infinity of the melodic *topos* as a given, and agreed to "let it be." His doing so was an admission that any well-attuned synthesis of the melodic *topos* by the geometric method of canonic science would be an impossibility, for such a synthesis could never succeed in accommodating all of the forms and genera of melody, this owing to the infinite divisibility of the melodic *topos*. At the same time, he intuited that even though the melodic *topos* was in itself illimitable, there was at any given moment in song some finite segment of it that could be represented to the ear's satisfaction. He found such finite segments to be dependent on the unchanging tonal functions that are productive of the various genera and species of melody. But when it came to expressing these finite segments by traditional geometric means, he ran head-long into the mathematical problems described earlier as a discontinuity between rational numbers.[101]

The geometric method, as Ptolemaïs of Cyrene realized, belongs to a domain utterly different from that of music: the domain of vision. This method, as practiced by the canonicians and handed down through succeeding centuries, was strictly metrical and became the official norm for all artistic endeavors. It was not until the Renaissance that artists were urged to reinvent metrical geometry if they hoped to represent what they intuited to be true of the domain of vision: an infinite vanishing

tetrachord – with the mathematical infinity of musical space. His motivation in this effort was to ensure that the resulting melodic segments of his divisions each had its own proper terminus.

[100] Cf. Chapter 5, note 53.

[101] To escape the contradiction between the empirical data of melody and discontinuity within the canonic ratios, Aristoxenus was led to create what he intuited to be true: a *continuum* in which every note takes its place just as in a series of integers or units.

point in three-dimensional space. To accommodate this phenomenon, artists had to invent a perspective geometry so as somehow to produce a "there" – a there that would impress the eye as an acceptable relation between objects.[102] In other words, artists had to create the unbroken continuity of different visual fields within the fixed limits of the canvas' finite dimensions. They had in effect to project themselves straight through the canvas into infinity. The geometric proportions between objects had, therefore, to be their principal concern. But such proportions would be impossible to achieve without a fixed unit of measure. The one most commonly used was the square.[103] Whatever the unit of measure and however complex the perspective methods employed, artists know for a certainty that in the end all paintings are a compromise with what the eye actually sees.

Aristoxenus recognized and accepted the same thing where the melodic *topos* was concerned. All attunements are a compromise with what the ear actually hears: a *continuum* of musical sound that escapes all traditional geometric representation. His solution was as ingenious as that of the Renaissance artists; he invented an arithmetic method that would reflect as nearly as possible the continuity of the melodic *topos*. It was a system of elements whereby one could pass from any one species of melody to another, from any one genus to another, and from any one *tonos* to another. It evolved from Aristoxenus' unique manipulation of the quantum, or unit of measure, to create a series of consecutive elements such that one was the equal of every other.[104]

[102] This sort of relation has been most aptly characterized by J. V. Field, *The Invention of Infinity*, p. 59 as "visually right and mathematically wrong."

[103] See Field (note 102), pp. 192–94 on the method of Girard Desargues, especially p. 194, Fig. 8.14.

[104] This made it possible for all the modulations to occur that are mentioned by such Aristoxenians as Cleonides *Introduction to Harmonics* 13 (Jan. 204. 19–206, 18); by genus, by system, by key, by melodic composition; Bacchius *Introduction to the Art of Music* I. 50 (Jan. 304. 6ff.); by system, by genus, by mode (*tropos*), by ethos, by rhythm, by tempo, by rhythmic position; Aristides Quintilianus *De mus*. I. 11 (Winnington-Ingram, 22. 11ff.). See Barker, II, p. 424, n. 126; Mathiesen, *Aristides*, pp. 88–92.

Thus, where painters have had to create by geometric means the infiniteness of perspective within the limits of a canvas' finite dimensions, Aristoxenus was led by his calculations of fixed units of measure to create the finite forms of melody from the infiniteness of the melodic *topos*. The forms (*eidē*) with reference to which he created his attunement belong to melody; for melody, like a soul which gives form to a body, is the end (*telos*) of all attunement.

8 The Infinite and the Infinitesimal

> The phenomena of melody
> seem to be infinite with respect
> to the sizes of musical intervals
> and the pitches of musical notes;
> but as to their functions, forms,
> and positions, they are finite and prescribed.
>
> Aristoxenus, *Harm. El.* III. 69

IN ALL THAT PERTAINS TO THE SCIENCE OF MELODY, ARISTOXENUS WAS a man *per se*, a man who stood very much alone. As a musician, he was an avowed conservative, ever at war with any kind of innovation that he thought of as violating the ancient forms of the musical art. He speaks, for example, of two ancient styles (*tropoi*) which he regarded as especially beautiful, styles which his contemporaries were, in his eyes, either too ignorant to appreciate, or too avid in their pursuit of a chromatic kind of sweetness even to remember. But these ancient styles had to be preserved at any cost, as he saw it, with all their ethical distinctions intact.[1] He seems, in fact, to have accepted the Damonian-Platonic view that these distinctions were ethical in nature, inasmuch as they appeared to

[1] *Harm. El.* I. 23 (Da Rios, 29. 17–30. 5). Cf. Barker, II, p. 141, n. 90, who sees this passage as referring to "styles of music current before the middle of the fifth century." These ancient styles were evidently of the Enharmonic genus and its precursor, the *spondeion* of Olympos. Both are distinctive for the prominence that they give to the ditone, or major third.

have a decided effect upon the listener.[2] The problem for Aristoxenus, as stated above, was to provide an affinity (*oikeiotēs*)[3] between all the ancient styles of melody (*tropoi*), all the pitch ranges of the voice and instruments (*tonoi*), and all the scales and genera (*systēmata* and *genē*) of theory, such that modulations (*metabolai*) from one style, key, and genus to another could be easily effected. There was one proviso, however: such modulations would be acceptable only if they did not disturb or compromise the characteristic forms of the individual melodic modes, or *tropoi*.

Aristoxenus could not but accept the Pythagorean proofs of the consonantal magnitudes, for these proofs defined to a certainty the limits of the consonances: fourth, fifth, and octave. They also demonstrated that the whole-tone is the excess of the fifth over the fourth. The mathematical laws that followed necessarily from these geometrically defined magnitudes also showed incontrovertibly that six whole-tones cannot be made to fit mathematically into the space of an octave, there being always something left over; that the superparticular ratios defining the fourth and the fifth are themselves mathematically indivisible; consequently, that the octave, music's most perfect consonance, is in fact incommensurable with its own constituents.[4] These facts yielded wonderfully fruitful mathematical concepts in the form of arithmetic classes. There is, for example, one such class the squares of whose numbers are all smaller than 2; and another such class the squares of whose numbers are greater than 2. But in neither class can a number be found whose square is equal to 2. All these abstract formulations flow from the number $\sqrt{2}$, that number which emerges on the mathematical division of

[2] Aristoxenus' most telling reference to the ethical theory is in *Harm. El.* II. 31 (Da Rios, 40. 13–41.2), where he refers to one type of music as hurtful to the moral character and another type that improves it. Cf. Barker, II, p. 148, n. 6, where he says that it is not certain to what extent Aristoxenus endorsed the ethical theory, "but the present passage shows that he did not reject [it] altogether."

[3] *Harm. El.* I. 7 (Da Rios, 12. 8–12). The question of affinity is one of great importance and has been vigorously debated by scholars for decades. See, for example, Barker, II, p. 131, n. 34. As he explains, the *tonoi*, or keys, do not themselves incorporate differing arrangements of notes and intervals, but are identical interval sequences at different relative pitches.

[4] See Chapter 4.

the whole-tone. But all such mathematical repartitions had no relevance for Aristoxenus when it came to his perception of musical space.

For Aristoxenus, musical space was homogeneous and isotropic in that the melodic movement produced in one pitch range could be reproduced in another pitch range without any variation of its properties. Moreover, this movement, once produced, could be repeated again and again, so long as the voice or instrument could conduct it within its own inherent limitations. The case has been stated by Helmholtz in much the same terms as those of Aristoxenus:[5]

> Every melodic phrase, every chord, which can be executed at any pitch, can also be executed at any other pitch in such a way that we immediately perceive the characteristic marks of their similarity. . . . Such a close analogy consequently exists in all essential relations between the musical scale and space, that even alteration of pitch has a readily recognized and unmistakable resemblance to motion in space.

This meant that there was a genuine affinity, or as Helmholtz has it, a close analogy, between scales, keys, and modes. The goal of Aristoxenus' system was to maintain this affinity between the scales of abstract pitch and their mode-bearing segments, so that these segments, or octave-species, could be reproduced in any pitch range without any variation in their distinctive properties. For musicians, the immediate practical result of this endeavor would be the possibility of modulating from one mode or scale to another, and the ability to transpose one melody in one pitch range to the same melody in another pitch range. But to make for these possibilities, there would have to be an affinity, or close analogy, between the scales and musical space

Aristoxenus' predecessors had recognized the need to deal with the affinity between scales, keys, and modes, but their efforts were judged by Aristoxenus to be wholly inadequate. This, he argued, was because they had no fundamental principle in place to back up their constructions. If they managed to touch on the subject at all, what they produced was purely accidental and not scientifically verifiable. Ironically enough,

[5] *On the Sensations of Tone*, p. 370. Unfortunately, Aristoxenus' own systematic treatment of the connection between the *tonoi* (keys) and *systēmata* (scales) has not survived. CF. Barker, II, p. 126, n. 3.

the charges leveled by Aristoxenus against his predecessors' work, and the reasons for them, are identical to those leveled against Aristoxenus himself by his modern critics. His theory is judged to be unscientific primarily because it lacks mathematical rigor. Yet Aristoxenus accomplished what no one had ever succeeded in doing before him: he found a way of measuring the rough data of aural sensation. He proposed as a firm ground for his type of measure a system of elements such that each could not be distinguished from its predecessor. It would be a linear series that allowed the passage from any one of the elements to any other. It was a *continuum* of one dimension.

In Aristoxenus' science of melody, a vital distinction is made between numbers for counting and numbers for measuring. It is a distinction that, for all its apparent simplicity, is, in reality, one of the most profound responses of the human mind to the universe and all its attributes. It is the distinction that separates Aristoxenus' theory from that of the Pythagoreans. For the concepts with which Aristoxenus deals are limits and continuity, or, as mathematicians have it, the infinite and the infinitesimal. These notions go back to Zeno, who insisted that the motion of an arrow is really impossible, if at any given moment in time the arrow must be at some instant, and that at such an instant there cannot be any motion. The same case can be made, as Aristoxenus in fact does, with the singing voice. For at any given instant, the voice appears to the ear to have stopped on a point of pitch; if the voice moves at all, it does so between pitches by increments. The incremental quantity that Aristoxenus selected for his working measure was what he called *amelodētos*, the number 1/2.[6] He had to invent a term for that number, as there was no word in the language to describe it: a measuring number. To Ptolemy, as noted earlier,[7] this number seemed to be nothing more than a ghostly wraith, neither finite nor actual, in any sense of the word. Yet, with this number in place, Aristoxenus could travel in his musical imagination from one note to any other, from *lichanos Meson* (F), say, to *mesē* (A), passing easily over the whole infinitude of possible pitches lying between them. It is this idea of connectedness that informed Aristoxenus' concept of continuity and limit, making it possible for him to construct a homogeneous and regularly linked system of tonalities that offered all the possibilities for modulation.

[6] See Chapter 5.
[7] See Chapter 6.

Aristoxenus' method is a pragmatic one for solving the most difficult of musical problems: how to accommodate the individual modes, each with its own local characteristics, into a musical *topos* afflicted with its own mathematical incommensurability. For, mathematically speaking, a contradiction seemed to emerge from the heart of the musical universe, even though the elements of music appeared to Aristoxenus to move in one concordant and ordered society. The Pythagoreans had long since known of the problem and tried to resolve it by forcing musical space to conform to the laws of mathematics. Aristoxenus' achievement was to bring musical space into correlation with what the ear perceives to be true: the space through which melody moves is as symmetrical as time itself appears to be. Like a modern *savant*, Aristoxenus saw that the problem with musical space was comparable to that of terrestrial time. For time, through which the seasons move with unchanging regularity, is afflicted with the same sort of incommensurability as is musical space: no convenient whole number of lunar months can be made to fit into one solar year. That the earth's diurnal rotations do not synchronize with its revolutions around the sun is not apparent to the senses, however; it is only when the intellectual faculty attempts to make lunar calendars that the discrepancy is revealed.

Accordingly, in contemplating music, which exemplified to him a miraculous order of events, Aristoxenus saw in its actual practice the same chaos and confusion that were afflicting the reckoning of time in fourth-century Greece, and the harmonicians were, in his estimation, doing nothing to alleviate the problem. As he says:[8]

> The harmonicians' rendering of the tonalities is exactly like the observance of the days of the month; as, for example, when the Corinthians observe one day as the tenth of the month, while the Athenians reckon it as the fifth, and others construe it as the eighth. Thus, some of the harmonicians say that the lowest of the tonalities is the Hypodorian, that a semi-tone above it is the Mixolydian, that a semi-tone above this is the Dorian, that a whole-tone above the Dorian is

[8] *Harm. El.* II. 37 (Da Rios, 46. 20–47. 6). As Gevaert, I. p. 250, observed, the absence of unity and the rampant individualism on the part of the various musical masters and schools of music were characteristic of the whole pre-Aristoxenian era. As he put it: "Around the epoch of Alexander, the theory and nomenclature of the keys are found in an indescribable disorder. All the authorities had in matters of this sort a different practice; some admitted only five keys, others went up to six; as for a practice universally followed, it simply did not exist."

> the Phrygian, and that the Lydian lies another whole-tone above the Phrygian. Others add to the aforementioned tonalities the Hypophrygian aulos tonality at the bass. Still others, basing their method on the boring of auloi, separate the three lowest tonalities – the Hypophrygian, the Hypodorian, and the Dorian – by three quarter-tones from one another. And they set the Phrygian at a distance from the Dorian by a whole-tone and the Lydian from the Phrygian again at three quarter-tones, as they do the Mixolydian from the Lydian.

Aristoxenus concludes by observing of the authors of these systems:[9] "But what is the principle they are looking at that makes them so willing to distribute the tonalities in this way? Of this, they have nothing to say."

Unfortunately, Aristoxenus' own system has not come down to us. What we have instead is that of Cleonides, who cites Aristoxenus as his authority. According to Cleonides, Aristoxenus set up a system of thirteen *tonoi*, or scales of transposition, whose distribution over musical space is not unlike that of the fifteen keys in use today. Each is separated from the next by a semitone. Taking the note *mesē* as a basis for comparison, the thirteen tonalities, each being a paradigmatic scale, or System *Ametabolon*, transposed a semitone down, appear as follows:[10]

	Mesē
Hypermixolydian	F
Hyperionian or higher Mixolydian	E
Hyperdorian or lower Mixolydian	E♭
Higher Lydian	D
Aeolian or lower Lydian	C♯
Higher Phrygian	C
Ionian or lower Phrygian	B
Dorian	B♭
Higher Hypolydian	A
Hypoaeolian or lower Hypolydian	G♯
Higher Hypophrygian	G
Hypoionian or lower Hypophrygian	F♯
Hypodorian	F_1

[9] *Harm. El.* II. 38 (Da Rios, 47. 13–15).
[10] Cleonides, *Isag. Harm.* 12 (Jan, 203–4).

The framework into which Aristoxenus made everything fit is one of his own construction; but he did not construct it at random. He constructed by measurement, and that is why he was able to fit the facts – the modal facts – into his framework without altering their essential characteristics. Ultimately, it is the doctrine of limits that underlies Aristoxenus' harmonic structure, and not any conscious use on his part of the infinitesimal. This is to say that Aristoxenus' *amelōdētos*, his $\frac{1}{12}$ of a whole-tone, involves neither the infinite nor the infinitesimal; for it is itself not a sum, but simply and strictly, the *limit* of a sum. As Aristoxenus intuited, an octave, a fifth, and a fourth are relatively finite pairs, though all three consist of terms which are absolutely infinite.

The Pythagoreans had demonstrated that melody, like all natural processes, betrays a fundamental discontinuity when its elements can be measured with mathematical precision.[11] But Aristoxenus, to his credit, restored melody to its primordial character – continuity of motion – by creating a new convention of measurement that conforms to melody's condition of symmetry. To accommodate this symmetry, Aristoxenus adopted the concept of melodic space or *topos*, a space inhabited by homogeneous objects – notes, consonances, dissonances, functions, and species. Between these objects he established spatial relations – continuity and intervallic distances. He abstracted from the properties of these objects only those that are determined by their spacelike relationships. These relationships define what Aristoxenus conceived of as his Science of Melody.

[11] Time, the paradigm of all natural processes; time "the moving image of eternity," is converted by music into a continual present. The seamless "now" which music inhabits, does not exist in its own right, but is an amazing characteristic of music. And space, the infinite progenitor of the universe, has no boundaries between the notes of music. A paradigm of infinity, music draws its power from the space it fashions for itself. Music thus astonishes nature: for what is otherwise inscrutable and ineffable, escapes through the mediation of music into the sphere of human existence and becomes the wonder-provoking phenomena that Aristoxenus so vigorously managed to describe.

ΣΦΡΑΓΙΣ

Εἰπέ τις, Εὐφίλητε, τεὸν μόρον, ἐς δέ με δάκρυ
ἤγαγεν, ἐμνήσθην δ᾽ ὀσσάκις ἀμφότεροι
ἥλιον ἐν λέσχῃ κατεδύσαμεν· ἀλλὰ σὺ μέν που
ἥρως ἀγαθὸς τετράπαλαι γενόμενος ἀπέθανεν·
αἱ δὲ τεαὶ ζώουσιν ἀηδόνες ἧσιν ὁ πάντων
ἁρπακτὴς ἀίδης οὐκ ἐπὶ χεῖρα βαλεῖ.

Bibliography

Abert, H., *Die Lehre vom Ethos in der griechischen Musik. Ein Beitrag zur Musikästhetik des Klassischen Altertums*. Leipzig: Breitkopf und Härtel 1899; repr. Wiesbaden: Hans Schneider Tutzing 1968.

Anderson, W., *Ethos and Eduction in Greek Music. The Evidence of Poetry and Philosophy*. Cambridge, MA: Harvard University Press 1966.

——— *Music and Musicians in Ancient Greece*. Ithaca, NY: Cornell University Press 1994.

Ansermet, E., *Die Grundlagen der Musik in menschlichen Bewusstsein*. Munich: Piper 1985.

Arnoux, G., *Musique Platonicienne: Ame du Monde*. Paris: Dervy-Livres 1960.

Balme, D.M., "*Genos and Eidos* in Aristotle's Biology," *CQ* n.s. xii (1962), 81–92.

Barbera, A., "Octave Species," *Journal of Musicology* 3 (1984), 229–41.

——— "Placing Sectio Canonis in Historical and Philosophical Contexts," *JHS* 104 (1984), 157–61.

——— "Greece," *New Harvard Dictionary of Music*, ed. D.M. Randel, 346–51. Cambridge, MA: Belknap Press 1986.

——— Ed., *Music Theory and its Sources: Antiquity and the Middle Ages*. Notre Dame, IN: University of Notre Dame Press 1990.

——— *The Euclidean Division of the Canon*. Lincoln: University of Nebraska Press 1991.

——— "Music and Perception: A Study in Aristoxenus," *JHS* xcviii (1978), 9–16.

——— "*`OI KALOYMENOI `ARMONIKOI*: The Predecessors of Aristoxenus," *PCPS* n.s. xxiv (1978), 1–21.

——— "*SUMFWNOI `APMONIKOI*: A Note on *Republic* 531C1–4," *CPh*. 73 (1978), 337–42.

——— "Methods and Aims in the Euclidean *Sectio Canonis*," *JHS* 101 (1981), 1–16.

——— "Aristides Quintilianus and Constructions in Early Music Theory," *C.Q.* n.s. 32 (1982), 184–97.

——— "Aristoxenus' Theorems and the Foundations of Harmonic Science," *Ancient Philosophy* 4 (1984), 23–64.

——— *Greek Musical Writings: I. The Musician and his Art*. Cambridge University Press 1984.

——— *Greek Musical Writings: II. Harmonic and Acoustic Theory*. Cambridge University Press 1989.

——— *Scientific Method in Ptolemy's "Harmonics."* Cambridge University Press 2000.

Barrow, J. D., *Pi in the Sky. Counting, Thinking, and Being*. Oxford: Clarendon 1992.

Beckmann, P., *A History of Π (PI)*. New York: The Galen Press 1971.

Bélis, A., *Aristoxène et Aristote: le Traité d'Harmonique*. Paris: Klincksieck, *Études et Commentaires* vol. 100. 1986.

Bevan, E. R., *The House of Ptolemy: A History of Egypt under the Ptolemaic Dynasty*. Chicago: Ares Publishers 1968.

Blum, D., *Casals and the Art of Interpretation*. London: Heinemann 1977.

Bowman, A. C., "The Foundations of Early Pythagorean Science; Archytas, Fragment I," *Ancient Philosophy* 2 (1982), 79–104.

Bowman, W. D., *Philosophical Perspectives on Music*. New York: Oxford University Press 1998.

Brumbaugh, R. S., *Plato's Mathematical Imagination. The Mathematical Passages in the Dialogues and their Interpretation.* Bloomington: The Indiana University Press 1954.

Burkert W., *Lore and Science in Ancient Pythagoreanism*, trans. E. L. Minar Jr. Cambridge, MA: Harvard 1972.

Caspar, M., *Ioannis Kepleri Harmonices Mundi Libri V, Vol. VI; Johannes Kepler, Gesammelte Werke*, ed. W. von Dyck and M. Caspar. Munich: Beck 1940.

Casson, L., *Libraries in the Ancient World*. New Haven: Yale University Press 2001.

Chaignet, E., *Pythagore et la philosophie pythagoricienne.* Paris: F. Vieweg 1873.

Chailley, J., *La musique grecque antique*. Paris: Les Belles Lettres 1979.

Cherniss, H., *Aristotle's Criticism of Presocratic Philosophy*. Baltimore: The Johns Hopkins Press 1935; repr. New York: Octagon Books 1964.

Clements, E., "The Interpretation of Greek Music," *JHS* 42 (1922), 133–66.

Coates, K., *Geometry, Proportion and the Art of Lutherie*. Oxford: Clarendon 1985.

Comotti, G., *Music in Greek and Roman Culture*. Trans. Rosaria V. Munson. Baltimore: The Johns Hopkins University Press 1989.

Cook, A., *Myth and Language*. Bloomington: Indiana University Press 1980.

——— *The Stance of Plato*. Lanham, MD: Bowman & Littlefield 1996.

Cook, N., *Music, Imagination, and Culture*. New York: Oxford University Press 1990.

Cooke, D., *The Language of Music*. New York: Oxford University Press 1959.

Cornford, F. M., *The Republic of Plato*. London: Oxford University Press 1941.

——— *Plato's Theory of Knowledge*. Indianapolis: Bobbs-Merrill Educational Publishing 1957.

——— *The Republic of Plato*. New York: Oxford University Press 1970.

Cousin, V., *Opera Patri Abaelardi*. 2 vols. Paris: Pottier de la Laine 1849–59.

Da Rios, R., *Aristoxeni Elementa Harmonica*. Romae: typis publicae officinae polygraphicae 1954.

Delatte, A., *Études sur la Littérature Pythagoricienne*. Paris: Edouard Champion 1915; rep. Genève: Slatkine 1974.

Derbyshire, J., *Prime Obsession: Bernhard Riemann and the Greatest Unsolved Problem in Mathematics*. Washington, DC: Joseph Henry Press 2003

Deubner, L., *Iamblichi De Vita Pythagorica Liber*. Leipzig: Teubner 1937.

Dicks, D. R., *Early Greek Astronomy to Aristotle*. Bristol: Thames and Hudson 1970.

Diels, H. and Kranz, W., *Die Fragmente der Vorsokratiker*. Dublin/Zürich: Weidmann 1966.

Dijksterhuis, E. J., *Archimedes*. Princeton: Princeton University Press 1987.

D'Ooge, M. L., *Nicomachus of Gerasa. Introduction to Arithmetic*. With Studies in Greek Arithmetic by F. E. Robbins and L. C. Karpinsky. New York and London: Macmillan 1926.

Duhem, P., *To Save the Phenomena. An Essay on the Idea of Physical Theory from Plato to Galileo*. Trans. E. Doland and Ch. Meschler. Chicago: University of Chicago Press 1969.

Düring, I., *Die Harmonielehre des Klaudios Ptolemaios*. Götesborgs Högskolas Ärsskrift, vol. 36, no. 1. Göteborg: Elanders 1930; repr. New York: Garland 1980.

——— *Porphyrios Kommentar zur Harmonielehre des Ptolemaios*. Götesborgs Högskolas Ärsskrift, vol. 38, no. 2. Göteborg: Elanders 1932; repr. New York: Garland 1980.

——— *Ptolemaios und Porphyrios über die Musik*. Götesborgs Högskolas Ärsskrift, vol. 40, no. 1. Göteborg: Elanders 1934; repr. New York: Garland 1980.

Dzielska, M., *Hypatia of Alexandria*. Trans. F. Lyra. Cambridge, MA: Harvard University Press 1995.

Fargue, Léon-Paul, *Maurice Ravel*. Paris: Domat 1949.

Field, J. V., *Kepler's Geometrical Cosmology*. Chicago: The University of Chicago Press 1988.

——— *The Invention of Infinity*. New York: Oxford University Press.

Fokker, A. D., "On the Expansion of the Musician's Realm of Harmony," *Acta Musicologica* 38 (1966), 197–202.

Fortlage, C., *Das Musikalische System Der Griecheschen In Seiner Urgestalt Aus Den Tonleitern Des Alypius*. Leipzig: Breitkopf und Härtel 1847; repr. Amsterdam, P. Schippers N. V. 1964

Frank, E., *Plato und die sogenannten Pythagoreer.* Halle: M. Niemeyer 1923.

Gaca, K. L., "The Reproductive Technology of the Pythagoreans," *Cl. Ph.* 95 (2000), 113–32.

Gevaert, Fr. Aug., *Histoire et Théorie de la Musique de L'Antiquité*, Vol. I, 1875; Vol. II, 1881 Gand: Annoot-Braeckman.

Gildersleeve, B. L., *Syntax of Classical Greek from Homer to Demosthenes*. 2 vols. New York: American Book Company 1900.

Godwin, J., ed., *The Harmony of the Spheres. A Sourcebook of the Pythagorean Tradition in Music*. Rochester, VT: Inner Traditions 1993.

——— *Cosmic Music. Musical Keys to the Interpretation of Reality*. Rochester, VT: Inner Traditions 1989.

Godwin, J., *Robert Fludd. Hermetic Philosopher and Surveyor of Two Worlds*. Grand Rapids, MI: Phanes Press.

Gombosi, O. J., *Tonarten und Stimmungen der Antiken Musik*. Kopenhagen: Ejnar Munksgaard 1939.

Grassi, T., "Musica, Mimica e Danza," *Studi Della Scuola Papirologica* 3 (1920), 117–35.

Greene, B., *The Elegant Universe. Superstrings, Hidden Dimensions, and the Quest for the Ultimate Theory*. New York: Vintage Books 2000.

Greene, M., *A Portrait of Aristotle*, Chicago: The University of Chicago Press 1963.

Guthrie, W. K. C., *A History of Greek Philosophy, I: The Earlier Presocratics and the Pythagoreans*. Cambridge University Press 1971.

Heath, Sir Th. L., *A History of Greek Mathematics, vol. I; From Thales to Euclid; vol. II: From Aristarchus to Diophantus*. Oxford: Clarendon 1921.

——— *The Thirteen Books of Euclid's Elements*, 3 vols. New York: Dover Publications 1956.

Heath, Sir Thomas, *Aristarchus of Samos. The Ancient Copernicus*. Oxford: Clarendon 1913.

Heath, Sir Th. L., *Greek Astronomy*. London: J. M. Dent & Sons, Ltd. 1932; repr. Dover 1991.

Heiberg, J. L., *Claudii Ptolemaii Syntaxis Mathematica*. 2 vols. Leipzig: Teubner 1898–1903.

Helmholtz, H. L. F., *On the Sensations of Tone as a Physiological Basis for the Theory of Music*. Trans. A. J. Ellis. New York: Dover Publications 1954.

Henderson, M. I., "The Growth of the Greek `ARMONIAI," *CQ* 36 (1942), 94–103.

——— "Ancient Greek Music," *New Oxford History of Music*, vol. 1, 336–403. London: Oxford University Press 1957.

Higgins, R. A. and Winnington-Ingram, R. P., "Lute Players in Greek Art," *JHS* 85 (1965), 62–71.

Hiller, E., *Theonis Smyrnae philosophi Platonici. Expositio rerum mathematicarum ad legendum Platonem utilium*. Leipzig: Teubner 1878; repr. New York: Garland 1987.

Hindemith, P., *A Composer's World*. Cambridge, MA: Harvard University Press 1952.

Hoche, R., *Nicomachi Geraseni Pythagorei Introductionis Arithmeticae Libri II*. Leipzig: Teubner 1966.

Hubbard, F., *Harpsichord Regulating and Repairing*. Boston: Tuner's Supply 1963.

Huntley, H. E., *The Divine Proportion. A Study in Mathematical Beauty*. New York: Dover Publications 1970.

Isacoff, S., *Temperament. The Idea that Solved Music's Greatest Riddle*. New York: Alfred A. Knopf 2001.

Jaeger, W., *Paideia: the Ideals of Greek Culture. Vol. I: Archaic Greece. The Mind of Athens*. Trans. Gilbert Highet. New York: Oxford University Press 1945.

James, Sir J., *Science and Music*. Cambridge University Press 1961.

James, J., *The Music of the Spheres. Music, Science and the Natural Order of the Universe*. New York: Grove Press 1993.

Jeppesen, K., *Counterpoint. The Polyphonic Vocal Style of the Sixteenth Century*. Trans. Glen Haydon. Englewood Cliffs: Prentice Hall 1939.

Johnson, C. W. L., "The Motion of the Voice, Å tÁj fwnÁj kínhsij, in the Theory of Ancient Music," *TAPA* 30 (1899), 42–55.

Jonker, G. H., *The Harmonics of Manuel Bryennius*. Groningen: Wolters-Noordhoff Publishing 1970.

Jourdain, R., *Music, the Brain, and Ecstasy*: *How Music Captures Our Imagination*. New York: William Morrow 1997.

Kemke, I., *Philodemi De Musica Librorum Quae Exstant*. Leipzig: Teubner 1884.

Kivy, Peter, *Music Alone. Philosophical Reflections on the Purely Musical Experience*. Cornell University Press 1990.

Klein, J. *Greek Mathematical Thought and the Origin of Algebra*. Trans. Eva Braun. Cambridge: M.I.T. Press 1968; repr. New York: Dover Publications 1992.

Kline, M., *Mathematics in Western Culture*. New York: Oxford University Press 1953.

Kucharski, P., *Étude sur la Doctrine Pythagoricienne de la Tétrade*. Paris: Les Belles Lettres 1952.

Kuhn, Th. S., *The Copernican Revolution. Planetary Astronomy in the Development of Western Thought*. Cambridge, MA: Harvard University Press 1957.

Laloy, L., *Aristoxène de Tarente: Disciple d'Aristote et la Musique de l'Antiquité Paris.* Société Francaise d'Imprimerie et de Librairie 1904; repr. Genève: Minkoff 1973.

Landels, J. G., *Music in Ancient Greece and Rome*. London: Routledge 1999.

Langer, S. K., *Philosophy in a New Key. A Study in the Symbolism of Reason, Rite, and Art*. Cambridge, MA: Harvard University Press 1957.

Lasserre, F., *Plutarque, de la musique: Texte, traduction, commentaire, précédés d'une étude sur l'éducation musicale dans la Grèce antique*. Olten and Lausanne: URS Graf-Verlag 1954.

Lavine, S., *Understanding the Infinite*. Cambridge, MA: Harvard University Press 1994.

Lawlor, R. and D. Qewnos Smurnaiou, *Mathematics Useful for Understanding Plato*. San Diego: Wizards Bookshelf 1979.

Levin, F. R., "The Hendecachord of Ion of Chios," *TAPA* 92 (1961), 295–307.

——— "Synesis in Aristoxenian Theory," *TAPA* 103 (1972), 211–34.

——— *The Harmonics of Nicomachus and the Pythagorean Tradition. American Classical Studies No. 1* University Park: The American Philological Association 1975.

——— "πληγη and τασιζ in the Harmonika of Klaudios Ptolemaios," *Hermes* 108 (1980), 205–29.

——— "Unity in Euclid's 'Sectio Canonis'," *Hermes* 118 (1990), 430–43.

——— *The Manual of Harmonics of Nicomachus the Pythagorean*. Grand Rapids, MI: Phanes Press 1994.

——— "Aspects of Ancient Greek Music," *Alexandria* 1 (1991), 251–92.

——— "Philolaus of Croton" and "Nicomachus of Gerasa" in *The New Grove Dictionary of Music II*. London: Methuen 1999.

——— "Ἄπειρία in Aristoxenian Theory," *Hermes* 135 (2007), 406–28.

Lippman, E. A., "Spatial Perception and Physical Location as Factors in Music," *Acta Musicologica* 35 (1963), 24–34.

——— *Musical Thought in Ancient Greece*. New York: Columbia University Press 1964.

Maas, M. and Snyder, J. M., *Stringed Instruments of Ancient Greece*. New Haven: Yale University Press 1989.

Maconie, R., *The Concept of Music*. Oxford: Clarendon 1990.

Macran, H. S., *The Harmonics of Aristoxenus*. Oxford: Clarendon 1902; repr. Hildesheim: G. Olms 1974.

Macurdy, G. H., *Hellenistic Queens. A Study of Woman-Power in Macedonia, Seleucid Syria, and Ptolemaic Egypt*. Chicago: Ares Publishers 1985.

Mathiesen, T. J., *Aristides Quintilianus: On Music in Three Books*. New Haven: Yale University Press.

Mathiesen, Thomas J., *Apollo's Lyre. Greek Music and Music Theory in Antiquity and the Middle Ages*. University of Nebraska Press 1999.

McClain, E. G., *The Pythagorean Plato. Prelude to the Song Itself*. York Beach: Nicolas-Hays 1984.

Meibom. M., *Antiquae Musicae Auctores Septem Graece et Latine*. 2 vols. Amsterdam: Elsevir 1652; rep. New York: Broude Brothers 1977.

Ménage, G., *Historia Mulierum Philosopharum*. Lyons: 1690.

Merleau-Ponty, J. and Morando, B., *The Rebirth of Cosmology*. trans. H. Weaver. Athens: Ohio University Press 1982.

Mersenne, M., *Harmonie Universelle, The Books on Instruments*, Paris, 1635, trans. R. E. Chapman, The Hague 1957.

Michaelides, S., *The Music of Ancient Greece: An Encyclopedia*. London: Faber and Faber 1978.

Michel, P.-H., *De Pythagore à Euclide. Contribution à l'Histoire des Mathematiques Préeuclidiénnes*. Paris: "Les Belles Lettres" 1950.

Monro, D. B., *The Modes of Ancient Greek Music*. Oxford: Clarendon 1894.

Morgenstern, S., ed., *Composers on Music. An Anthology of Composers' Writings from Palestrina to Copland*. New York: Pantheon Books 1956.

Morris, R., *Achilles in the Quantum Universe. The Definitive History of Infinity*. London: Souvenir Press 1998.

Mountford, J. F., "Greek Music and its Relation to Modern Times." *JHS* 40 (1920), 13–42.

——— in *The Greek Aulos*, by Kathleen Schlesinger. "Introduction," pp. xv–xxvii.

Moutsopoulos, E., *La Musique dans l'oeuvre de Platon*. Paris: Presses Universitaires de France 1959.

Najock, D., *Drei anonyme griechische Traktate über die Musik. Eine Kommentierte Neuausgabe des Bellermanschen Anonymus*. Göttinger musikwissenschaftliche Arbeiten, vol. 2. Kassel: Bärenreiter 1972.

Neubecker, A. J., *Die Bewertung der Musik bei Stoikern und Epicurean*. Berlin: Akademie-Verlag 1956.

——— *Über Die Musik IV. Buch. Text, Übersetzung und Kommentar*. Napoli: Istituto Italiano Per Gli Studi Filosofici 1986.

Neugebauer, O., *The Exact Sciences in Antiquity*. Providence, RI: Brown University Press 1957.

Newton, I., *Mathematical Principles of Natural Philosophia*. Trans. A. Motte; rev. F. Cajori. Berkeley and Los Angeles: 1974.

Nietsche, F. *The Birth of Tragedy in the Philosophy of Nietsche*. New York: The Modern Library n.d.

Norton, R. *Tonality in Western Culture. A Critical and Historical Perspective*. University Park: The Pennsylvania State University Press 1984.

O'Meara, D. J., *Pythagoras Revived. Mathematics and Philosophy in Late Antiquity*. Oxford: Clarendon 1989.

Owen, George E., *The Universe of the Mind*. Baltimore: The Johns Hopkins Press 1971.

Pearson, L., *Aristoxenus. Elementa Rhythmica. The Fragment of Book II and the Additional Evidence for Aristoxenian Rhythmic Theory*. Oxford: Clarendon 1990.

Philip, J. A., *Pythagoras and Early Pythagoreanism*. University of Toronto Press 1966.

Pine-Coffin, R. S., *Saint Augustine Confessions*. Baltimore: Penguin Books 1961.

Pistelli, H., *Iamblichus. In Nichomachi Arithmeticam Introductionem Liber*. Stuttgart: Teubner 1975.

Piston, W., *Harmony*. New York: W. W. Norton 1941.

Pöhlmann, Egert, *Denkmäler altgriechischer Musik*. Nürnberg: Hans Carl 1970.

Pöhlmann, E., and West, M. L., *Documents of Ancient Greek Music*. Oxford: Clarendon 2001.

Poincaré, H., *Science and Hypothesis*: New York: Dover Publications 1952.

Pomeroy, S. B., "Technikai kai Mousikai: The Eduction of Women in the Fourth Century and in the Hellenistic Period," *AJAH* 2 (1977), 51–68.

——— *Goddesses, Whores, Wives, and Slaves. Women in Classical Antiquity*. New York: Schocken Books 1975.

——— *Women in Hellenistic Egypt from Alexander to Cleopatra*. Detroit, MI: Wayne State University Press 1984.

Proust, M., *Remembrance of Things Past*, 2 vols., trans. C. K. Scott Moncrief. New York: Random House 1934.

Raffman, D., *Language, Music, and Mind*. Cambridge, MA: MIT Press 1993.

Redfield, John., *Music: A Science and an Art*. New York: Alfred A. Knopf 1930.

Reinach, Th., *La Musique Grecque*. Paris: Payot 1926.

Richter, Lukas., *Zur wissenschaftslehre von der Musik bei Platon und Aristoteles*. Berlin: Akademie-Verlag 1961.

Richter, L., *Momente der Musikgeschichte Antike und Byzanz.* Salzburg*:* Verlag Mueller-Speiser 2000.

Riley, M., "Theoretical and Practical Astrology: Ptolemy and His Colleagues," *TAPA* 117 (1987), 235–56.

Robbins, F. E., *Ptolemy. Tetrabiblos*. The Loeb Collection Library. Cambridge, MA: Harvard University Press 1940.

Ross, W. D., *ARISTOTELOUΣ FUSIKH AKPOASIS. Aristotle's Physics*. Oxford: Clarendon 1936; repr. 1960.

Ross, Sir D., *Aristotle*. London: Methuen 1923.

Rothstein, E., *Emblems of the Mind. The Inner Life of Music and Mathematics*. New York: Random House 1995.

Ruelle, Ch.-E., *Traductions de traités musicaux*, 6 vol. Paris: *Collection des auteurs grecs relatif à la musique* 1871–1898.

Ruelle, Ch.-E., "La Monocorde, Instrument de Musique," *REG* 10 (1897), 309–12.

Russell, B., *Introduction to Mathematical Philosophy*. New York: Macmillan 1919.

——— *Principles of Mathematics*. New York: W. W. Norton 1902.

——— *A History of Western Philosophy*. New York: Simon and Schuster 1945.

Sacks, Oliver, *Musicophilia, Tales of Music and the Brain*. New York: Alfred Knopf 2007.

Sautoy, M. du, *The Music of the Primes*. New York: HarperCollins 2003.

Schlesinger, K., *The Greek Aulos: A Study of Its Mechanism and of its Relation to the Modal System of Ancient Greek Music*. London: Methuen 1939; repr. Groningen: Bouma's Boekhuis 1970.

Schoenberg, A., *Theory of Harmony*. trans. Roy E. Carter. London: Faber and Faber 1978.

Schopenhauer, A., *The World as Will and Representation*. Trans. E. F. J. Payne. Vol. I. New York: Dover Publications 1959.

Scruton, R., *The Aesthetics of Music*. Oxford: Oxford University Press 1997.

Seashore, Carl E., *Psychology of Music*. New York: McGraw-Hill, 1938; repr. Dover 1967.

Sloboda, J. A., *The Musical Mind. The Cognitive Psychology of Music*. Oxford: Clarendon 1985.

Solomon, J., "The Diastaltic Ethos," *CPh* 76 (1981), 93–100.

——— "Cleonides: *EISAGOGE HARMONIKE*: Critical Edition, Translation, and Commentary." Ph.D. dis., University of North Carolina: Chapel Hill, 1980.

——— "The Seikilos Inscription: A Theoretical Analysis," *AJPh* 107 (1986), 455–79.

Solomon, Jon, "*EKBOLE* and *EKLUSIS* in the Musical Treatise of Bacchius," *Symbolae Osloensis* 55 (1980), 111–26.

Solomon, J., *Ptolemy "Harmonics": Translation and Commentary*. Leiden: Brill 2000.

Sorabji, R., *Time, Creation and the Continuum. Theories in Antiquity and the Early Middle Ages*. Ithaca, NY: Cornell University Press 1983.

Spitzer, L., *Classical and Christian Ideas of World Harmony. Prolegomena to an Interpretation of the Word "Stimmung."* Ed. Anna Granville Hatcher. Baltimore: The Johns Hopkins Press 1963.

Stahl, W. H., with Johnson, R. and Burge, E. L., *Martianus Capella and the Seven Liberal Arts. Records of Civilization: Sources and Studies, no. 84.* New York: Columbia University Press 1971–77.

Stanford, W. B., *The Sound of Greek: Studies in the Greek Theory and Practice of Euphony*, Sather Classical Lectures, vol. 38. University of California Press 1966.

Steinmayer, O., "Bacchius Geron's *Introduction to the Art of Music*," *Journal of Music Theory* 29 (1985), 271–98.

Stephenson, B., *The Music of the Heavens. Kepler's Harmonic Astronomy*. Princeton University Press 1994.

Stravinsky, I., *An Autobiography*. New York: Simon and Schuster 1936.

Strunk, O., *Source Readings in Music History: Antiquity and the Middle Ages*. New York: W. W. Norton 1965.

Tannery, P., "Inauthenticité de la 'Division du Canon' attribuée à Euclide," *Memoires scientifiques* III (1915), 213–19.

——— *Quadrivium de Georges Pachymere ou = Syntagma ton tessaron mathmaton arithmetikes, mousikes, yeometrias kai astronomias*, rev. R. P. E. Stephanou. A. A. Studi e Testi, vol. 94. Vatican City: Biblioteca Apostolica Vaticana 1940.

Tarn, W. W., *Hellenistic Civilization*. Cleveland and New York: The World Publishing Company 1952.

——— *Antigonas Gonatas*. Oxford: Clarendon 1913.

——— "Queen Ptolemais and Apama," *CQ* 23 (1929), 138–41.

Taub, L. C., *Ptolemy's Universe. The Natural Philosophical and Ethical Foundations of Ptolemy's Astronomy*. Chicago: Open Court 1993.

Taylor, A. E., *A Commentary on Plato's Timaeus*. Oxford: Clarendon 1928.

Thesleff, H., *An Introduction to the Pythagorean Writings of the Hellensitic Period*. Äbo: Äbo Akademi 1961.

——— *The Pythagorean Texts of the Hellenistic Period*. Äbo: Äbo Akademi 1965.

Tillyard, H. J. W., "Inscriptions: Laconia, Geraki," *BSA* 11 (1904–5), 111–12.

Tillyard, H. J. W. and Wace, A. J. B., "The History of Demetrius the Fair," *BSA* 11 (1904–05), 113–19.

Van der Waerden, B. L., *Science Awakening I. Egyptian, Babylonian, and Greek Mathematics*. Trans. A. Dresden. Dordrecht: Noordhoff International Publishing 1975.

——— "Die Harmonielehre der Pythagoreer," *Hermes* 78 (1943), 163–99.

Vincent, A. J. H., "Notice sur divers manuscritsgrecs relatifs à la musique, comprenant une traduction française et des commentaires," *Notice et extraits des manuscrits de la bibliotèque du Roi et autres biblioteches*, vol. XVI, 2. Paris: Imprimerie royale 1847.

Wachsmuth, T.C. and Hense, O., *Stobaei Eclogae* vols. I–III. Leipzig: Teubner 1884.

Wallis, Ch.G., *The Harmonies of the World by Johannes Kepler*. Chicago: University of Chicago Press 1952.

Wasserstein, A., "Theaetetus and the History of the Theory of Numbers," *CQ* n.s. 8 (1958), 165–77.

Watts, W., *Augustine. Confessions*. 2 vols. The Loeb Classical Library. Cambridge, MA: Harvard University Press 1912; repr. 1997.

Wehrli, F., "Aristoxenos," *Die Schule des Aristoteles*. Basle: Benno Schwabe 1945.

Werner, E., *The Sacred Bridge. The Interdependence of Liturgy and Music in Synagogue and Church during the First Millenium*. New York: Columbia University Press 1959.

West, M.L., *Ancient Greek Music*. Oxford: Clarendon 1992.

White, M.J., *The Continuous and the Discrete. Ancient Physical Theories from a Contemporary Perspective*. Oxford: Clarendon 1992.

Wilkinson, L. P., "Philodemus on Ethos in Music," *CQ* 32 (1938), 174–81.

Williams, C.F. Abdy, *The Story of Notation*. New York: Haskell House Publishers 1969 (repr. of 1903 ed.).

Willis, J. *Martianus Capella*. Leipzig: Teubner 1983.

Wilson, A.M., *The Infinite in the Finite*. New York: Oxford University Press 1995.

Winnington-Ingram, R.P., "Aristoxenus and the Intervals of Greek Music," *CQ* 26 (1932), 195–208.

——— *Mode in Ancient Greek Music*. Cambridge: Cambridge University Press 1936.

Wittgenstein, L., *Notebooks, 1914–1916*, ed. G.H. von Wright and G.E.M. Anscombe. Oxford: Blackwell 1961.

Wood, A., *The Physics of Music*, rev. J. M. Bowsher. London: Chapman and Hall 1975.

Zeidler, B.H., *The History of Woman Philosophers*. Lanham, MD: University Press of America 1984.

Ziegler, K. and Pohlenz, M., *Plutarchi Moralia*, vol. vi. Fasc. 3. Leipzig: Teubner 1953.

Zuckerkandl, V., *Sound and Symbol. Music and the External World*; trans. W. R. Trask. Bollingen Series xliv. Princeton University Press 1956.

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